Instructor's Manual

For *Sustainable Transportation Systems Engineering*

Sample Syllabus, Homework Solutions, and Exam Problems with Solutions

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Francis Vanek, Largus Angenent, James Banks, Ricardo Daziano, Mark Turnquist

Assisted by:

Alexandra Cheng, Scott Cloutier, Ryan Cummiskey, Zhijie Dong, Mike Hyland, Bingyan Huang, Ben Kemper, Hyun Park, Julie Schwartz, Kunrawee Tangmitpracha, Molly Tierney, Kecheng Xu, Ema Yamamoto

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Introduction

This instructor's manual includes a sample course syllabus, answers to all end-of-chapter exercises, and sample exam problems with solutions. It is our goal to periodically update the manual with new problems. Please make sure you are working from the most recent version of the manual by checking for a new version at:

www.lightlink.com/francis/STSEInstructorManual.pdf or

www.lightlink.com/francis/STSEInstructorManual.docx

Suite of available materials

There are additional supporting materials for teaching with the book beyond this document. These include:

- "Problems-only" version of the manual: since this manual is quite lengthy, there is a shorter version that contains only the problems without the solutions, to allow users navigate between problems more easily.
- Book PowerPoint show: a PowerPoint show with all figures, tables, and photographs from the book. In general, the tables are in text form so that the numbers can be copied and pasted into a spreadsheet for manipulation.
- Teaching PowerPoint show: a PowerPoint show that provides a basic framework for teaching a course based on the book. It is deliberately not as long as a standard-length college course to allow instructors room to add their own content.
- Spreadsheet workbook with solver and optimization models. For certain homework problems, a workbook is provided with a tab for each problem and then a solver or optimization model contained within the tab.

Sample syllabus

The sample syllabus is written for a 14-week semester with two 75-minute lectures each week. It can be modified to fit other schedule formats. It is assumed that the instructor might invite a guest speaker or screen a documentary film for one or two meetings, and that the material in the syllabus could still be covered. A lecture meeting or two might be devoted to in-class examinations, although that was not done in the Cornell class. Also, Chap.4 on transportation demand and Chap.6/Appendix D on transportation infrastructure were not covered in the course because these topics are covered in other classes at Cornell, although an instructor could swap these materials in by taking out other materials. In any case, the instructor is encouraged to use the sample syllabus as a template and adapt it to his or her own course.

Comments about homework exercises and exam problems

The exam problems shown were given in class in an open-book test, but the exam could be closed-book with some equations or parameters provided. Exam problems could also be used as additional homework problems.

For the exercises and exam problems, instructors are encouraged to modify the exercises as they are posed to the students in problem sets, both to meet individual goals for the course, and also to prevent posing exactly the same problem from one year to the next. In some cases, comments have been added in a section at the bottom of some exercise to help the instructor add depth to the problem that is being posed.

Many worksheets use the Excel Solver function to find the solution to a problem, so the security settings in Excel should be set to allow macros to function. Matlab or WolframAlpha is an alternative to using the Solver or What-If functions in Excel.

In some cases, answers as written in the Instructor's Manual have been shortened to three significant digits, for brevity. Instructor and/or student answers may not agree exactly with the numbers shown due to rounding. Each instructor can of course decide their own standard for minimum number of significant digits.

The instructor's manual is provided in both Word and PDFformat. Instructors are welcome to copy and paste text from the manual in making up homework problems, to facilitate adapting the problems to the specific purposes of a given course, and to introduce variability into homework assignments from year to year so that students cannot simply copy homework from the year before.

Open-ended class research problems

Along with quantitative problems, a sample of "class research problems" is provided after the homework problem section. The intention of these problems is for each member of the class to find their own individual answer, from which a class compilation is formed to create a more general answer to a question. For example, by asking a class of 20 to 40 students to each compile a list of light-duty vehicles and their overall length in feet, it is possible to arrive at a typical average length of 15.5 to 16.5 ft, which supports the standard figure used in the homework problems. Instructors are encouraged to use the discussion that happens in class as a guide to developing new questions that can be posed as the semester progresses.

Related homework and exam problems from the broader "library" of problems from Cornell instructors

The Cornell authors of this book and their colleagues are aware that the contents of the book, while broad, do not cover every possible topic. We ourselves sometimes bring in other problems not covered in the book and support them with separate material, so a selection of these problems is provided as a service to interested instructors. The following topics are covered:

- Gravity modeling
- Congestion pricing
- Elasticity of transportation demand
- Network equilibrium models

- Stochastic assignment model

Wherever these problems appear, they are clearly marked as such. Note that the problems are provided on an "as-is" basis and instructors should already be familiar enough with their content to independently provide the necessary supporting material to students.

Upholding academic integrity and soliciting feedback

A note about unauthorized duplication of the manual: in recent years, the use of the internet for students to commit academic fraud by sharing unauthorized solutions has become a serious problem. Therefore, distribution of this electronic manual to students is prohibited. We leave it up to the discretion and judgment of instructors as to how they wish to use the information contained in the manual to explain the solution of homework and exam problems to students, without simply providing the entire manual for them to view.

Lastly, we are very interested in developing the best instructor's manual possible for supporting your teaching needs. If you have corrections or other feedback, please email Francis Vanek at fmv3@cornell.edu.

Note: Thank you in advance for upholding academic integrity by not sharing this manual with students.

Sample Syllabus

Course number:

Course Title: “Sustainable Transportation Systems Engineering”

Syllabus

Time:

Room:

Instructor:

Instructor email:

Overview

The goal of this course is to explain to students the ways in which transportation technologies and systems can be used to respond to challenges currently facing society, the economy, and the natural environment. The recommended prerequisite course is an introductory transportation engineering course, although students can take this course without the recommended prerequisite if they are willing to do some outside review of the main material covered in the introductory course. The course will make use of engineering economics, statistics, and transportation network modeling skills. The majority of the lecture content will be technical and quantitative, although there will also be some amount of lecture time dedicated to big-picture policy issues.

As one of the courses in the CEE catalog that fulfills the design requirement for the major, part of the focus of CEE 4630 will be on design aspects of transportation technologies and systems. The course curriculum will consider the rudimentary design of transportation systems that address the needs of livable cities, green freight, and incorporating alternative energy sources. These goals will be the central focus of both the mid-semester individual technical report and final team project. With a focus on solving transportation problems that exist today with a view toward creating a better system for the future, it is natural that the course combines elements of technology, design, and good citizenship.

Elements of the course

As the course title suggests, the *methodology* for the course is a mixture of engineering techniques applied to individual technologies, and techniques that bring together technologies as components into a cohesive system that delivers the outcomes that society desires. Transportation systems are in turn situated in a *context* for which we will make use in the course of the three-dimensional sustainable development model. The three dimensions of ecological, economic, and social sustainability provide the goals that both the future technologies and future systems are aiming to achieve. The first week of the course discusses this context and also provides some fundamental building blocks for transportation technology that will be useful in later parts of the semester.

The course is divided into three *topic areas* that will be discussed through the semester. The first area is *urban passenger transportation*. With the majority of the human population in both industrial and industrializing countries living in or near urban centers (both small and large), it is clear that transportation practices in urban centers have a key role to play in any solution to sustainability. We will discuss how private (individually owned cars and light trucks), public (bus, urban rail, taxi and paratransit), and non-motorized modes can contribute to this solution. We will also briefly discuss intercity passenger transportation at the end of the unit with the remaining time.

The second topic area is *freight transportation*. Unlike passenger transportation, the majority of freight transportation volume is incurred over long intercity distances, so that the focus is on large regional or national networks, rather than individual urban areas. Both future freight technologies and systems-level solutions are considered. Urban freight issues will be considered as well, although to a lesser degree.

The third topic is *alternatives for transportation energy*. For both passenger and freight applications, having resources that are both available for the long-term and sustainable in terms of their impacts is essential for future viability. Thus transportation energy options are discussed as a single unit for all types of transportation, although the application will affect the nature of the solution: urban versus intercity, surface versus air or water, as well as passenger versus freight. Unlike topics one and two, which included efforts to use conventional petroleum derived energy sources more efficiently, this section focuses on developing new sources.

Assessment

The breakdown of the grade is as follows:

Mid-semester technical report on urban passenger transport systems—20%

Final team report—30 % Final cumulative exam—30%

Homeworks—18%

Online end-of-semester course evaluation—2%

Key components of the overall assessment are described as follows:

Homework

There will be a total of 7 homework assignments due throughout the semester. One of these assignments (homework #7) will be a short team term project presentation to the class on an emerging future transportation topic of your group's choosing (see below). The other 6 are conventional written homework assignments. The lowest homework score will be dropped. Otherwise, late homework is not accepted without a medical or family emergency. Most homework problems are either numerical or open-ended short questions; occasionally I may ask the class to do research on some topic and bring in a "data point" for an "open-ended research question."

End-of-semester evaluation

Two percent of the grade is given for filling out the online evaluation at the end of the semester, based on records obtained from the registrar after the evaluation period closes. In other words, your answers are not seen by me until after grades are submitted; just the fact that you completed the evaluation automatically gets you 2 out of 100 points toward your final grade. I appreciate your feedback in helping me improve the course and I look forward to your comments.

Mid-semester technical report: Sustainable design for urban passenger system

Students will work individually to write a report based on a design solution for urban passenger transportation, based in a real-world city of their choosing (U.S. or non-U.S. are both OK). The report will be due after we have completed the urban passenger unit in lecture, so that students can benefit from all lecture materials as they complete the report. I will ask you to submit your title and two or three sentences about it before you start, to make sure your topic is suitable. The title is due **Thursday, Sept. 12.** If you do not have a topic by then, I will work with you to help you decide one as soon as possible after that.

Final-term project: Sustainable design and alternative energy for passenger or freight system

Students will form **groups of two or three** to conduct a term research project on a sustainable transportation topic of your choice. The teams will then submit a short project proposal with the working title of the project, list of team members, and a one-paragraph description of the project to initiate work. I will accept descriptions by **Tuesday, Oct. 31.** If you do not have a topic by Oct. 31, I will work with you to help you decide one as soon as possible after that. The final paper will be due at the end of the semester on **Friday, Nov. 30.**

Although the topic is up to the team, each report should present a rudimentary design solution for bringing alternative energy to a transportation system (e.g., implementing alternative transportation energy in Ithaca) and cover some information about the underlying technology, its economics, and its energy and environmental benefits. The main difference, therefore, between the scope of the midterm and final projects is that the latter included energy and environment, which we will be covering in the last stage of the course. I am available to help students find partners and think about subjects.

Note that the final term project topic must be different than the midterm for each team member. If I have concerns, I will discuss when I see your topic description.

Homework #7 Assignment

Each team will fulfill the homework #7 requirement by presenting a short persuasive PowerPoint show about a "future transportation technology or system," lasting no more than 5 minutes, to be given at the beginning of a Tuesday class of your choice. The following is a list of suggested topics, or you can also come up with your own. Along with the talk please provide a web-link to an article or website to which students can refer for more information.

Dynamic ride-sharing	Twitter-based carpooling	Convertible flying cars
Self-driving cars	Maglev trains	LA to SF hyperloop concept
Urban freight tunnels	Airship freight delivery	Personal rapid transit
Fast-ship freight concept	Electric-assist bicycles	Etc. etc.

I will give a sample presentation early in the semester for the Segway personal transporter, as an example. The presentation will then be scored by all of those students present. After the talk on Tuesday, Nov. 26 I will announce to all the three teams with the highest scores. These teams will then be invited to give a short talk about their project topic on the last day of class 12/5. Teams that present will have their overall grade raised by ½ letter grade, in appreciation of the extra effort required to make a second presentation. If any of the top three teams declines, I will keep going down the list until I fill my roster of three teams.

When your team is ready to schedule a presentation slot, you may request any Tuesday that is available (except the last one, Tuesday, Dec. 3, 2011). If there are more teams than slots, or if your team opts not to give a talk, those teams that do not have a chance to present can meet the requirement by submitting a short PowerPoint show and article link at the end of the semester, although they will not be in the competition for the highest score award. Academic integrity must be observed: vote rigging in the scoring, etc., is not allowed.

Final exam

The final exam for the course is open book and open note. Content will be cumulative for the entire course. Calculators required. You may not use laptops, cell phones, etc.

Guest speakers

I am planning on two guest speaker slots for this semester, exact dates to be confirmed. I do not take attendance at lectures in general and expect you to make arrangements if you miss a class. However, since guest speakers are making an effort to come and see us, I do require attendance at these meetings. I will give plenty of advance notice about their dates.

Required textbook for course

The required reading for the course is *Sustainable Transportation Systems Engineering: Evaluation and Implementation*, written by myself along with several co-authors, and published by McGraw-Hill. It is available from the Cornell Campus Store. There is no additional course pack, but occasional supplemental readings that I have authored will be posted to the blackboard site.

For those of you especially interested in the policy side of transportation systems, optional readings in *An Introduction to Sustainable Transportation* by Schiller, Bruun, and Kenworthy (hereafter Schiller et al, 2010) are included in the reading list below as well. These readings were required in previous years in the course, and although I am not making the book available through the campus store, you are of course welcome to purchase your own copy on-line. Note that there is a paperback version available, which is quite a bit cheaper than the hard cover version. Schiller et al will be available on reserve in Uris Library.

Also, for students who are especially interested in gaining more technical background on the topics in the course, the background papers and chapters that were used in previous years for the units on passenger, freight, and energy are listed along with the corresponding units.

Course blackboard site

A Blackboard website is available to support the course (http://blackboard.cornell.edu). All PowerPoint slides screened in lecture will be available on the site. Answers to questions that students ask about the homework, project assignment, and final exam will be made available in the Piazza website (www.piazza.com) to which there should be a link from Blackboard. For everyone's convenience, please post your questions about assignments there. General questions about the course can be emailed to me.

Course Outline and readings

Note: all required readings are in the textbook. The Schiller et al readings are optional. Also, run-time of units may change slightly from what is shown as the semester progresses; depending on the interests of the class, we can slow down or speed up the pace for specific topics.

Unit 1: Introduction—contemporary issues facing the transportation system (2 weeks)

Assignment: HW1, Exercises 1-1, 2-1, 2-2, 2-3, 5-3.

- Review of society/economy/environment requirements
- Primer on transportation technologies and systems, using international development of high-speed rail as an example

Required readings:

    - Chapter 1: Introduction
    - Chapter 2: Background on Energy Security and Climate Change
    - Chapter 5: Background on Transportation Systems and Vehicle Design

Optional readings: (*Note: you can skim these as time allows during the course of the semester. At times I will point to specific examples that support lectures or homeworks.*)

    - Chapter 3: Systems Tools
    - Appendix C (electronic chapter, available on Blackboard site or at www.mhprofessional.com)

- Schiller et al Chapter 1: Introduction: A highly mobile planet and its challenges

Unit 2: Urban passenger transportation (4.5 weeks)

Assignments: HW2, Exercises 7-1, 8-1, 8-2, 8-4, 9-1; HW3, Exercises 9-2, 9-4, 10-1, 10-2. Midterm design report, Friday, Oct. 11 (before Fall Break).

- Goals for sustainable transportation and mix of tools available
- Role of information technology in improving system function
- Potential and limitations of urban public transit as a solution
- Automobile based solutions: car sharing and telecommuting

Required readings:

- Chapter 7: Overview of Passenger Transportation Systems (OK to skim Sec.7-4)
- Chapter 8: Urban Public Transportation and Multimodal Solutions
- Chapter 9: Personal Mobility and Personal Accessibility

Optional readings:

- Chapter 10: Intercity Passenger Transportation (I will point to specific passages as time allows)
- Optional readings from Schiller et al:
  - Chapter 3: History of Sustainable and Unsustainable Transportation
  - Chapter 4: Modes, Roads, and Routes
  - Chapter 9: Exemplars of Sustainable Transportation

Unit 3: Enhanced freight transportation and green logistics (3 weeks)

Assignment: HW4, Exercises 11-1, 11-2, 12-1, 12-2, 12-4.

- Sustainability issues facing freight transportation
- Advantages and disadvantages of freight transportation modes
- Using information technology to maximize efficiency of freight
- Network-level analysis of opportunities to reduce environmental burdens and improve sustainability

Required readings:

- Chapter 12: Modal and Supply ChainManagement Approaches
- Chapter 13: Spatial and Geographic Aspects of Freight

Optional readings:

- Schiller et al Chapter 11: Introduction to Freight Transportation Systems (review for background)
- Schiller et al Chapter 5: Moving Freight, Logistics, and Supply Chains in a More Sustainable Direction

Unit 4: Alternative energy sources for transportation and course conclusion (4.5 weeks)

Assignments: HW5, Exercises 12-6, 13-3, 13-4, 14-1, 14-4, 14-6; HW6, Exercises 15-1, 15-2, 15-6, 16-1, 16-3, 16-5; Final team paper, last day of class.

- Review of opportunities and limitations for solving the transportation energy problem through more efficient use of fossil fuels
- Introduction to emerging energy alternatives to replace petroleum
- Opportunities for use of biofuels in transportation
- Electric vehicles, plug-in hybrids, matching renewable energy sources to transportation energy demand, and implications for the electric grid
- Summary of course, term paper presentations, and final exam review

Required readings:

- Chapter 14: Overview of Alternative Fuels and Platforms
- Chapter 15: Electricity and Hydrogen as Alternative Fuels
- Chapter 16: Bioenergy Resources and Systems
- Chapter 17: Conclusion: Toward Sustainable Transportation Systems

Optional reading:

- Schiller et al Chapter 10: Conclusion: Growing More Exemplars

References: Additional resources for the course

Full-length books on reserve for course:

- Bruun, E. (2013) *Better Public Transit System: Analyzing Investments and Performance*, Routledge, London.
- Chowdhury, Mashrur A. and A. W. Sadek. (2003) *Intelligent Transportation Systems: Planning Requirements for ITS*. Artech House Publishers, Norwood, MA.
- Mackay, David J. C. (2009) *Sustainable Energy—Without the Hot Air.* UIT Cambridge Limited, Cambridge, UK.
- Papacostas, C. and P. Prevedouros. (2001) *Transportation Engineering & Planning*. Prentice-Hall, Englewood Cliffs, NJ.
- Schiller, P., E. Bruun, and J. Kenworthy. (2010) *An Introduction to Sustainable Transportation: Policy, Planning, and Implementation*. Earthscan, London.
- Sperling, D. and D. Gordon. (2009) *Two Billion Cars: Driving toward Sustainability*. Oxford University Press.
- Vanek, F., L. Albright, and L. Angenent. (2012) *Energy Systems Engineering: Evaluation and Implementation*, 2nd ed, McGraw-Hill, New York.
- Vuchic, V. (2005). *Urban Transit: Operations, Planning and Economics*. Wiley, New York.
- Vuchic, V. (2007). *Urban Transit: Systems and Technology*. Wiley, New York.

Additional journal papers of interest:

- Ang-Olson, Jeffrey, Schroeer, Will (2002). “Energy efficiency strategies for freight trucking potential impact on fuel use and greenhouse gas emissions”. T*ransportation Research Record*. n. 1815, pp. 11-18.
- Hill, J., E. Nelson, D. Tilman, S. Polasky, D. Tiffany. (2006). Environmental, economic, and energetic costs and benefits of biodiesel and ethanol fuels. Proceedings of the National Academy of Sciences of the USA. v. 103, n. 30, pp. 11206-11210.
- Kempton, W. and J. Tomic. (2005). “Vehicle-to-grid power fundamentals: Calculating capacity and net revenue.” *Journal of Power Sources.* v.144, pp. 268-279.
- Kempton, W. and J. Tomic. (2005) “Vehicle-to-grid power implementation: From stabilizing the grid to supporting large-scale renewable energy.” *Journal of Power Sources*. v.144, pp. 280-294.
- Korpela, J., K. Kylaheiko, and A. Lehmusvaara. (2001) “The effect of ecological factors on distribution network evaluation.” *International Journal of Logistics*, 4:2.
- McKinnon, A. and Y. Ge (2004). “Use of a synchronised vehicle audit to determine opportunities for improving transport efficiency in a supply chain. *International Journal of Logistics: Research and Applications*, 7:3, pp. 219-238.
- Peng, D. and D. Zhong. (2008) Optimization Model for Integrated Logistics Network Design in Green Manufacturing System. 2008 International Conference on Information Management, Innovation Management and Industrial Engineering.
- Saricks, C. et al. (2003) “Fuel consumption of heavy-duty trucks potential effect of future technologies for improving energy efficiency and emissions”. *Transportation Research Record 1842.* pp. 9-19.
- Shaheen, A. Susan, Adam P. Cohen, J. Darius Roberts. (2006). Carsharing in North America: Market growth, current developments, and future potential. *Transportation Research Record*. n. 1986, pp 116-124.
- Shafizadeh, Kevan R; Debbie A. Niemeier, Patricia L. Mokhtarian, and Ilan Salomon. (2007). “Costs and benefits of home-based telecommuting: a Monte Carlo simulation model incorporating telecommuter, employer, and public sector perspectives”. *Journal of Infrastructure Systems*. 13:1, pp. 12-26.

Solutions to Exercises

Chapter 1 Introduction

Problem 1-1

Write one paragraph describing a transportation technology or system with which you have personal experience in a specific metropolitan area or region, in either the U.S. or a foreign country. Was it successful? Why or why not? To answer this question, write a second paragraph evaluating its success (or lack thereof). Your answer may include qualitative or anecdotal information, quantitative data (including numbers such as cost, capacity, and so on), or a mixture of the two.

<u>Open-ended solution.</u>

Sample answer from an anonymous Cornell University student:

During the fall of 2011, my family and friends were not able to pick me up from Cornell to bring me home for Thanksgiving. Instead of staying in my apartment, I decided to look for a carshare opportunity through Zimride. I found an available seat in a Cornell student's car from Ithaca to Philadelphia for $30 round trip, which equates to about 13 cents per mile. When I arrived in Philadelphia, I was dropped off at the 30th street station where I took a train into Trenton, then the Northeast Corridor line into New Brunswick. The fare was about $10. From the New Brunswick station, I was picked up by my best friend's older sister who drove me home free of charge.

I think my travels that day were very cost efficient, but not time efficient. Overall I spent about 10 hours in transit, most of which were spent in train stations or on the trains themselves. I could have arrived in my hometown in a third of the time if I drove straight there from Cornell, assuming I didn't hit traffic. I would call the transportation system I used that day a great success and would recommend it to others as long as time is not a constraint.

Problem 1-2

Note to instructor on the limited accuracy of this problem: Because there is no comprehensive database of passenger-km (pkm) and tonne-km (tkm) for all countries of the world that could be used for this problem, to the best of our knowledge, the values used are representative, estimated values. The goal of the problem is to give the student an "order-of-magnitude" idea of the relative role of different country groups, rather than predict future growth pathways exactly.

The underlying population trajectories for the four country groups ("low income," etc.) discussed in the future scenarios presented in this chapter are given below for the period 2000 to 2100, with population given in millions of people throughout the table. Create your own trajectories for world passenger and/or freight volumes by changing any or all of the following values: (a) assignment of population to each of the four country groups, (b) size of total world population, (c) per capita transportation volumes in different years. Explain why you made the changes you did, and discuss how total world transportation and its division between country groups would be different from the values presented in the body of the chapter.

Year	High Income High Volume	High Income Low Volume	Middle Income	Low Income	Total
2000	500	800	2500	2200	6000
2010	500	800	3032	2668	7000
2020	500	800	3457	3043	7800
2030	500	800	3777	3323	8400
2040	500	800	3989	3511	8800
2050	500	800	4069	3581	8950
2060	500	800	4096	3604	9000
2070	500	800	4096	3604	9000
2080	500	800	4096	3604	9000
2090	500	800	4096	3604	9000
2100	500	800	4096	3604	9000

<u>Solution.</u>

This problem is open-ended and could be answered with various assumptions. As an example, for both passenger and freight, the following assumptions are used to test their effect on total transportation demand in the year 2100:

1. Population figures for four world regions for years through to 2100 are unchanged.
2. Low-income countries continue to languish economically during the 21st century, as the gap between them and the rest of the countries widens. Therefore, population grows to 3.604 billion in the year 2100 as shown, but per capita passenger and freight remain constant.
3. Mid-income countries are unchanged.
4. Both types of high-income countries see increasing levels of per capita passenger and freight demand in response to increasing wealth. Per capita values grow linearly by 30% between years 2000 and 2100.

Passenger side result.

First, to reconstruct the result in the chapter for mid-income countries using a starting value of 5000 pkm/capita, start year of 2000, end year of 2050, and growth of 10,000 pkm, plugging in to the formula for the triangle function and calculating for the year 2010:

$$f(2010) = 5K + 10K\left[2\left(\frac{2010-2000}{2050-2000}\right)^2\right] = 5.8K = 5{,}800\, pkm/cap/yr$$

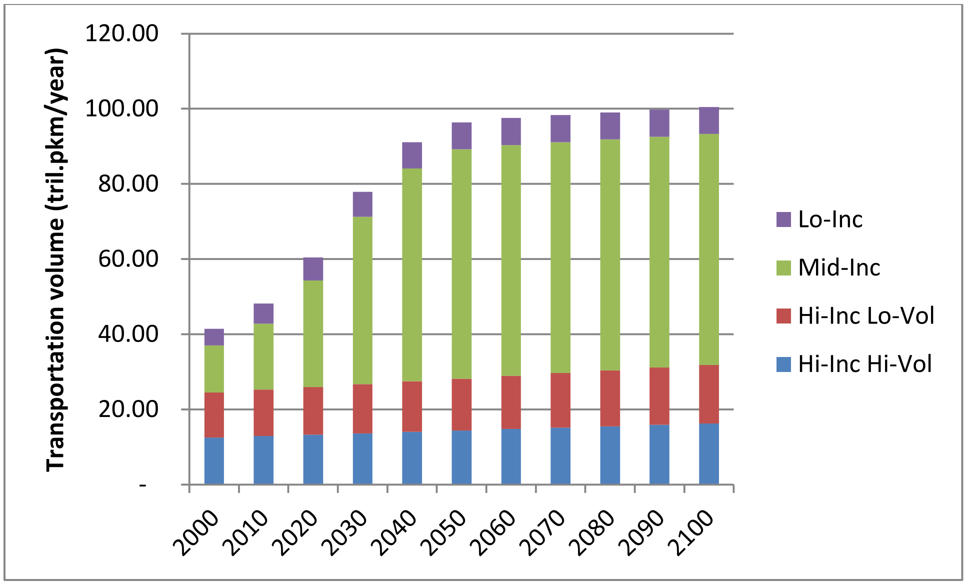
The process is repeated for other years. Next, the high-income countries grow by 30% total to the year 2100, or by 4.5K and 7.5K for the low-volume and high-volume countries, respectively. Thus the amount per decade is 0.45K and 0.75K, respectively. The low-income countries remain at 2000 pkm/capita/year for the duration. Decade by decade per capita values are the following:

Year	Hi Income Hi Volume	Hi Income Low Volume	Middle Income	Low Income
2000	25	15	5	2
2010	25.75	15.45	5.8	2
2020	26.5	15.9	8.2	2
2030	27.25	16.35	11.8	2
2040	28	16.8	14.2	2
2050	28.75	17.25	15	2
2060	29.5	17.7	15	2
2070	30.25	18.15	15	2
2080	31	18.6	15	2
2090	31.75	19.05	15	2
2100	32.5	19.5	15	2

Multiplying per capita activity by population gives the following results in table form:

Year	Hi Income Hi Volume	Hi Income Low Volume	Middle Income	Low Income	Total
2000	12.50	12	12.50	4.40	41.40
2010	12.88	12.36	17.59	5.34	48.16
2020	13.25	12.72	28.35	6.09	60.41
2030	13.63	13.08	44.56	6.65	77.92
2040	14.00	13.44	56.65	7.02	91.11
2050	14.38	13.80	61.04	7.16	96.37
2060	14.75	14.16	61.44	7.21	97.55
2070	15.13	14.52	61.44	7.21	98.29
2080	15.50	14.88	61.44	7.21	99.02
2090	15.88	15.24	61.44	7.21	99.76
2100	16.25	15.60	61.44	7.21	100.49

Alternatively, in graphic form:



Freight side results.

First, to reconstruct the result in the chapter for mid-income countries using a starting value of 4000 tkm/capita, start year of 2000, end year of 2050, and growth of 4000 pkm, plugging in to the formula for the triangle function and calculating for the year 2010:

$$f(2010) = 4K + 4K\left[2\left(\frac{2010-2000}{2050-2000}\right)^2\right] = 4.32K = 4{,}320 tkm/cap/yr$$

The process is repeated for other years. Next, the high-income countries grow by 30% total to the year 2100, or by 5.1K and 2.4K for the low-volume and high-volume countries, respectively. Thus the amount per decade is 0.51K and 0.24K, respectively. The low-income countries remain at 2000 tkm/capita/year for the duration. Thus the per capita values for tkm/year are the following:

Year	Hi Income Hi Volume	Hi Income Low Volume	Middle Income	Low Income
2000	17	8	4	2
2010	17.51	8.24	4.32	2
2020	18.02	8.48	5.28	2
2030	18.53	8.72	6.72	2
2040	19.04	8.96	7.68	2

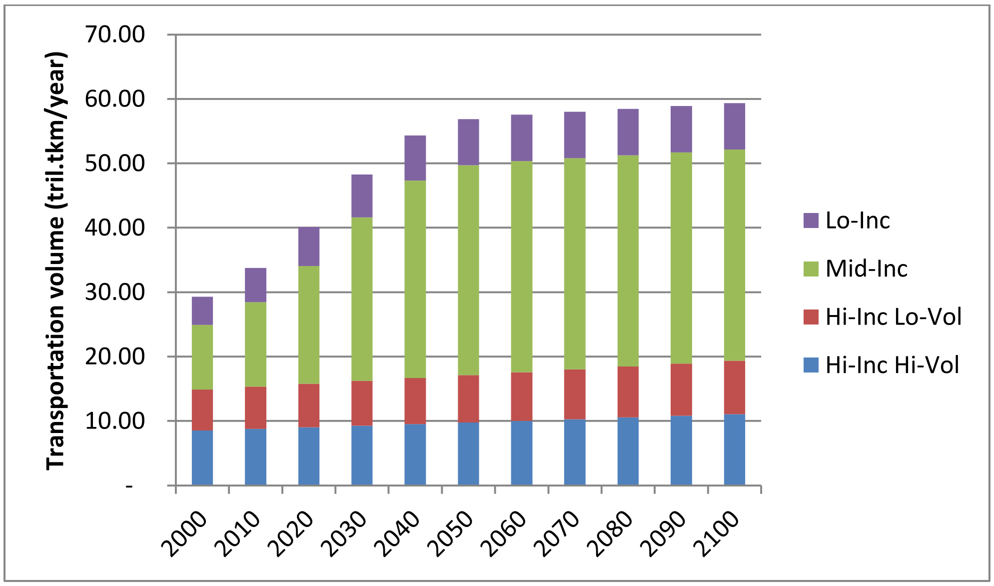
2050	19.55	9.2	8	2
2060	20.06	9.44	8	2
2070	20.57	9.68	8	2
2080	21.08	9.92	8	2
2090	21.59	10.16	8	2
2100	22.1	10.4	8	2

Multiplying per capita activity by population gives the following results in table form:

Year	Hi Income Hi Volume	Hi Income Low Volume	Middle Income	Low Income	Total
2000	8.50	6.40	10	4.40	29.30
2010	8.76	6.59	13.10	5.34	33.78
2020	9.01	6.78	18.26	6.09	40.13
2030	9.27	6.98	25.38	6.65	48.27
2040	9.52	7.17	30.64	7.02	54.35
2050	9.78	7.36	32.55	7.16	56.85
2060	10.03	7.55	32.77	7.21	57.56
2070					

	10.29	7.74	32.77	7.21	58.00
2080	10.54	7.94	32.77	7.21	58.45
2090	10.80	8.13	32.77	7.21	58.90
2100	11.05	8.32	32.77	7.21	59.34

The results are shown next in graphical form:



Discussion for both passenger and freight.

The assumptions in this scenario are similar for passenger and freight so the outcome is similar. In both the cases, total pkm and tkm in the year 2100 declines compared to the scenario in the chapter, because the reduction in low-income pkm and tkm is greater than the increase in high-income pkm and tkm. This is due in part to the larger population in the low-income countries. If these countries are excluded from access to modern transportation, total resource consumed by the transportation sector worldwide goes down (e.g., from 140 to ~100 trillion pkm/year), but the distribution is more inequitable. In the year 2100, the ratio of pkm/capita between the most intensive (high-income high-volume) and low income is 16:1. For freight, the tkm/capita ratio is 11:1.

Other variations on this solution are also possible, although they are not presented here. Another United Nations population trajectory has global population stabilizing at 11 billion instead of 9 billion in the year 2100, with many individual country pathways adjusted upward accordingly.

Chapter 2 Background on Energy Security and Climate Change

Problem 2-1.

Suppose the estimated ultimate recovery (EUR) for world conventional oil resources is 3.5 trillion barrels. Find on the internet or other source data on the historical growth in world oil production from 1900 to the present. Then use the Gaussian curve technique applied to annual production to predict: (a) the year in which the consumption peaks, (b) the world output in that year, and (c) the year following the peak in which the output has fallen by 90% compared to the peak.

Note to instructor: As a convenience, the annual world production for the period 1900 to 2006 is provided in a table below. It can be copied into a spreadsheet for manipulation by students.

Year	bil bbl/ year	Year	bil bbl/year	Year	bil bbl/year
1900	0	1940	2.1	1980	21.74
1901	0	1941	2.3	1981	20.46
1902	0	1942	2.5	1982	19.51
1903	0	1943	2.7	1983	19.44
1904	0	1944	2.9	1984	19.89
1905	0	1945	3.1	1985	19.70
1906	0	1946	3.3	1986	20.51
1907	0	1947	3.5	1987	20.67
1908	0	1948	3.7	1988	21.42
1909	0	1949	3.9	1989	21.82
1910	0.1	1950	4.1	1990	22.08
1911	0.2	1951	4.45	1991	21.97
1912	0.3	1952	4.8	1992	21.94
1913	0.4	1953	5.15	1993	21.96
1914	0.5	1954	5.5	1994	22.30
1915	0.6	1955	5.85	1995	22.77
1916	0.7	1956	6.2	1996	23.27

1917	0.8	1957	6.55	1997	24.00
1918	0.9	1958	6.9	1998	24.44
1919	1	1959	7.25	1999	24.06
1920	1	1960	7.66	2000	25.00
1921	1.05	1961	8.19	2001	24.86
1922	1.1	1962	8.89	2002	24.52
1923	1.15	1963	9.54	2003	25.35
1924	1.2	1964	10.29	2004	26.47
1925	1.25	1965	11.07	2005	26.94
1926	1.3	1966	12.03	2006	26.84
1927	1.35	1967	12.92		
1928	1.4	1968	14.10		
1929	1.45	1969	15.22		
1930	1.5	1970	16.75		
1931	1.4	1971	17.71		
1932	1.3	1972	18.67		
1933	1.4	1973	20.32		
1934	1.5	1974	20.34		
1935	1.6	1975	19.28		
1936	1.7	1976	20.93		
1937	1.8	1977	21.79		
1938	1.9	1978	21.96		
1939	2	1979	22.88		

Solution.

Let S be the shape parameter for the Hubbert curve and t_m the year in which the maximum occurs. Then for each year t, the value of P_{est} can be calculated using the following equation:

$$P = \frac{Q_{\inf}}{S\sqrt{2\pi}} \exp\left[-(t_m - t)^2/(2S^2)\right] \qquad (5\text{-}1)$$

We have used a solver to find values S = 42.6 years and t_m = 2027 that minimize root mean squared deviation (RMSD). The solver's settings are the following:

- Minimize the sum of all error square terms for years 1900, 1901…2006.
- To achieve this minimum, manipulate spreadsheet cells containing values of S and t_m.
- Since S and t_m are by definition positive, it is not necessary to allow negative values.
- No other constraints are required.

As a hand-calculated example of how RMSD is calculated, the appropriate values for calculating RMSD for select years (1910, 1940, 1970, 2000) are given in the following table. As an example for the year 2010, the estimated production is calculated as follows:

$$P(2010) = \frac{3.5 bil.bbl}{(42.6)(2\pi)^{0.5}} \exp\left[-(2027-1910)^2 / \left(2 \cdot (42.6)^2\right)\right] = 0.7 bil.bbl.$$

Repeating gives the values shown in the table:

T	P_{actual} (10^9 barrel)	P_{est} (10^9 barrel)	$(P_{\text{actual}} - P_{\text{est}})^2$
1910	0.1	0.7	0.4
1940	2.1	4.0	3.6
1970	16.8	13.2	12.4
2000	25.0	26.8	2.68

Summing the square of error terms for all 107 values from years 1900 to 2006 gives a value of 447.2. The value of the RMSD is then calculated using the following equation:

$$RMSD = \sqrt{\frac{1}{n}\left(\sum_i \left(P_{actual,i} - P_{estimated,i}\right)^2\right)}$$

$$\text{RMSD} = \sqrt{\frac{1}{107}(447.2)} = 2.044$$

Answer to part (a): year of maximum is 2027.

Part (b): To calculate output in 2027, substitute Q_{inf} = 3.5 trillion, S = 42.6 years, and t_m = 2027 into Eq. (5-1) gives

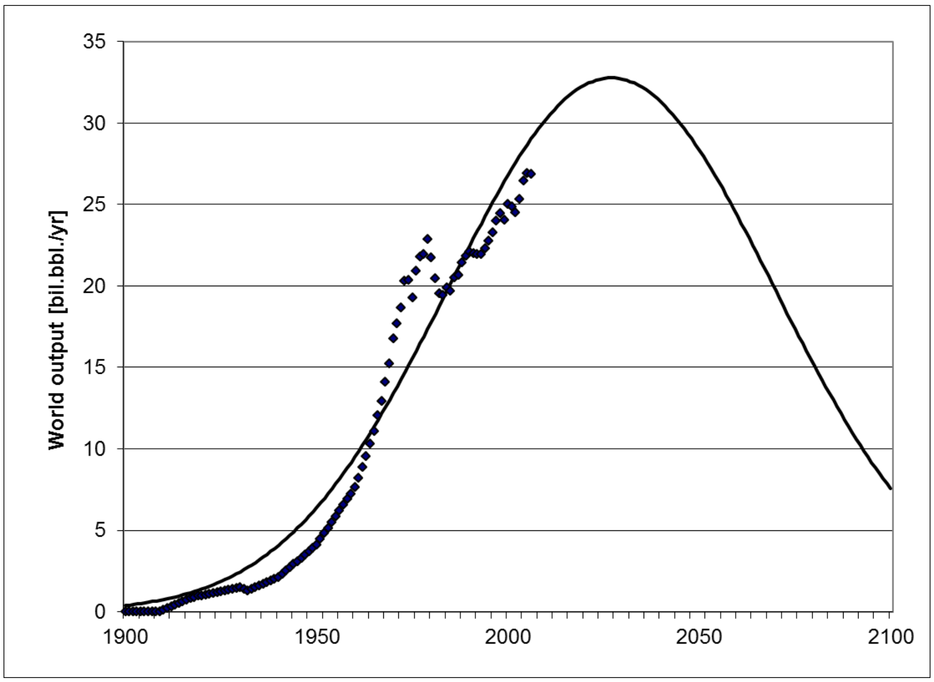
$$P = \frac{3.5 \times 10^{12}}{42.6\sqrt{2\pi}} \exp(0) = 3.27 \times 10^{10} \text{ bbl}$$

Thus the output in the peak year is 32.7 billion barrels.

Part (c): 10% of the peak output value is 3.27 billion barrels. This value occurs in the year 2119. The value can be confirmed using the production equation:

$$P = \frac{3.5 \times 10^{12}}{42.6\sqrt{2\pi}} \exp(-(2027 - 2119)^2 / (2 \cdot 42.6)^2) = 3.26 \times 10^9 = \sim 3.27 \times 10^9 \text{bbl}$$

As an illustration, the figure below compares observed and estimated production:



Problem 2-2.

Repeat Problem 2-1 using cumulative production data and the logistics function, also known as the technological substitution function.

Solution.

Part (a): Year in which production peaks: The problem is solved by fitting the modeled cumulative consumption to the observed curve up to the year 2006, and then extrapolating the model into the future. In this case, the problem is solved using absolute values of consumption; the solution with percent values follows the same steps. The technological substitution curve with known values of F = 3.5 × 10^{12} barrels and unknown values of c_1 and c_2 is of the form:

$$f(t') = \frac{F \cdot e^{(c_1 + c_2 t')}}{1 + e^{(c_1 + c_2 t')}}$$

Here t' is the number of years since the start year (t = 1900 in this case). The approach to solving the problem is to set up a spreadsheet table (or equivalent approach in a mathematical software package such as Matlab) where each row represents 1 year, and then the square of the error term is calculated for each year based on the difference between observed and modeled (using the substitution curve equation) cumulative production values for that year. The sum of the square of the error terms is used to calculate RMSD. Solving for c_1 and c_2 that minimize RMSD gives values of $c_1 = -5.9954$, $c_2 = 0.04946$, and RMSD = 29.757. For example, with these values, the row for the year 1910 (i.e., t' = 10 years elapsed since the start of the data stream) with values in billions of barrels per year is as follows:

Calculation of modeled cumulative consumption:

$$f(10) = \frac{3.5 tril.bbl \cdot e^{(-5.9954+0.0494610)}}{1 + e^{(-5.9954+0.0494610)}} =$$

Values in row for year 2010:

Year	t'	Observed	Modeled	Error^2
1910	10	0.1	14.2	199.8

Extrapolating the modeled curve out makes it possible to create a table of cumulative and annual values in units of billion barrels around the year 2022, which is the year when cumulative production surpasses 50% of 3.5 trillion or 1.75 trillion barrels:

Year	Production	
	Cumulative	Annual
2020	1698	43.20
2021	1741	43.27
2022	1784	43.28
2023	1828	43.24
2024	1871	43.14

Therefore, since the year 2022 has the highest annual value in the table, this is the year of peak production.

Part (b): Again, from the table, the output in that year is 43.28 billion barrels.

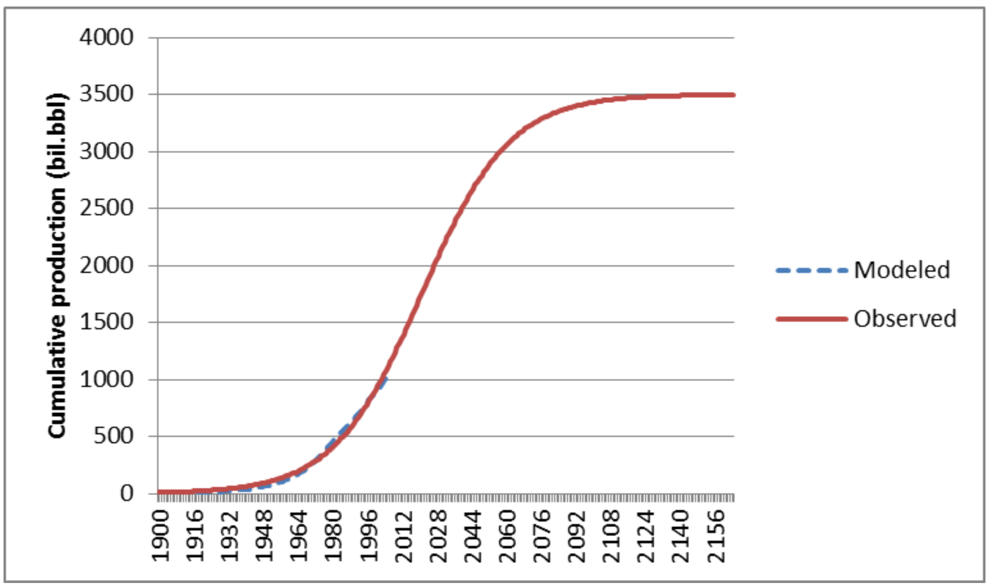
Part (c): The desired year is when output has dropped below 10% of the value in the peak year, thus 4.33 billion barrels. The table below gives values around the years 2094 to 2098.

Year	Production	
	Cumulative	Annual

2094	3407	4.59
2095	3411	4.38
2096	3416	4.18
2097	3420	3.98
2098	3,23	3.80

From the table, output falls below 4.33 billion in the year 2096.

For reference, although the problem does not require it, a graph of observed and modeled output:



Note to instructor: comparison of the two approaches shows that they produce different peak years, namely 2027 for the annual production data and 2022 for the cumulative production. It may be useful to discuss with the students the fact that two different curves fit to the same data need not exactly agree on the peak year.

Problem 2-3.

It is a well-known fact that thanks to the greenhouse effect caused by greenhouse gases (GHGs) such as CO_2, the surface temperature of the planet earth is noticeably higher than if there were no layer of GHGs in our atmosphere. The current average world temperature is approximately 288°K or 15°C, before taking account of ongoing impact from climate change. In this problem you will estimate the impact of GHGs by considering the case of our planet without them. Solar energy arrives from the sun at the average rate of 1372 W/m^2, of which 30% is reflected back into space and 70% penetrates the atmosphere and provides energy to the earth. The earth's radius is, on average, approximately 6400 km. Use the following additional information: with no greenhouse effect, the surface temperature of the planet T is the value where the amount of energy coming from the sun is in equilibrium with the amount of energy leaving due to black body radiation. The flux of black body radiation $F_{BB} = \sigma T^4$, where σ = 5.67 × 10^{-8} W/m^2-°K^4 and T is measured in units of degrees kelvin, or K. Hint: in order to solve correctly, take into account the fact that the flux of energy from the sun is spread over the surface of the planet earth, which is spherical, while the radiation intercepted by the earth is in the shape of a disc.

a. What is the predicted value of T with no greenhouse effect in degrees K?
b. What is the value of T in degrees centigrade? What is one practical impact of such a temperature for life on the planet?

Part (a): Let R be the average radius of the planet earth. The arriving energy flux from the sun must first be adjusted using the ratio of the area of a disc to the area of a sphere, because the energy from the sun arrives through an area the size of a disc with diameter R, but energy leaves the surface of the earth across an area with physical area the size of a sphere with diameter R. Let S_0 be the solar constant of 1372 W/m^2. Let P be total power and p be power per unit area.

$$P_{arriving} = P_{leaving}$$
$$A_{disc} p_{arriving} = A_{sphere} p_{leaving}$$
$$(\pi R^2)(0.7)S_0 = (4\pi R^2) p_{leaving}$$
$$p_{leaving} = (0.7)S_0\left(\frac{\pi R^2}{4\pi R^2}\right) = \frac{0.7(1372)}{4} = \frac{960.4}{4} = 240.1 W/m^2$$

Now set $P_{leaving}$ equal to the black body radiation formula. Then rearrange to solve for T:

$$\sigma T^4 = p_{leaving}$$
$$T = \left(\frac{p_{leaving}}{\sigma}\right)^{0.25} = \left(\frac{240.1}{5.67 \times 10^{-8}}\right)^{0.25} = 255.1K$$

Answer: **255.1 K.**

Part (b): Since 273 °K = 0°C, the temperature in Celsius is **−18°C**. The implication is that the freezing temperature would be a severe constraint on the ability of most currently known life forms to survive, so that any life on earth would need to have evolved quite differently.

Problem 2-4.

Adequacy of energy for the world: In 2008 world total energy consumption was estimated at 520 exajoules, or EJ. 1 EJ = 10^{18} joule; the amount of energy is equivalent to 493 quadrillion Btu, or Quads, although the rest of this problem is solved in metric units. Solar energy arrives from the sun at the average rate of 1372 W/m^2, of which 30% is reflected back into space and 70% penetrates the atmosphere and provides energy to the earth. The earth's radius is, on average, approximately 6400 km. Recall that 1 W = 1 J/s.

a. What is the ratio of the energy reaching the planet to the total world energy consumption?
b. Without revealing the exact answer, it is safe to say that the amount of energy reaching the earth from the sun is orders of magnitude larger than the amount used around the world. Since this is the case, why is only a small fraction of energy used by humans extracted directly from arriving sunlight? Give one possible reason, short answer form, in one or two sentences. Hint: there are many possible correct reasons.

Solution.

Part (a): Multiplying the arriving power per area by the size of the cross-section of the earth and the number of seconds per year gives total energy:

$$E_{year} = \left(1372\frac{W}{m^2}\right)(0.7)(\pi)\left(6.4\times10^6 m\right)^2\left(3.1536\times10^7 \sec\right) = 3.9\times10^6 EJ / yr$$

The ratio is therefore:

$$\frac{E_{year}}{E_{consumed}} = \frac{3.9\times10^6}{520} = 7495:1$$

Part (b): The reasons concern both the complexity and cost of using solar energy, relative to fossil fuels. Solar energy is diffuse (meaning it must be concentrated from a wide area to deliver large amounts of power) and intermittent (meaning systems must be created to manage both high and low arriving solar output relative to demand). For this and other reasons, solar is complex and also expensive in terms of direct economic cost, compared to fossil fuels such as gas, oil, or coal.

Problem 2-5.

Create an "IPCC-like" stabilization scenario using the following simplified information. Starting in the year 2004, the concentration of CO_2 in the atmosphere is 378 ppm, the global emissions of CO_2 are 7.4 Gt carbon per year, the total carbon in the atmosphere is 767 Gt, and emissions are growing by 4% per year, for example, in 2005 emissions reach 7.7 Gt, 8.0 Gt in 2006, and so on. Note that concentration is given in terms of CO_2, but emissions and atmospheric mass are given in Gt carbon. The oceans absorb 3 Gt net (absorbed minus emitted) each year for the indefinite future. The change or decline of emissions is influenced by "global CO_2 policy" as follows: in year 2005, the emission rate declines by 0.1 percentage points to 3.9%, and after that and up to the point that concentration stabilizes, the change is 0.1% multiplied by the ratio of the previous year's total emissions divided by 7.4 Gt, that is

$$\text{Chg\%}_t = \text{Chg\%}_{t-1} - \frac{(0.1)(\text{total emissions}_{t-1})}{7.4}$$

After concentration stabilizes, emissions are 3 Gt/year, so that emissions are exactly balanced by ocean absorption. To illustrate, (Chg%)2004 = 4%, (Chg%)2005 = 3.9%, and so on. Also, concentration can be calculated as 378 ppm · (total carbon in atmosphere/767 Gt). (a) What is the maximum value of concentration reached? (b) In what year is this value reached? (c) What is the amount of CO_2 emitted that year? (d) Plot CO_2 concentration and emissions per year on two separate graphs. Hints: Use a spreadsheet or other software to set up equations that calculate emission rate, change in emissions, and concentration in year *t* as a function of values in year *t* – 1. (e) Compare the shape of the pathway in this scenario, based on the graphs from part (d), with actual IPCC scenarios, and discuss similarities and differences. (f) The scenario in this problem is a simplification of how a carbon stabilization program might actually unfold in the future. Identify two ways in which the scenario is simplistic.

Solution.

Parts (a) and (b): The approach to this problem is to understand the calculation in the first 1-year step from 2004 to 2005, and then to repeat the calculations in a spreadsheet or math software package until the concentration stabilizes.

In the first year, take *t* = 2005, *t* – 1 = 2004. Then total emissions in *t* – 1 are given as 7.4 Gt carbon per year, and the change percent value $Chg\%_{t-1}$ is 4.0%. We can therefore use the above formula to calculate $Chg\%_{2005}$:

$$Chg\%_{2005} = Chg\%_{2004} - \frac{(0.1\%)(TotEmits_{t-1})}{7.4}$$
$$= 4.0\% - \frac{(0.001)(7.4)}{7.4} = 3.9\%$$

Emissions for 2005 therefore increase to 7.7 Gt C. Concentration in ppm in 2005 is then the following:

$$Conc_{2005} = 378\,ppm\frac{771.4}{767} = 380.2\,ppm$$

Repeating for the transition from 2005 to 2006:

$$Chg\%_{2006} = Chg\%_{2005} - \frac{(0.1\%)(TotEmits_{2005})}{7.4}$$
$$= 3.9\% - \frac{(0.001)(7.7)}{7.4} = 3.79\%$$
$$Conc_{2006} = 378\frac{776.1}{767} = 382.5$$

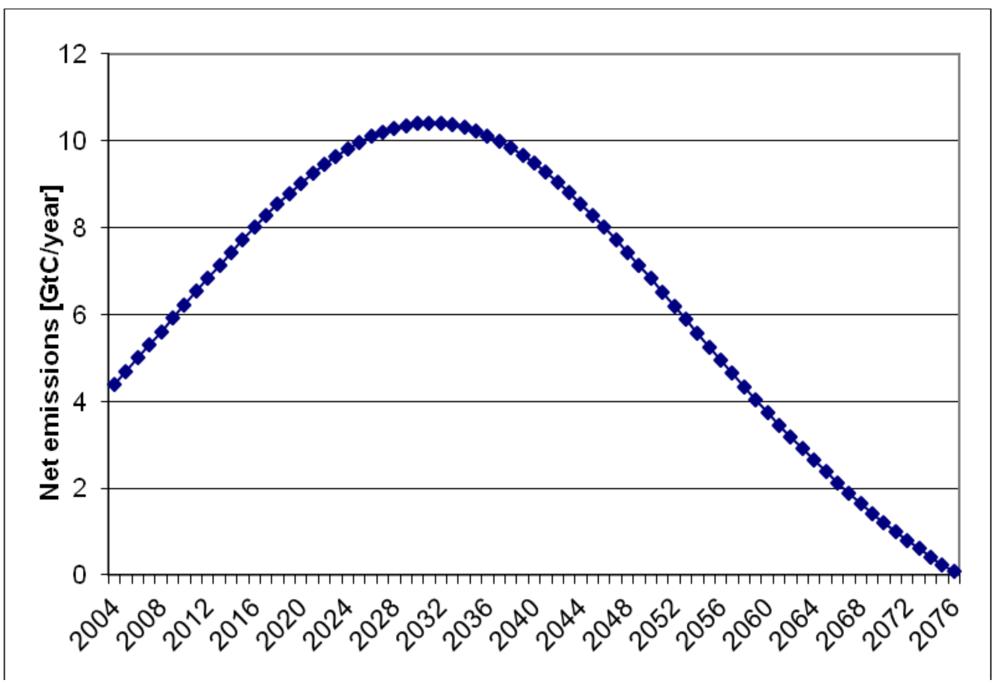
This process is repeated in a spreadsheet table in the following form, shown only up to year 2008 for brevity:

Year	Emits (Gt C)	Total in Atmosphere (Gt C)	Concentration (ppm)	Net Emits (Gt C)	Change (pct)
2004	7.40	767	378	4.4	4%
2005	7.70	771.4	380.2	4.70	3.90%
2006	8.00	776.1	382.5	5.00	3.79%
2007	8.30	781.1	384.9	5.30	3.68%
2008	8.60	786.4	387.6	5.60	3.56%

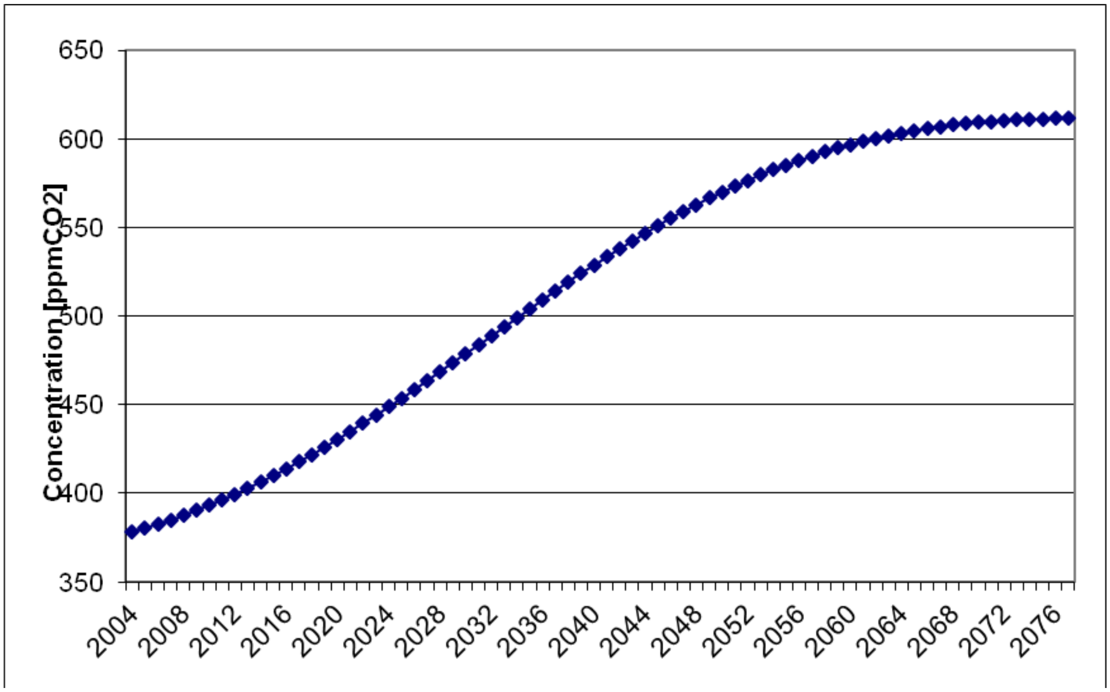
The change percent value eventually goes into negative territory in the year 2031 and the concentration begins to taper off. In the year 2077, the concentration reaches 611.5 ppm where it stabilizes, and the change percent changes from –5.26% in year 2076 to 0% in 2077, so that emissions and uptake of CO_2 are in equilibrium thereafter.

Part (c): Emissions are 3.0 Gt in 2077, so net emissions are hence zero.

Part (d): Graphs of net emissions per year after subtracting the uptake value:



Concentration value:



Part (e): Many answers are possible. For example, the scenario stabilizes relatively quickly compared to IPCC, even though the concentration is high (610 ppm). Also, the IPCC scenarios tend to have a "smooth landing" at the tail, whereas this scenario changes abruptly.

Part (f): Again, many answers are possible. For instance: (i) the assumption of constant absorption of CO_2 by the oceans.(ii) The assumption of a "smooth" representation of policy. It is likely to be more lumpy, and also to respond to the effects of climate change as they unfold.

Problem 2-6.

One concern held by some scientists is that while investments required to slow global climate change may be expensive, over the long term the loss of "ecosystem services" (pollination of crops, distribution of rainfall, etc.) may have an even greater negative value. Consider an investment portfolio which calls for $100 billion per year invested worldwide from 2010 to 2050 in measures to combat climate change. The value per year is the net cost after taking into account return from the investments such as sales of clean energy in the marketplace. If the investments are carried out, they will prevent the loss of $2 trillion in ecosystem services per year from 2050 to 2100. Caveats: the analysis is simplified in that (a) in 2050 the loss of ecosystem services jumps from 0 to $2 trillion, when in reality it would grow gradually from 2010 to 2050 as climate change becomes more widespread, (b) the world could not ramp up from 0 to $100 billion in investment in just 1 year in 2010, and (c) there is no consideration of what happens after the year 2100. Also, the given value of investment in technology to prevent a given value of loss of ecosystem services is hypothetical. The potential loss of ecosystem services value is also hypothetical, although at a plausible order of magnitude relative to a published estimate of the value of ecosystem services (R.Costanza, R. d'Arge, R. de Groot, et al, 1997, “The Value of the World’s Ecosystem Services and Natural Capital”, *Nature*, Vol. 387, pp. 253–260).

a. Calculate the NPV of the investment in 2010, using a discount rate of 3% and of 10%, and also using simple payback.

b. Discuss the implications of the results for the two cases.

Solution.

Part (a): The general approach is to take the ecosystems services 2050 to 2100, discount them to the year 2050 (if applicable), and then discount this single lump sum back to the year 2010. The simple payback case is the least complicated, as follows:

NPV = –Value of investment + Value of benefits

= –1E11(40) + 2E12(50) = 7.6E13

In other words, $76 trillion in net benefit.

For the other two cases at 3% and 10%, the calculation is the following (it is assumed that the discounting factors are calculated in the usual way):

NPV @ 3% = –Value of investment + Value of benefits
= –1E11(*P/A*, 3%, 40) + 2E12(*P/A*, 3%, 50)(*P/F*, 3%, 40) = 1.35E13

NPV @ 3% = –Value of investment + Value of benefits
= –1E11(*P/A*, 10%, 40) + 2E12(*P/A*, 10%, 50)(*P/F*, 10%, 40) = –5.49E11

Of interest is the fact that with sufficiently high discount rate, in this case 10%, the strong positive value with no discount rate becomes a negative NPV of –$549 billion.

Part (b): Implications: using a high discount rate may result in a decision that violates the intergenerationality principle of sustainable development.

In other words, failure to invest during 2010 to 2050 time period leads to major loss of services 2050 to 2100, but the NPV suggests that this outcome is acceptable. Other interpretations are also possible.

Problem 2-7.

An oil company refines crude oil valued at $102/barrel and sells it to motorists at its retail outlets. The price is $3.90/U.S. gallon ($1.03/L). On a per unit basis (e.g., per gallon or per liter), by what percentage has the price per gallon of liquid increased going from crude oil before refining to final sale to the motorist? A barrel of oil contains 42 U.S. gallons.

Solution.

Each barrel of oil contains 42 gal, so the price of crude oil per gallon is $102/42 = $2.43. The percent increase in price is therefore:

$$\frac{(\$3.90 - \$2.43)}{\$2.43} = \frac{\$1.43}{\$2.43} = 60.5\%$$

Problem 2-8.

Estimated ultimate recovery (EUR) of world oil supplies: Suppose that thanks to advances in technology the EUR for world conventional oil resources have expanded to 5 trillion barrels. Download data on the historical growth in incremental and cumulative world oil production from 1900 to 2006 from the blackboard site. Then use the Gaussian curve to fit a modeled curve to the data that minimizes RMSD. Answer the following questions:

c. The year that t_m is reached
d. The value of the width parameter S
e. A graph of observed and modeled annual production for years 1900 to 2200 (of course the observed curve will end after the year 2006)

Solution.

The given data on total oil availability gives $Q_{inf} = 5 \times 10^{12}$ bbl. The Gaussian curve with unknown values of t_m and S is of the form:

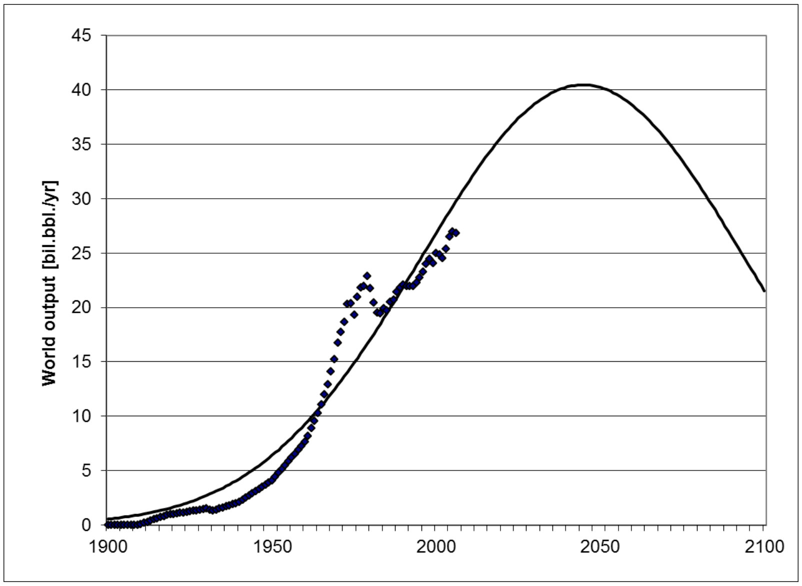
$$P = \frac{Q_\infty}{S\sqrt{2\pi}} \exp\left[-(t_m - t)^2/(2S^2)\right]$$

The approach to solving the problem is to set up a spreadsheet table (or equivalent approach in a mathematical software package such as Matlab) where each row represents 1 year, and then the square of the error term is calculated for each year based on the difference between observed and modeled (using the Gaussian curve equation) production values for that year. The sum of the square of the error terms is used to calculate RMSD. Solving for t_m and S that minimize RMSD gives values of $t_m = 2045$ and $S = 49.3$. For example, with these values, the row for the year 1910 with values in billions of barrels per year is as follows:

Year	Observed	Modeled	Error^2
1910	0.1	0.97	0.76

Answers:

a. $T_m = 2045$
b. $S = 49.3$ years
c. The graph is the following:



Problem 2-9.

Repeat 2-8, but use the cumulative production and logistics curve technique (see the "Cumulative U.S. oil production" section in Chapter 1 in the course pack) to answer the following:

a. Predict the year in which the consumption reaches 50% of the EUR value, including the best-fit values of c_1 and c_2,
b. the world output in that year, and
c. the year following the peak in which the output reaches 90% of EUR.
d. Provide a graph of the projected versus actual curves, again for the years 1900 to 2200?
e. Short answer: Do the curves in questions parts b and c agree with each other? Comment briefly on the difference (1 sentence answer is fine).

Solution.

Part (a): The problem is solved by fitting the modeled cumulative consumption to the observed curve up to the year 2006, and then extrapolating the model into the future. In this case, the problem is solved using absolute values of consumption; the solution with percent values follows the same steps. The technological substitution curve with known values of $F = 5 \times 10^{12}$ bbl and unknown values of c_1 and c_2 is of the form:

$$f(t') = \frac{F \cdot e^{(c_1 + c_2 t')}}{1 + e^{(c_1 + c_2 t')}}$$

The approach to solving the problem is to set up a spreadsheet table (or equivalent approach in a mathematical software package such as Matlab) where each row represents 1 year, and then the square of the error term is calculated for each year based on the difference between observed and modeled (using the substitution curve equation) cumulative production values for that year. The sum of the square of the error terms is used to calculate RMSD. Solving for c_1 and c_2 that minimize RMSD gives values of $c_1 = -6.186$ and $c_2 = 0.04678$. For example, with these values, the row for the year 1910 (i.e., $t' = 10$ years elapsed since the start of the data stream) with values in billions of barrels per year is as follows:

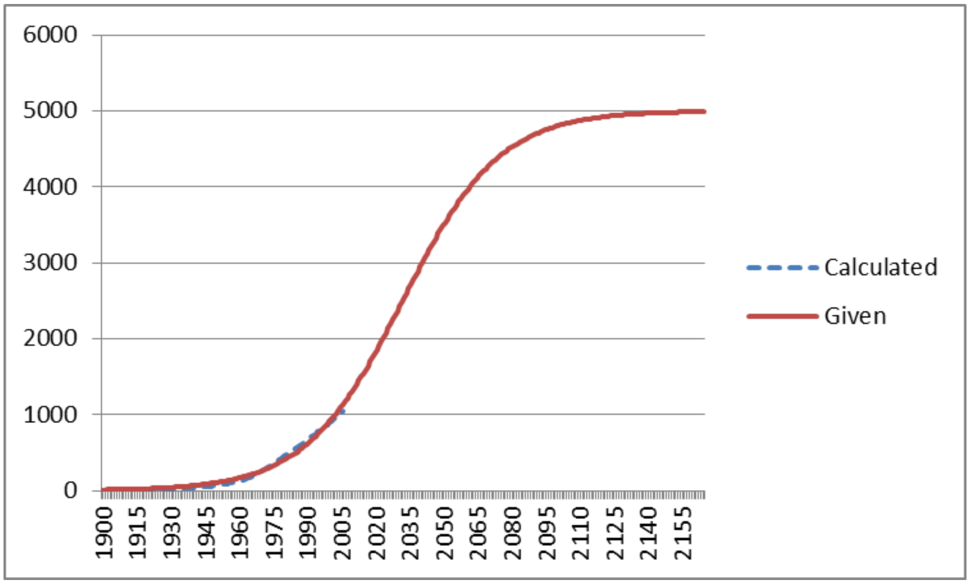
Year	t'	Observed	Modeled	Error^2
1910	10	0.1	16.38	265.1

Extrapolating the modeled curve out shows that the cumulative production first surpasses 2.5×10^{12} bbl in the year 2033.

Part (b): The cumulative production in 2033 is 2545 billion barrels. In 2032, it is 2486 billion barrels. Therefore, the annual production is the difference, or 59 billion barrels.

Part (c): Since the value of EUR is 5 trillion barrels, 90% of this value is 4.5 trillion barrels. The cumulative production first surpasses this value in the year 2080.

Part (d): The graph is the following:



Part (e): The two approaches do not agree exactly, the Gaussian curve puts the maximum in the year 2045, and the cumulative curve puts the peak 12 years earlier, in 2033. There is no underlying theoretical reason why the two approaches should result in the same peak year, so some variation is to be expected.

Problem 2-10.

Estimated ultimate recovery (EUR) of world oil supplies: Suppose the EUR for world conventional oil resources is 4 trillion barrels, the EUR for tar sands another 2 trillion, and the EUR for shale oil another 2 trillion, for a total of 8 trillion. Download data on the historical growth in incremental and cumulative world oil production from 1900 to the present from the blackboard site. Then use the Gaussian curve to fit a modeled curve to the data that minimized RMSD. Include in your answer the year that t_m is reached, the value of the width parameter *S*, and a graph of observed and modeled annual production for years 1900 to 2200.

<u>Solution.</u>

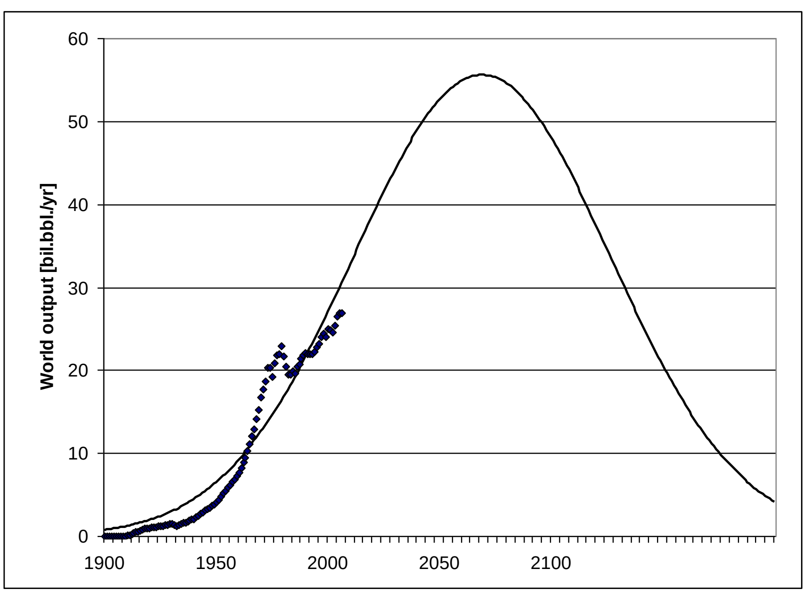
The given data on total oil availability gives $Q_{inf} = 8 \times 10^{12}$ bbl. The Gaussian curve with unknown values of t_m and S is of the form:

$$P = \frac{Q_\infty}{S\sqrt{2\pi}} \exp\left[-(t_m - t)^2/(2S^2)\right]$$

The approach to solving the problem is to set up a spreadsheet table (or equivalent approach in a mathematical software package such as Matlab) where each row represents 1 year, and then the square of the error term is calculated for each year based on the difference between observed and modeled (using the Gaussian curve equation) production values for that year. The sum of the square of the error terms is used to calculate RMSD. Solving for t_m and S that minimize RMSD gives values of $t_m = 2069$ and $S = 57.4$. For example, with these values, the row for the year 1910 with values in billions of barrels per year is as follows:

Year	Observed	Modeled	Error^2
1910	0.1	1.20	1.21

The graph is the following:



Problem 2-11.

Repeat 2-10, but use the cumulative production and logistics curve technique to answer the following:

a. Predict the year in which the consumption reaches 50% of the EUR value, including the best-fit values of $c_1$and c_2,
b. the world output in that year, and
c. the year following the peak in which the output reaches 90% of EUR.
d. Provide a graph of the projected versus actual curves, again for the years 1900 to 2200?
e. Short answer: Do the curves in parts b and c agree with each other (1 sentence answer is fine)?

Solution.

Part (a): The problem is solved by fitting the modeled cumulative consumption to the observed curve up to the year 2006, and then extrapolating the model into the future. In this case, the problem is solved using absolute values of consumption; the solution with percent values follows the same steps. The technological substitution curve with known values of $F = 8 \times 10^{12}$ bbl and unknown values of c_1 and c_2 is of the form:

$$f(t') = \frac{F \cdot e^{(c_1 + c_2 t')}}{1 + e^{(c_1 + c_2 t')}}$$

The approach to solving the problem is to set up a spreadsheet table (or equivalent approach in a mathematical software package such as Matlab) where each row represents 1 year, and then the square of the error term is calculated for each year based on the difference between observed and modeled (using the substitution curve equation) cumulative production values for that year. The sum of the square of the error terms is used to calculate RMSD. Solving for c_1 and c_2 that minimize RMSD gives values of $c_1 = -6.528$ and $c_2 = 0.04469$. For example, with these values, the row for the year 1910 (i.e., $t' = 10$ years elapsed since the start of the data stream) with values in billions of barrels per year is as follows:

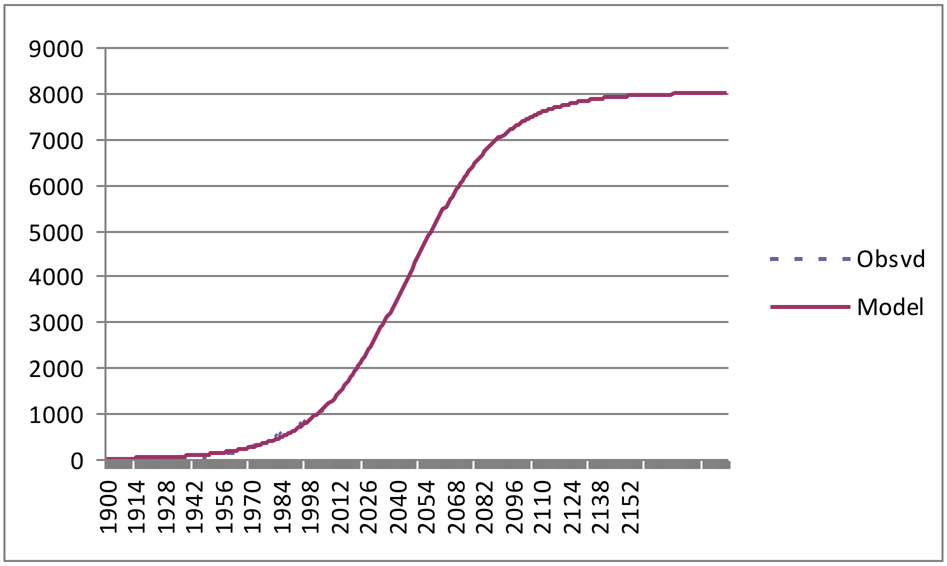
Year	t'	Observed	Modeled	Error^2
1910	10	0.1	18.24	328.9

Extrapolating the modeled curve out shows that the cumulative production first surpasses 4×10^{12} bbl in the year 2047.

Part (b): The cumulative production in 2047 is 4081 billion barrels. In 2046, it is 3992 billion barrels. Therefore, the annual production is the difference, or 89 billion barrels.

Part (c): Since the value of EUR is 8 trillion barrels, 90% of this value is 7.2 billion barrels. The cumulative production first surpasses this value in the year 2096.

Part (d): The graph is the following:



Part (e): The two approaches do not agree exactly, the Gaussian curve puts the maximum in the year 2069, and the cumulative curve puts the peak 22 years earlier, in 2047.

Chapter 3 Systems Tools

Problem 3-1.

Consider a technical project with which you have been involved, such as an engineering design project, a research project, or a term project for an engineering course. The project can be related to transportation systems, or to some other field. Consider whether or not the project used the systems approach. If it did use the approach, describe the ways in which it was applied, and state whether the use of the approach was justified or not. If it did not use the approach, do you think it would have made a difference if it had been used? Explain.

Solution.

This question is open-ended and based on personal experience, so there is no single model answer. However, in general the more complex the project, the more likely it is that the extra early-stage effort of the systems approach will be justified. In terms of misfit projects, both complex projects that underperformed for lack of a systems approach, and simple projects that were burdened unnecessarily by a systems approach, are possible.

Problem 3-2.

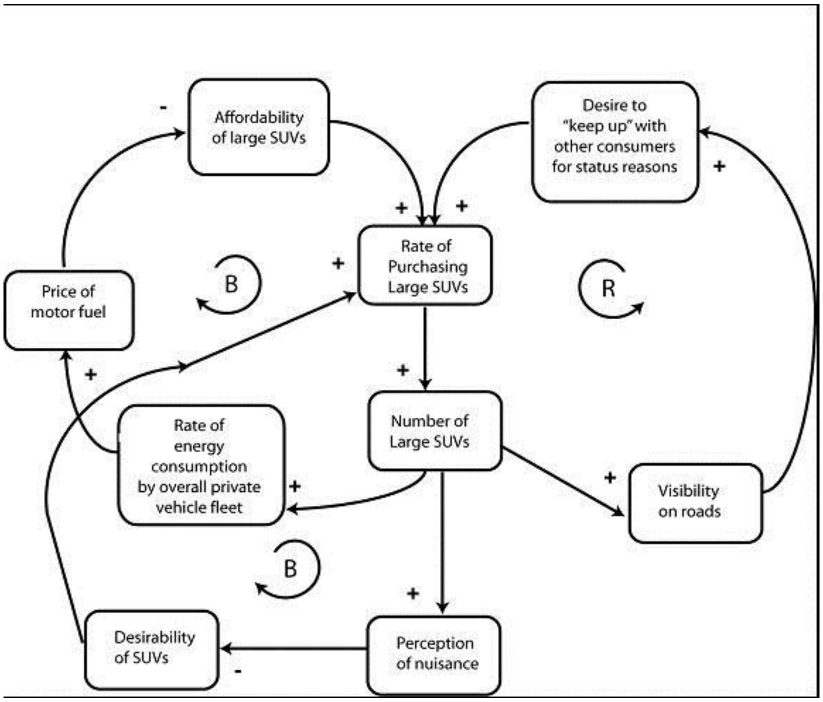
Causal loop diagrams: In recent years, one phenomenon in the private automobile market of many countries of the world, especially the wealthier ones, has been the growth in the number of very large sport utility vehicles and light trucks. Create two causal loop diagrams, one "reinforcing" and one "balancing," starting with "number of large private vehicles" as the initial component.

Solution.

This problem is open-ended in the sense that the causal loop diagram could incorporate various factors. Suppose that we limit it to three effects:

1. Consumer effect: consumers see other drivers with large SUVs and want their own.
2. Nuisance effect: large SUVs are eventually blamed for congestion and become unpopular.
3. Price effect: increasing fuel consumption drives up the cost of fuel, reducing affordability.

The following diagram is the result:



Problem 3-3.

The values of world energy consumption by 25 year increments for the period 1850 to 1975 are given below. Fit a curve to the data points using the exponential growth function, and then use this curve to predict the value of energy consumption in 2000. If the actual value in 2000 is 419 EJ, by how many EJ does the projected answer differ from the actual value in units of EJ?

1850	25 EJ
1875	27 EJ
1900	37 EJ
1925	60 EJ
1950	100 EJ
1975	295 EJ

1975	295 EJ

Solution.

Fit the exponential curve to the given data by setting parameters a and b to minimize root mean squared deviation (RMSD). Using a solver gives values a = 4.096 and b = 0.0340 resulting in the following table:

Term	Year	Energy use		Error
		Actual	Estimated	
1	1850	25	4.1	437.0
2	1875	27	9.6	303.1
3	1900	37	22.5	211.4
4	1925	60	52.6	54.8
5	1950	100	123.2	537.1
6	1975	295	288.5	42.8

Summing error terms in the right column gives a value of 1568.2, leading to RMSD = 16.26. Now solve for the value in the year 2000, i.e., with 150 years elapsed since the year 1850:

$$E(150) = 4.096 \cdot \exp(0.0340 \cdot 150) = 675.5$$

Thus, the curve predicts 675.5 EJ of world energy consumption, significantly higher than the observed 419 EJ, suggesting that there has been a tapering off of growth in energy consumption relative to the exponential growth observed up through 1975.

Problem 3-4.

Current global energy consumption trends combined with per capita energy consumption of some of the most affluent countries, such as the U.S., can be used to project the future course of global energy demand over the long term.

a. Using energy consumption and population data from Chap. 2, calculate per capita energy consumption in USA in 2008, in units of GJ/person.

b. If the world population ultimately stabilizes at 10 billion people, each with the energy intensity of the average U.S. citizen in 2008, at what level would the ultimate total annual energy consumption stabilize, in EJ/year?

c. From data introduced earlier, it can be seen that the world reached the 37 EJ/year mark in 1900, and the 100 EJ/year mark in 1950. Using these data points and the logistics formula, calculate the number of years from 2008 until the year in which the world will reach 97% of the value calculated in part (b).

d. Using the triangle formula, pick appropriate values of the parameters *a* and *b* such that the triangle formula has approximately the same shape as the logistics formula.

e. Plot the logistics and triangle functions from parts (c) and (d) on the same axes, with year on the *X* axis and total energy consumption on the *Y* axis.

f. Discussion: the use of the U.S. as a benchmark for the target for world energy consumption has been chosen arbitrarily for this problem. If you were to choose a different country to project future energy consumption, would you choose one with a higher or lower per capita energy intensity than USA? What are the effects of choosing a different country?

Solution.

(a) Per capita consumption in 2004 is 117 EJ/(298M) = 392.6 GJ/capita.

(b) World consumption would then be (392.6 GJ/capita)(10B) = 3926 EJ/year.

(c) Since there are two unknowns, c_1 and c_2, for the logistics curve formula, and 2 years with known data, we can solve by hand. First solving for c_1 using data from the year 1900:

$$p(0) = \frac{\exp(c_1)}{1+\exp(c_1)} = \frac{37}{3926} = 0.00942$$
$$p(0) = \exp(c_1)(1 - p(0))$$
$$\exp(c_1) = p(0)/(1 - p(0))$$
$$c_1 = \ln[p(0)/(1 - p(0))] = -4.655$$

Now solve for c_2 using data from the year 1950:

$$p(50) = \frac{\exp(c_1 + c_2 t)}{1+\exp(c_1 + c_2 t)} = \frac{100}{3926} = 0.0255$$
$$p(50) = \exp(c_1 + c_2 t)(1 + p(50))$$
$$\exp(c_1 + c_2 t) = p(50)/(1 + p(50))$$
$$c_1 + c_2 t = \ln[p(50)/(1 + p(50))] = -3.644$$
$$c_2 = \frac{4.655 - 3.644}{50} = 0.0202$$

Using these values, the year where $p(t)$ exceeds 97% is when t = 403, or in year 2303. Thus, it is 299 years from the year 2004. The calculation is as follows:

$$p(303) = \frac{\exp(c_1 + c_2 t)}{1+\exp(c_1 + c_2 t)} = \frac{\exp(-4.655 + 0.0202 \cdot 403)}{1+\exp(-4.655 + 0.0202 \cdot 403)} = 97.03\%$$

(d) For the triangle function, we could choose years 1900 and 2350 as beginning and end point, i.e., a = 1900, b = 2350. The choice of end year is somewhat arbitrary since the logistics curve is approaching $p(t)$ = 1 in the period 2300 to 2350, so other values for b could be chosen as well. The triangle function therefore reaches $p(t)$ = 0.5 in the year 2125. Values of $p(t)$ are calculated using the standard triangle function formula:

$$p(t) = 2\left(\frac{t-a}{b-a}\right)^2 \text{ if } a \le t \le \frac{a+b}{2}$$

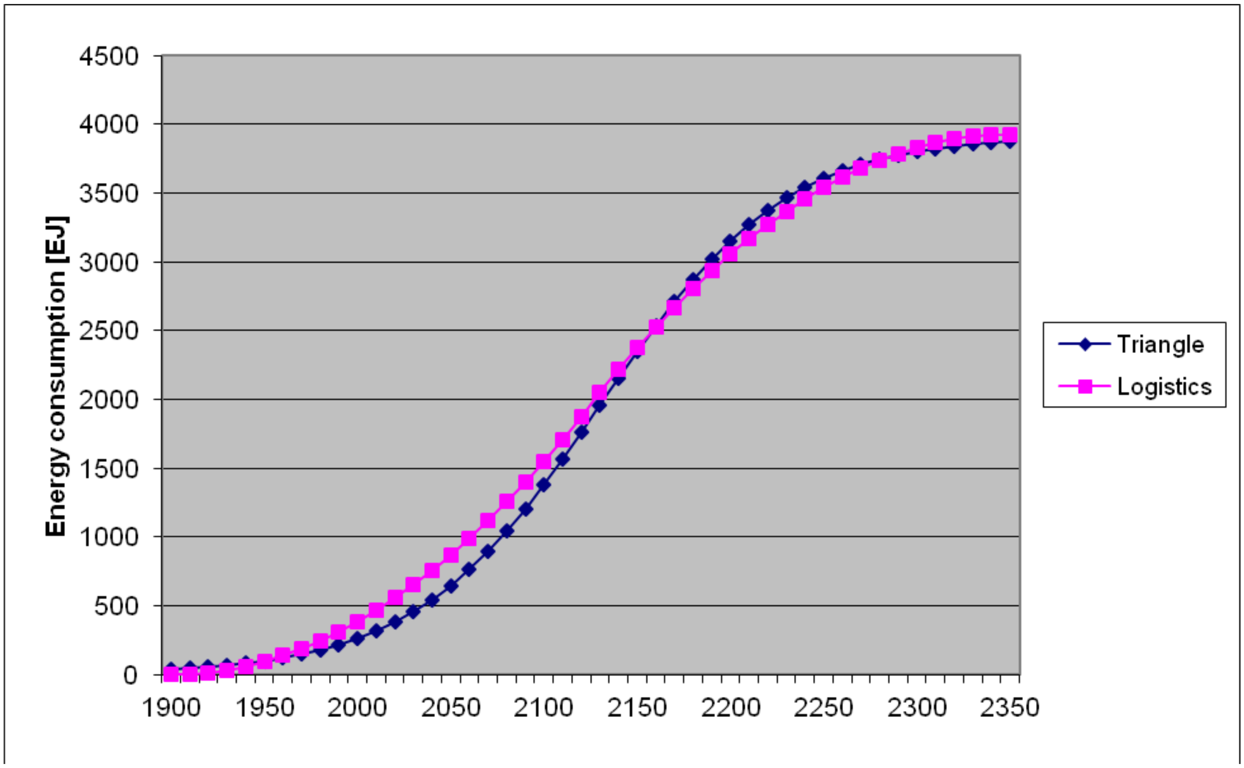
$$p(t) = 1 - 2\left(\frac{b-t}{b-a}\right)^2 \text{ if } \frac{a+b}{2} \le t \le b$$

(e) A table can be created to provide data for calculating projected energy use, using the logistics or triangle functions. The table below is truncated for brevity, and shows the beginning and end. The years 2300 to 2310 are shown in detail to illustrate the surpassing of the 97% penetration figure in the year 2303.

Year	Logistics			Triangle	
	t value	*p*(*t*)	Energy use (EJ)	*p*(*t*)	Energy use (EJ)
1900	0	0.0094	37.00	-	-
1910	10	0.0115	45.19	0.0010	3.88
1920	20	0.0141	55.17	0.0040	15.51
1930	30	0.0171	67.32	0.0089	34.90
1940	40	0.0209	82.08	0.0158	62.04
1950	50	0.0255	100.00	0.0247	96.94
1960	60	0.0310	121.70	0.0356	139.60
2300	400	0.9686	3803	0.9753	3829
2301	401	0.9692	3805	0.9763	3833
2302	402	0.9698	3808	0.9772	3837
2303	403	0.9704	3810	0.9782	3841
2304	404	0.9710	3812	0.9791	3844
2305	405	0.9715	3814	0.9800	3848
2306	406	0.9721	3817	0.9809	3851
2307	407	0.9726	3819	0.9817	3854
2308	408	0.9732	3821	0.9826	3858
2309	409	0.9737	3823	0.9834	3861
2310	410	0.9742	3825	0.9842	3864

2350	450.0000	0.9883	3880	1.0000	3926

From the values in the complete table, the following figure is created:



Again, note that a choice of *b* in the vicinity of but not equal to *b* = 2350 would only slightly change the shape of the triangle curve and the good agreement with the logistics curve would remain.

(f) Discussion question: Although the answer is open ended, there would be advantages of choosing a European or east Asian country with per capita consumption on the order of ~200 GJ/capita. The world would arrive at the steady state for energy consumption sooner, and there would be less burden in terms of total energy input needed, extent of energy infrastructure, burden on the environment, etc.

Problem 3-5.

Multi-criteria decision making: A construction firm that is building a passenger transportation terminal facility is evaluating two materials to choose the one that has the least effect on the environment. The alternatives are a traditional "natural" material and a more recently developed "synthetic" alternative. Energy consumption in manufacturing is used as one criterion, as are the effect of the material on natural resource depletion and on indoor air quality, in the form of off-gassing of the material into the indoor space. Cost per unit is included in the analysis as well. The scoring for each criterion is calculated by assigning the inferior material a score of 100, and then assigning the superior material a score that is the ratio of the superior to inferior raw score multiplied by 100. For example, if material A emits half as much of a pollutant as material B, then material B scores 100 and material A scores 50. Using the data given below, choose the material that has the overall lower weighted score. (Note that numbers are provided only to illustrate the technique and do not indicate the true relative worth of synthetic or natural materials.)

Data for exercise 3-5:

	Units	Natural	Synthetic	Weight
Energy	MJ/unit	0.22	0.19	0.24
Natural resource	kg/unit	1000	890	0.15
Indoor AQ	gram/unit	0.1	0.9	0.21
Cost	$/unit	$1.40	$1.10	0.40

Solution.

We will use energy as an example for the calculation, and then calculate all other scores following the same pattern. From the data, the natural material has higher energy consumption per unit and thus earns a score of 100 in this category. The synthetic material then earns the following:

$$Score = (100)\frac{0.19}{0.22} = 86$$

Similar computations lead to the following: for natural resource use, natural = 100, synthetic = 89; for indoor air quality, synthetic = 100, natural = 11; for cost, natural = 100, synthetic = 79.

Given the weights of the four categories, we can calculate the overall score of natural as below:

$$0.24(100) + 0.15(100) + 0.21(11) + 0.4(100) = 81$$

Similarly, we can calculate the overall score of synthetic as below:

$$0.24(86) + 0.15(89) + 0.21(100) + 0.4(79) = 87$$

Thus, natural is preferred to synthetic. Note that the wide difference in off-gassing has an important impact.

Problem 3-6.

Reconstruct the values for the Kaya identity in Table 3-4. (a) Calculate the three ratios on the right hand side (RHS) of the Kaya equation for each of the three countries, in units of $kgCO_2$/GJ, GJ/$GDP, and $GDP/person, respectively. Note that 10^9 GJ = 1 EJ, i.e., 1 billion gigajoules = 1 exajoule. (b) For each country, verify mathematically that when the three factors are multiplied together with the population, the CO_2 emissions rate results, i.e., the identity holds. A slight discrepancy due to rounding is fine. (c) Discussion: for each country, explain which factors on the RHS of the equation the national leaders of that country might be able to change to reduce CO_2 emissions, which factors might actually increase and thereby aggravate CO_2 emissions, and which factors are likely to remain the same.

Solution.

Part (a): Values are calculated for China as an example, and can be repeated for the other countries. GDP/P = $2241 tril./1.299 bil. = $1725/person. E/GDP = 63 tril. MJ/$2.241 tril. = 28 MJ/$GDP = 0.028

GJ/\$GDP. CO_2/E = 4.706 tril.kg/28 MJ = 0.075 kgCO_2/MJ = 75 kgCO_2/GJ. Repeating for the other countries, the following table of values arises, with P in units of millions, GDP/P in \$/cap, E/GDP in MJ/\$, and CO_2/E in kgCO_2/GJ:

	P	GDP/P	E/GDP	CO2/E	CO2
China	1299	1725	28	75	4707
India	1065	740	21	68	1113
US	293	42846	8.4	56	5912

Part (b): For China,

$$CO_2 emission = \frac{1299 \times 1725 \times 28 \times 75}{1{,}000{,}000} = 4705$$

For India,

$$CO_2 emission = \frac{1065 \times 740 \times 21 \times 68}{1{,}000{,}000} = 1125$$

For U.S.,

$$CO_2 emission = \frac{293 \times 42846 \times 8.4 \times 56}{1{,}000{,}000} = 5905$$

Part (c): For China, population might increase only very slowly in future years due to the one-child policy. E/GDP and CO_2/E might decline as part of larger efforts to reduce CO_2 emissions, and GDP/P might actually increase as wealth grows. For India, E/GDP and CO_2/E might decrease to reduce CO_2 emissions. GDP/P and population are likely to increase. For U.S., E/GDP, CO_2/E might be able to change to reduce CO_2 emissions. Population is likely to increase due to high demand for immigration to the U.S. The U.S. is already a wealthy country, so GDP/P may stay more or less constant, or grow slightly.

Problem 3-7.

We have a long-term need as a society to reduce the level of CO_2 emissions, and in this exercise you will consider a calculation based on total energy consumption in an economy. The underlying equation is called the "Kaya equation" (named after the Japanese economist Yoichi Kaya), or alternatively the Kaya identity, and has the following form:

$$CO_2 Emissions = \left(\frac{Emissions}{Energy}\right)\left(\frac{Energy}{GDP}\right)\left(\frac{GDP}{Persons}\right)Persons$$

The particular focus of the problem is China, India, and the U.S., in the year 2011. Below is found a table with the data needed to calculate the factors for the Kaya equation for each country. For comparison, the same values for the year 2004 appear in problem 3-6. Note that between 2004 and 2011 values have changed, China in particular is quite different in 2011.

Data table for 2011:

Country	P	GDP	E	CO_2
	[Million]	[Bill.$]	[EJ]	[Bill.kg]
China	1344	7322	115.6	8715
India	1241	1873	24.9	1726
U.S.	311.8	15534	102.9	5491

a. Calculate the three ratios on the right hand side of the Kaya equation for each of the three countries, in units of $kgCO_2$/GJ, GJ/$GDP, and $GDP/person, respectively. Note that 10^9 GJ = 1 EJ, i.e., 1 billion gigajoules = 1 exajoule.

<u>Solution.</u>

The problem can be solved using a spreadsheet and creating a table with the factors in columns and a row for each country. The result is the following:

Country:	GDP/P	E/GDP	CO_2/E
	[$/cap]	GJ/$GDP	kg/GJ
China	5448	0.0158	75.4
India	1509	0.0133	69.3
US	49,820	0.0066	53.4

b. For each country, verify mathematically that when the three factors are multiplied together with the population, the CO_2 emission rate results, i.e., the identity holds. A slight discrepancy is fine.

Solution.

Calculation for China, India, and U.S. are as follows:

$$E_{China} = (1344M)\left(\frac{\$5448GDP}{pers}\right)\left(\frac{0.0158GJ}{\$GDP}\right)\left(\frac{75.4kgCO_2}{GJ}\right) = 8.72Bil.kg \cong 8.715Bil.kg$$

$$E_{India} = (1241M)\left(\frac{\$1509GDP}{pers}\right)\left(\frac{0.0133GJ}{\$GDP}\right)\left(\frac{69.3kgCO_2}{GJ}\right) = 1.73Bil.kg \cong 1.726Bil.kg$$

$$E_{USA} = (311.8M)\left(\frac{\$49820GDP}{pers}\right)\left(\frac{0.0066GJ}{\$GDP}\right)\left(\frac{53.4kgCO_2}{GJ}\right) = 5.47Bil.kg \cong 5.491Bil.kg$$

c. Discussion: For each country, explain which factors on the RHS of the equation the national leaders of that country might be able to change to reduce CO_2 emissions, which factors might actually increase and thereby aggravate CO_2 emissions, and which factors are likely to remain the same.

Solution.

The purpose of this part of the question is to have you think about the implications of the Kaya identity, so there is no one right answer—a variety of answers are possible. For population, you might mention that it is growing in India and USA, the latter because of immigration as well as birth rates exceeding death rates. In China, the "one-child policy" may work to dampen population growth. Both China and India could be expected to have increasing GDP per capita, since the focus of their economies is to improve standard of living by increasing wealth. In the U.S., GDP per capita might continue to grow, but it may grow only slowly if at all, since it is already a wealthy country. The E/GDP ratio for China and India compared to USA shows that they are relatively inefficient in this measure, so it could be expected that they would improve as they modernize. The U.S. is already quite efficient at 6.6 GJ/$GDP, so it might improve some more, but perhaps relatively modestly if at all. Instead, all three countries would be expected to reduce CO_2 per GJ, because of the pressure of the need to do something about climate change, especially in the form of increasing the share of low- or zero-carbon energy sources, such as renewables, nuclear, or fossil with carbon capture and sequestration (CCS).

Problem 3-8.

A region has two types of motorcycles: (1) full size, called "motorcycles" hereafter, and (2) motor scooters, called "scooters" hereafter. Vehicle kilometers (vkm) and average fuel intensity (L/km) for the years 2005 and 2010 are provided in the table below:

Metric	Activity [million vkm]		Intensity [L/km]	
Year	2005	2010	2005	2010
Motorcycle	36	25	0.059	0.052
Scooter	19	27	0.024	0.021

Part (a): Calculate the actual and trended fuel consumption in millions of liters in 2010.

Part (b): Use Divisia analysis to calculate the structure term and intensity terms that explain the difference between actual and trended energy consumption in 2010. Report both structure and intensity terms in millions of liters.

Solution.

Part (a): The following table shows how the amount of fuel consumed by both modes in 2010 was obtained.

Year	Activity (vkm)	Fuel (liter)	Intensity (liter/km)
2005	55M	2.58M	0.04691
2010	52M	1.867M	0.0359

The intensity term for 2005 was calculated as follows:

$$Intensity = \frac{(36)(0.059)+(19)(0.024)}{36+19} = 0.04691\_L/km$$

The intensity term for 2010 can be calculated in the same way. The actual fuel consumption in 2010 is therefore as below:

$$Actual\_Fuel\_Consumption = (52E+6)vkm \times (0.0359)L/km = 1.867E+6L$$

The trended fuel consumption for 2010 is calculated from 2005 intensity and 2010 activity.

$$Trended\_Fuel\_Consumption = (52E+6)vkm \times (0.04691)L/km = 2.44E+6L$$

Part (b): The original table with activity and intensity given in the problem statement is as below:

Metric:	Activity [million vkm]		Intensity [L/km]	
Year	2005	2010	2005	2010
Motorcycle	36	25	0.059	0.052
Scooter	19	27	0.024	0.021

The first step in calculating the structure and intensity terms is to determine the amount of fuel consumed by each source in each year. The following table displays the amount of fuel consumed by the scooter and the motorcycle in 2005 and 2010, as well as their combined amount of fuel consumed in each year.

Table: Fuel Consumption (liters)

Year	2005	2010
Motorcycle	2.124M	1.3M

Scooter	0.456M	0.567M
Combined	2.58M	1.867M

The number of combined vehicle kilometers travelled in 2005 was 55 million (36M+19M).

The number of combined vehicle kilometers travelled in 2010 was 52 million (25M+27M).

Given the total number of vkm per year we can now calculate the percent share for each activity source. The table below provides the share for each activity source for each year:

Share	2005	2010
Motorcycle	65.45%	48.08%
Scooter	34.55%	51.92%

For example, $2005\_Motorcycle\_Share = (36E+6)vkm \div (55E+6)vkm = 65.45\%$)

Now, terms for the Divisia analysis are then generated from the given data according to the presentation in Chap. 2 in the book. The key equation is the following:

$$\Delta e_t = \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2} + (s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$
$$= \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2}\right] + \sum_{i=1}^{n}\left[(s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$

To calculate the structure and intensity terms implied by this equation, it is necessary to calculate using a spreadsheet a table of input values for each of the activity sources.

For the Motorcycle:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$(e_t + e_{t-1})/2$	$(s_t + s_{t-1})/2$
2005	0.059	65.45%				
2010	0.052	48.08%	−0.007	−17.37%	0.0555	56.8%

For the Scooter:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$(e_t + e_{t-1})/2$	$(s_t + s_{t-1})/2$
2005	0.024	34.55%				
2010	0.021	51.92%	*−0.003*	17.37%	0.0225	43.2%

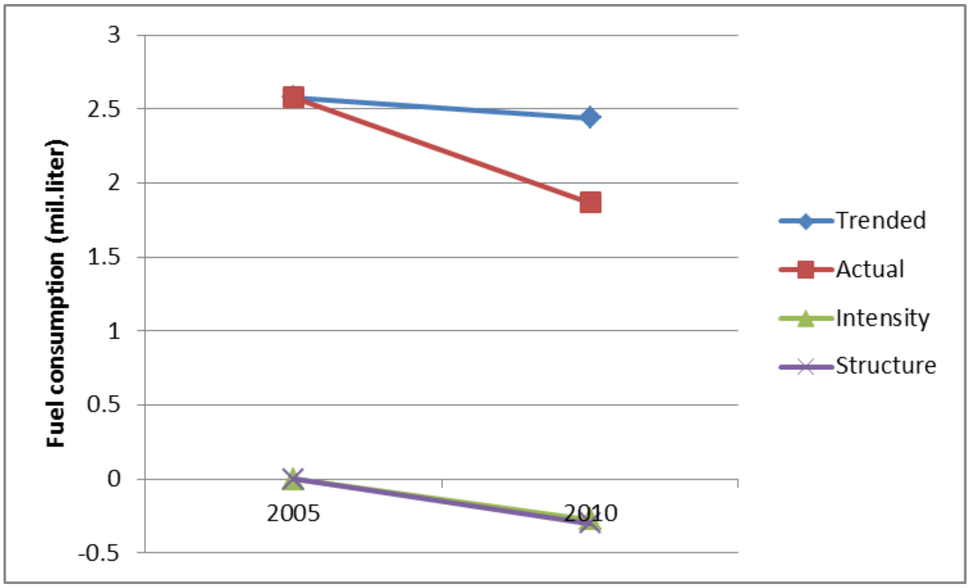
Using the above equation, we can now calculate the individual terms needed for the Divisia analysis, including the intensity and structure terms for the motorcycle and scooter modes for the year 2010.

Year	Intensity (L/km)		Structure (L/km)	
	Motorcycle	Scooter	Motorcycle	Scooter
2005	0	0	0	0
2010	*−0.0040*	*−0.0013*	−0.0096	0.0039
Combined 2010	*−0.0053*		−0.0057	

The "combined" row shows the contributions of the intensity and structure terms in the Divisia analysis. These two terms should be multiplied by the combined number of vkm in 2010 (52 million) to see how to account for the difference between the trended fuel consumption (439 million liters) and the actual fuel consumption in 2010 (1.87 million liters). The table below shows the contributions of the intensity and Structure terms. The table is in units of millions of liters.

Trended	2.439
Intensity	−0.274
Structure	−0.298
Sum	1.867
Actual	1.867
Difference	-

Hence, the structure term is −0.298 million liters and the intensity term is −0.274 million liters. Although not required in the problem, the figure below illustrates the Divisia analysis graphically:


Fuel consumption (mil.liter)
3
2.5
2
1.5
1
0.5
0
-0.5
2005
2010
Trended
Actual
Intensity
Structure

Chapter 4 Individual Choices and Transportation Demand

Problem 4-1.

Preference essentials:

a. The conditional indirect utility function of a nonchosen alternative cannot be higher than the conditional indirect utility function of the actually chosen alternative. Explain.
b. Alternatives of discrete choice models need to be mutually exclusive. Does this mean that behavior such as choosing to use a combination of modes (park and ride) cannot be modeled using discrete choice?
c. In a binary choice context, if a consumer is indifferent between the two alternatives, how would the choice probabilities look like?

Solution.

a. If individuals are utility maximizers, then the chosen alternative $\mathrm{j_i}$ is such that $\mathrm{j_i} = \arg\max_{\mathrm{j}} \mathrm{v_j}\left(\mathbf{q}_{\mathrm{ij}}, \mathrm{I_i} - \mathrm{p_{ij}}\right)$, which is equivalent to $\mathrm{v_{j_i}}\left(\mathbf{q}_{\mathrm{ij_i}}, \mathrm{I_i} - \mathrm{p_{ij_i}}\right) - \mathrm{v_j}\left(\mathbf{q}_{\mathrm{ij}}, \mathrm{I_i} - \mathrm{p_{ij}}\right) \geq 0, \forall \mathrm{j} \neq \mathrm{j_i}$. This rules out the possibility that, for any nonchosen $\mathrm{j} \neq \mathrm{j_i}$, $\mathrm{v_j}\left(\mathbf{q}_{\mathrm{ij}}, \mathrm{I_i} - \mathrm{p_{ij}}\right)$ could be greater than $\mathrm{v_{j_i}}\left(\mathbf{q}_{\mathrm{ij_i}}, \mathrm{I_i} - \mathrm{p_{ij_i}}\right)$.
b. Mode combinations can be modeled using discrete choice. A combination of mode can be modeled as an additional, mutually exclusive alternative. For example, an individual is modeled to choose among {car, subway, park and ride}.
c. In a binary choice context, there are two available alternatives. If individual i is indifferent between alternatives 1 and 2, then $\mathrm{P_{i1}} = \mathrm{P_{i2}} = 0.5$. Note that this result is equivalent as saying that the individual is tossing a coin to decide which alternative to choose.

Problem 4-2.

Show that the maximum likelihood estimator of $\boldsymbol{\beta}$ in the standard regression problem $y_i = \mathbf{x}_i'\boldsymbol{\beta} + \varepsilon_i, \varepsilon_i \sim \mathcal{N}(0, \sigma^2)$, or $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$ in matrix form, is $\widehat{\boldsymbol{\beta}}_{\mathrm{MLE}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$.

Solution.

Note that $y_i \sim \mathcal{N}(\mathbf{x}_i'\boldsymbol{\beta}, \sigma^2)$, as a result the likelihood function takes the following form:

$$\ell(\boldsymbol{\beta}, \sigma^2; \mathbf{y}|\mathbf{X}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - \mathbf{x}_i'\boldsymbol{\beta})^2}{2\sigma^2}\right),$$

and the loglikelihood can be written in matrix form as below:

$$\mathcal{L}(\boldsymbol{\beta}, \sigma^2; \mathbf{y}|\mathbf{X}) = -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})$$

The maximum likelihood estimate can be found by maximizing the loglikelihood function $\max_{\boldsymbol{\beta},\sigma^2} \mathcal{L}(\boldsymbol{\beta}, \sigma^2; \mathbf{y}|\mathbf{X})$. If we consider the first order condition (likelihood equation) with respect to $\boldsymbol{\beta}$, we find the following:

$$\frac{\partial \mathcal{L}(\widehat{\boldsymbol{\beta}}_{\text{MLE}}, \sigma^2; \mathbf{y}|\mathbf{X})}{\partial \boldsymbol{\beta}} = \frac{\mathbf{X}'(\mathbf{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}_{\text{MLE}})}{\sigma^2} = 0 \Rightarrow \widehat{\boldsymbol{\beta}}_{\text{MLE}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \ \blacksquare$$

Problem 4-3.

Consider $V_{\text{in}} = \beta_p p_{\text{in}} + \beta_q q_{\text{in}}$, where p_{in} is price of alternative *i* as experienced by individual *n*, and q_{in} is a qualitative attribute.

a. Suppose that we manage to find the prices as a function of q_{in} – , i.e., $p_{\text{in}} = p(q_{\text{in}})$ – that make the utility level constant, that is, $\beta_p p(q_{\text{in}}) + \beta_q q_{\text{in}} = V^0$, where V^0 is constant. What do you think is the economic interpretation of the pair $(p(q_{\text{in}}), q_{\text{in}})$?
b. Differentiate the expression $\beta_p p(q_{\text{in}}) + \beta_q q_{\text{in}} = V^0$ with respect to q_{in} and derive an expression for $p'(q_{\text{in}})$. What is the economic interpretation of your result?
c. Repeat (b) but consider now a nonlinear function for the utility $V_{\text{in}} = V(p(q_{\text{in}}), q_{\text{in}})$.

Solution.

a. The pair $[\text{p}(\text{q}_{\text{in}}), \text{q}_{\text{in}}]$ represents an indifference curve, i.e., the combination of prices and attributes that make utility constant. Because we assume compensatory behavior, price responds to changes in the attributes in such a way so that the consumer's choice does not change.
b.

$$\frac{\partial[\beta_p p(q_{\text{in}}) + \beta_q q_{\text{in}}]}{\partial q_{\text{in}}} = \frac{\partial V^0}{\partial q_{\text{in}}}$$

$$\beta_p \frac{\partial p(q_{\text{in}})}{\partial q_{\text{in}}} + \beta_q = 0$$

$$\frac{\partial p(q_{\text{in}})}{\partial q_{\text{in}}} = p'_{\text{in}}(q_{\text{in}}) = -\frac{\beta_q}{\beta_p}$$

The economic interpretation of this result is that $\text{p}'_{\text{in}}(\text{q}_{\text{in}})$ is the marginal rate of substitution between price and the qualitative attribute. This is the same as saying that variations in price to keep utility constant is the **willingness to pay** for changes in q_{in} that improve utility (such as willingness to pay for emission reductions or for performance improvements).

c.

$$V(p(q_{\text{in}}), q_{\text{in}}) = V^0$$

$$\frac{\partial V(p(q_{\text{in}}), q_{\text{in}})}{\partial q_{\text{in}}} = 0$$

Using implicit differentiation:

$$\frac{\partial V_{\text{in}}}{\partial q_{\text{in}}} + \frac{\partial V_{\text{in}}}{\partial p_{\text{in}}} \frac{\partial p_{\text{in}}}{\partial q_{\text{in}}} = 0$$

$$p'_{\text{in}}(q_{\text{in}}) = -\frac{\partial V_{\text{in}}/\partial q_{\text{in}}}{\partial V_{\text{in}}/\partial p_{\text{in}}}.$$

Problem 4-4.

Look at the point estimates shown in Example 4-4.

a. Comment on the sign and statistical significance of the parameters.
b. Write the general expression for the utility function of each of the four alternatives of the model: SGV, AFV, HEV, HFC.
c. Write the general expression for the choice probability of each of the four alternatives of the model: SGV, AFV, HEV, HFC.
d. Suppose that an individual faces a choice situation where the attributes take the following values:

Attribute	SGV	AFV	HEV	HFC
Purchase price ($10,000)	2.1	2.2	3.2	4.2
Fuel cost ($100/month)	1.3	1.5	0.7	0.9
Fuel network (density ratio, from 0: no stations to 1: current network)	1.0	0.6	1.0	0.2
HOV access (0: No, 1: Yes)	0.0	0.0	0.0	1.0
Power (ratio compared to current car)	1.0	0.9	1.0	0.8

Calculate the choice probabilities for this individual.

e. Recalculate the choice probabilities of part (d) if hydrogen fuel cars receive a subsidy of $2500.

Solution.

a. The parameters for the two price components (purchase price and fuel cost) have negative signs. This negative marginal utility indicates that the consumer would prefer less a vehicle with a higher cost (everything else held constant). The opposite effect can be seen for fuel availability, access to an express lane, and power, which appear as desirable attributes with positive marginal utilities. All the parameters are significantly different from zero at the 5% confidence level. This can be seen in the following table:

Attribute	Point Estimate	Standard Error	Asymptotic t-stat ($\beta/s.e$)
CNG Constant (β_{AFV})	−4.500	0.661	−6.81
HEV Constant (β_{HEV})	−1.380	0.633	−2.18
HFC Constant (β_{HFC})	−2.100	0.644	−3.26
Price (β_{price})(10,000CAD$)	−0.856	0.210	−4.08
Fuel Cost (β_{fuelcost})(100CAD$/month)	−0.826	0.198	−4.17
Fuel availability (β_{avail}) (%)	1.360	0.186	7.31

Express lane access (β_{express})	0.156	0.068	2.29
Power (β_{pow}) (%)	2.700	0.655	4.12

Note that all values for the asymptotic t-statistic are greater than 1.96, which is the critical value at the 5% confidence level (meaning that we can reject the hypothesis that individual parameters are equal to zero).

b.

$$U_{i,SGV} = \beta_{price}price_{i,SGV} + \beta_{fuel\ cost}fuel\ cost_{i,SGV} + \beta_{avail}avail_{i,SGV} + \beta_{express}express_{i,SGV} + \beta_{pow}pow_{i,SGV} + \varepsilon_{i,SGV} = V_{i,SGV} + \varepsilon_{i,SGV}$$

$$U_{i,AFV} = \beta_{AFV} + \beta_{price}price_{i,AFV} + \beta_{fuel\ cost}fuel\ cost_{i,AFV} + \beta_{avail}avail_{i,AFV} + \beta_{express}express_{i,AFV} + \beta_{pow}pow_{i,AFV} + \varepsilon_{i,AFV} = V_{i,AFV} + \varepsilon_{i,AFV}$$

$$U_{i,HEV} = \beta_{HEV} + \beta_{price}price_{i,HEV} + \beta_{fuel\ cost}fuel\ cost_{i,HEV} + \beta_{avail}avail_{i,HEV} + \beta_{express}express_{i,HEV} + \beta_{pow}pow_{i,HEV} + \varepsilon_{i,HEV} = V_{i,HEV} + \varepsilon_{i,HEV}$$

$$U_{i,HFC} = \beta_{HFC} + \beta_{price}price_{i,HFC} + \beta_{fuel\ cost}fuel\ cost_{i,HFC} + \beta_{avail}avail_{i,HFC} + \beta_{express}express_{i,HFC} + \beta_{pow}pow_{i,HFC} + \varepsilon_{i,HFC} = V_{i,HFC} + \varepsilon_{i,HFC}$$

c.

$$P_{i,SGV} = \frac{\exp(V_{i,SGV})}{\exp(V_{i,SGV}) + \exp(V_{i,AFV}) + \exp(V_{i,HEV}) + \exp(V_{i,HFC})}$$

$$P_{i,AFV} = \frac{\exp(V_{i,AFV})}{\exp(V_{i,SGV}) + \exp(V_{i,AFV}) + \exp(V_{i,HEV}) + \exp(V_{i,HFC})}$$

$$P_{i,HEV} = \frac{\exp(V_{i,HEV})}{\exp(V_{i,SGV}) + \exp(V_{i,AFV}) + \exp(V_{i,HEV}) + \exp(V_{i,HFC})}$$

$$P_{i,HFC} = \frac{\exp(V_{i,HFC})}{\exp(V_{i,SGV}) + \exp(V_{i,AFV}) + \exp(V_{i,HEV}) + \exp(V_{i,HFC})}$$

d.

$$P_{i,SGV} = \frac{3.2825}{3.2825 + 0.0126 + 0.5287 + 0.0213} = 0.8537$$

$$P_{i,AFV} = \frac{0.0126}{3.2825 + 0.0126 + 0.5287 + 0.0213} = 0.0033$$

$$P_{i,HEV} = \frac{0.5287}{3.2825 + 0.0126 + 0.5287 + 0.0213} = 0.1375$$

$$P_{i,HFC} = \frac{0.0213}{3.2825 + 0.0126 + 0.5287 + 0.0213} = 0.0055$$

According to the model, the individual has a 13.75% probability of choosing a hybrid. Note that $\mathrm{P_{i,SGV}} + \mathrm{P_{i,AFV}} + \mathrm{P_{i,HEV}} + \mathrm{P_{i,HFC}} = 1$.

Problem 4-5.

Consider a conditional logit model with a linear specification of utility.

a. Show that the likelihood equation takes the form as below:

$$\frac{\partial \mathcal{L}(\widehat{\boldsymbol{\beta}}; \mathbf{y}|\mathbf{X})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{N} \sum_{j} (y_{ij} - P_{ij})\mathbf{x}_{\text{in}} = 0$$

b. Show that the choice elasticities are as below:

$$E_{P_{ij},x_{kij}} = \beta_k x_{kij}(1 - P_{ij})$$

$$E_{P_{il},x_{kij}} = -\beta_k x_{kij} P_{ij}$$

c. Use the cross choice elasticity $E_{P_{il},x_{kij}} = -\beta_k x_{kij} P_{ij}$ to explain that the conditional logit imposes proportional substitution.

Solution.

a. We know that $P_{ij} = \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}) / \sum_l \exp(\mathbf{x}_{il}'\boldsymbol{\beta})$ *and that the loglikelihood is as below:*

$$\mathcal{L}(\widehat{\boldsymbol{\beta}}; \mathbf{y}|\mathbf{X}) = \sum_{i=1}^{N} \sum_{j} y_{ij} \ln P_{ij}$$

From the first-order condition:

$$\frac{\partial \mathcal{L}(\widehat{\boldsymbol{\beta}}; \mathbf{y}|\mathbf{X})}{\partial \boldsymbol{\beta}} = \frac{\partial}{\partial \boldsymbol{\beta}}\left[\sum_{i=1}^{N} \sum_{j} y_{ij}\mathbf{x}_{ij}'\boldsymbol{\beta} - \sum_{i=1}^{N} \sum_{j} y_{ij} \ln \sum_{l} \exp(\mathbf{x}_{il}'\boldsymbol{\beta})\right]$$

$$= \sum_{i=1}^{N} \sum_{j} y_{ij}\mathbf{x}_{ij} - \sum_{i=1}^{N} \sum_{j} y_{ij} \frac{1}{\sum_l \exp(\mathbf{x}_{il}'\boldsymbol{\beta})} \sum_{l} \exp(\mathbf{x}_{il}'\boldsymbol{\beta})\mathbf{x}_{il}$$

$$= \sum_{i=1}^{N} \sum_{j} y_{ij}\mathbf{x}_{ij} - \sum_{i=1}^{N} \sum_{j} y_{ij} \sum_{l} P_{il}\mathbf{x}_{il}$$

$$= \sum_{i=1}^{N} \sum_{j} y_{ij}\mathbf{x}_{ij} - \sum_{i=1}^{N} \sum_{l} P_{il}\mathbf{x}_{il} \sum_{j} y_{ij}$$

$$= \sum_{i=1}^{N}\sum_{j} y_{ij}\mathbf{x}_{ij} - \sum_{i=1}^{N}\sum_{l} P_{il}\mathbf{x}_{il}$$

$$= \sum_{i=1}^{N}\sum_{j} (y_{ij} - P_{ij})\mathbf{x}_{\text{in}} = 0 \ \blacksquare$$

b.

$$E_{P_{ij},x_{kij}} = \frac{\partial P_{ij}}{\partial x_{kij}}\frac{x_{kij}}{P_{ij}} = \frac{x_{kij}}{P_{ij}}\left[\frac{\partial \exp(\mathbf{x}'_{ij}\boldsymbol{\beta}) \,/\, \sum_l \exp(\mathbf{x}'_{il}\boldsymbol{\beta})}{\partial x_{kij}}\right]$$

$$= \frac{x_{kij}}{P_{ij}}\left[\beta_k \frac{\exp(\mathbf{x}'_{ij}\boldsymbol{\beta})}{\sum_l \exp(\mathbf{x}'_{il}\boldsymbol{\beta})} - \beta_k \frac{\exp(\mathbf{x}'_{ij}\boldsymbol{\beta})}{(\sum_l \exp(\mathbf{x}'_{il}\boldsymbol{\beta}))^2}\exp(\mathbf{x}'_{ij}\boldsymbol{\beta})\right]$$

$$= \frac{x_{kij}}{P_{ij}}\beta_k\left[P_{ij}(1 - P_{ij})\right]$$

$$= \beta_k x_{kij}(1 - P_{ij}) \blacksquare$$

$$E_{P_{il},x_{kij}} = \frac{\partial P_{il}}{\partial x_{kij}}\frac{x_{kij}}{P_{il}} = \frac{x_{kij}}{P_{il}}\left[\frac{\partial \exp(\mathbf{x}'_{il}\boldsymbol{\beta}) \,/\, \sum_h \exp(\mathbf{x}'_{ih}\boldsymbol{\beta})}{\partial x_{kij}}\right]$$

$$= \frac{x_{kij}}{P_{il}}\left[-\beta_k \frac{\exp(\mathbf{x}'_{il}\boldsymbol{\beta})}{\left(\sum_h \exp(\mathbf{x}'_{ih}\boldsymbol{\beta})\right)^2}\exp(\mathbf{x}'_{ij}\boldsymbol{\beta})\right]$$

$$= \frac{x_{kij}}{P_{il}}\left[-\beta_k P_{il} P_{ij}\right]$$

$$E_{P_{il},x_{kij}} = -\beta_k x_{kij} P_{ij} \ \blacksquare$$

c. From $\mathrm{E}_{\mathrm{P_{il}},\mathrm{x_{kij}}} = -\beta_k \mathrm{x_{kij}} \mathrm{P_{ij}}$ we can see that an improvement in alternative j (due to a percent change in $\mathrm{x_{kij}}$) changes the choice probability of all other alternatives by the same percentage (substitution among alternatives is constant).

Problem 4-6.

Consider individuals choosing among a compact sedan (1), a small SUV (2), a compact hybrid (3), and a hybrid SUV (4).

a. Derive the choice probability of a nested logit model with a hypothesized nesting structure with two sets based on vehicle class. The first nest contains compact vehicles ($\pi_{\text{Compact}} = \{\text{compact sedan, compact hybrid}\}$), while the second nest contains SUVs ($\pi_{\text{SUV}} = \{\text{small SUV, hybrid SUV}\}$)
b. Propose a statistical test to validate the proposed nested structure.
c. Derive an expression of the consumer surplus.

d. How would you determine which model is best, the one with the nesting structure based on vehicle class of part (a), or the one with the nesting structure based on technology of Example 4-5.

Solution.

a.

$$P_{i,\text{compact hybrid}} = P_{i,\text{п}_{\text{compact}}} P_{i,\text{compact hybrid}|\text{п}_{\text{compact}}}$$

$$P_{i,\text{compact hybrid}|\text{п}_{\text{compact}}} = \frac{\exp(\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}})}{\exp\left(\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}\right) + \exp(\mathbf{x}'_{i,\text{compact sedan}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}})}.$$

$$P_{i,\text{п}_{\text{compact}}} = \frac{\exp(\text{EMU}_{\text{п}_{\text{compact}}})}{\exp\left(\text{EMU}_{\text{п}_{\text{compact}}}\right) + \exp(\text{EMU}_{\text{п}_{\text{SUV}}})}$$

$$= \frac{\left(e^{\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}} + e^{\mathbf{x}'_{i,\text{compact sedan}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}}\right)^{\lambda_{\text{п}_{\text{compact}}}}}{\left(e^{\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}} + e^{\mathbf{x}'_{i,\text{compact sedan}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}}\right)^{\lambda_{\text{п}_{\text{compact}}}} + \left(e^{\mathbf{x}'_{i,\text{hybrid SUV}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{SUV}}}} + e^{\mathbf{x}'_{i,\text{small SUV}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{SUV}}}}\right)^{\lambda_{\text{п}_{\text{SUV}}}}}.$$

The joint probability is then:

$$P_{i,\text{compact hybrid}}$$
$$= \frac{\exp(\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}})\left(e^{\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}} + e^{\mathbf{x}'_{i,\text{compact sedan}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}}\right)^{\lambda_{\text{п}_{\text{compact}}}-1}}{\left(e^{\mathbf{x}'_{i,\text{compact hybrid}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}} + e^{\mathbf{x}'_{i,\text{compact sedan}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{compact}}}}\right)^{\lambda_{\text{п}_{\text{compact}}}} + \left(e^{\mathbf{x}'_{i,\text{hybrid SUV}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{SUV}}}} + e^{\mathbf{x}'_{i,\text{small SUV}}\boldsymbol{\beta}/\lambda_{\text{п}_{\text{SUV}}}}\right)^{\lambda_{\text{п}_{\text{SUV}}}}}$$

b. We can test the nested logit structure against a conditional logit. If $\mathcal{L}_{\text{NL}}(\widehat{\boldsymbol{\beta}};\mathbf{y}|\mathbf{X}_\Delta)$ is the loglikelihood of the nested logit, and $\mathcal{L}_{\text{CL}}(\widehat{\boldsymbol{\beta}};\mathbf{y}|\mathbf{X}_\Delta)$ is the loglikelihood of the conditional logit, then we can perform the following likelihood ratio test:

$$\text{LR} = -2\left[\mathcal{L}_{CL}(\widehat{\boldsymbol{\beta}};\mathbf{y}|\mathbf{X}_\Delta) - \mathcal{L}_{NL}(\widehat{\boldsymbol{\beta}};\mathbf{y}|\mathbf{X}_\Delta)\right] \sim \chi^2_2$$

Since there are two restrictions ($\lambda_{\text{п}_{\text{compact}}} = 1, \lambda_{\text{п}_{\text{SUV}}} = 1$), then the chi square distribution has two degrees of freedom. If $\text{LR} > \chi^2_{2,\text{crit},5\%}$ then the conditional logit model can be rejected.

c.

$$\text{EMU}_{root} = \frac{1}{\alpha}\ln\left[\exp\left(\text{EMU}_{\text{п}_{\text{compact}}}\right) + \exp(\text{EMU}_{\text{п}_{\text{SUV}}})\right]$$

d. The two nested logit models cannot be written as a restricted version of each other. For this kind of problems, we could use the Bayesian information criterion. Because the two models in this case have exactly the same number of parameters, we can select the model with the largest likelihood.

Problem 4-7.

Consider the estimates of Example 4-15.

a. Interpret the signs of the marginal utilities. Are they reasonable?
b. "The likelihood of consumers buying low-emission vehicles increases when the appropriate refueling/recharging network increases." Comment on this assertion.
c. Calculate the willingness to pay for a car that produces 1 g/mi less of CO_2.
d. A Volkswagen Jetta produces 193 g/km of CO_2. The Prius produces 113 g/km. Based on the results of the model, how much more could cost the Prius just based on its environmental benefits.
e. Find online the average price of a new Jetta versus the average price of a new Prius. Do you think that the result you found in 4-7 (d) explains this actual price difference?

Solution.

a. The signs are reasonable. Positive signs are associated with beneficial attributes (engine power, network), meaning that an increase in this attribute leads to an increase in the utility. Conversely, negative signs are associated with the detrimental attributes (price, fuel costs, CO_2) that will lead to a decrease in utility as these attributes increase.

b. This statement is intuitively logical, namely as refueling stations for alternatively fueled vehicles become more common, people will be more likely to purchase said vehicles (thus the positive marginal utility). However, simply availability of service stations alone is not sufficient to determine if low-emission vehicles will become more popular and the relative importance of this factor (i.e,. the magnitude of the marginal utility), also playing a role is the refueling time. Thus, even if many stations are available but refueling takes 1 hour or more (as is the case with electric vehicles), low-emission vehicles may still be a very small fraction of the vehicle population.

c.

$$\text{WTP}_{\Delta CO2} = -\frac{\beta_{CO2}}{\beta_{price}} = -\frac{-0.0041}{-0.0369}$$
$$\text{WTP}_{\Delta CO2} = -0.1111[1000\ €/(g/km)] = -111.1[€/(g/km)]$$

Therefore, the data shows that people are willing to pay an additional €111.1 for a vehicle that reduces CO_2 emissions by 1 g/km.

d.

$$\Delta\text{price} = \text{WTP}_{\Delta CO2}\Delta CO2$$
$$\Delta\text{price} = -111.1[€/(g/km)] \times (113 - 193)[g/km]$$
$$\Delta\text{price} = €8900$$

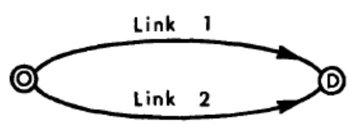
Therefore, a Prius can cost almost €9000 more than a Jetta based on lower CO_2 emissions alone.

From http://www.toyota.com/priusv/: The price of a new Prius V starts at $26,400.
From http://configurator.vw.com/ihdcc/configurator.html#10105/10501/58: The price of a new Jetta starts at $16,500.
The difference in price is <$10,000. Converting to euros (€1 = $1.40), this amount becomes <€7,150. Therefore, the price difference is close to, but not as large as, the difference predicted by the model. The discrepancy in the projections may be result of different markets, where consumers in Germany valued carbon reductions more greatly than in the U.S.. Other differences like the cost of the battery and location of manufacturing can also affect the retail price of the vehicles, but we do not have that data to draw more definitive conclusions.

Problem 4-8.

Note to instructor for Problems 4-8 and 4-9: This problem may require additional background for students on network equilibria. As a resource, you may wish to refer to Sheffi, Y, "Urban Transportation Networks." As of 6/10/14 this resource was available for free download at sheffi.mit.edu/sheffi_urban_trans_networks.pdf.

A network with nodes O for origin and D for destination and links 1 and 2 is shown below. The time requirement t_1 on link 1 is $t_1 = 2 + 0.03x_1$ and the time requirement t_2 on link 2 is $t_2 = 3 + 0.02x_2$, where x_1 and x_2 are the flow values allocated to links 1 and 2, respectively. Flow from O to D is 300 units (these could be vehicles, passengers, etc.). In terms of the equilibrium calculation, you have already established that in Iteration 1 the objective value is 1950 with all flow on link 1 and 1800 in Iteration 2 with all flow on link 2, so you can now proceed to Iteration 3 where you compute the final equilibrium values for x_1 and x_2.



a. [12 points] Find the values of x_1 and x_2 that meet the equilibrium flow conditions, the time required on each link a that flow level, and the objective value.

<u>Solution.</u>

Since some fraction of the 300 will go to 1 and the rest to 2, we can integrate across one function from 0 to λ and the other from 0 to (300 – λ), and then solve for the value of λ that minimizes the value of the objective:

$$\int_0^{\lambda} (2 + 0.03x)dx + \int_0^{300-\lambda} (3 + 0.02x)dx$$

$$= (2x + 0.015x^2)\Big|_0^{\lambda} + (3x + 0.01x^2)\Big|_0^{300-\lambda}$$

$$= 2\lambda + 0.015\lambda^2 + 3(300-\lambda) + 0.01(300-\lambda)^2$$
$$= 0.025\lambda^2 - 7\lambda + 1800$$
$$\frac{df}{d\lambda} = 0.05\lambda - 7 = 0 \rightarrow \lambda_{\min} = \frac{7}{0.05} = 140$$

Therefore the following values arise:

$$x_1 = \lambda = 140$$
$$x_2 = 160$$
$$t_1 = 2 + 0.03(140) = 6.2$$
$$t_2 = 3 + 0.02(160) = 6.2$$
$$Obj = 2(140) + 0.015(140)^2 + 3(160) + 0.01(160)^2 = 1310$$

Alternatively, you can set $t_1 = t_2$ and solve for the value of x_1 and x_2 where both paths have equal travel time. In a more complex network this method will not work, but it gives the correct answer here and is accepted for full credit. Solving for two unknowns with two equations:

$$2 + 0.03x_1 = 3 + 0.02x_2$$
$$x_1 + x_2 = 300$$
$$x_1 = 140$$
$$x_2 = 160$$
$$t_1 + t_2 = 6.2$$

Now, suppose in a different scenario we begin with no flow on the network and load it gradually by increments of 1, in other words, O → D demand is equal to 1, 2, 3, ….and so on.

b. [1 point] State which link the flow will use initially, link 1 or link 2?

Solution:

Link 1 (because it has a lower time value at x = 0, 1, …etc.)____

c. [2 points] At what value of O → D demand will the network begin to use both Links 1 and 2? Show your work.

Solution:

Solve for value of x_1 on Link 1 for which the travel time is exactly equal to travel time with x_2 = 0:

$$2 + 0.03x_1 = 3$$

$$x_1 = \frac{1}{0.03} = 33.3 =\sim 33$$

Thus at a flow of around 33 units, of alternatively 34 units, the flow will begin to be split between the two links.

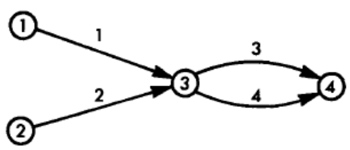
Problem 4-9.

Consider the following four-link, four-node network, see figure below. Times and demand levels are the following:

$t_1 = 3, t_2 = 2, t_3 = 2 + 0.02x_3, t_4 = 4 + 0.01x_4$

$q_{14} = 500, q_{24} = 300$

Hint: At each iteration, assume travel times based on no-load travel conditions. Your solution should therefore have three iterations.



Solution:

Iteration 1

For initial flow, use "zero volume" travel times which implies shortest path 1 to 3 using links 1 and 3 and shortest path 2 to 3 using links 2 and 3.

So, initial flow: x_1 = 500; x_2 = 300; x_3 = 800; x_4 = 0

Link	Vol	Time	Integral
1	500	3	1500
2	300	2	600
3	800	18	8000
4	0	4	0
		Z=	10100

Iteration 2

Travel times at initial flow: t_1 = 3; t_2 = 2; t_3 = 18; t_4 = 4 which implies shortest path 1 to 4 uses links 1 and 4; shortest path 2 to 4 uses links 2 and 4.

So, trial flow: x_1 = 500; x_2 = 300; x_3 = 0; x_4 = 800

Link	Vol	Time	Integral
1	500	3	1500
2	300	2	600
3	0	2	0
4	800	12	6400
		Z=	8500

Iteration 3

Solve the optimization problem:

$$\min \int_0^3 500dx + \int_0^2 300dx + \int_0^{800-800\lambda} (2+0.02x)dx + \int_0^{800\lambda} (4+0.01x)dx$$

$$\text{such that } 0 \le \lambda \le 1$$

Note that the first two integrals are constants, so we only need to minimize the last terms, which is as below:

$$\int_0^{800-800\lambda} (2+0.02x)dx + \int_0^{800\lambda} (4+0.01x)dx$$
$$= (2x + 0.01x^2)\Big|_0^{800-800\lambda} + (4x+0.005x^2)\Big|_0^{800\lambda}$$
$$= 2(800-800\lambda) + 0.01(800-800\lambda)^2 + 4(800\lambda) + 0.005(800\lambda)^2$$

By derivation over lambda (λ), we get $2(-800) + 0.02(800-800\lambda)(-800) + 800 \times 4 + 0.01 \times 800\lambda \times 800 = 0 \Rightarrow 19200\lambda = 11200 \Rightarrow \lambda = 0.583$

Then we get the flow for iteration 3. $x_3 = 800 - 800\lambda = 333, x_4 = 800\lambda = 467$

Link	Vol	Time	Integral
1	500	3	1500

2	300	2	600
3	333	8.66	1775
4	467	8.67	2958
		Z=	6833

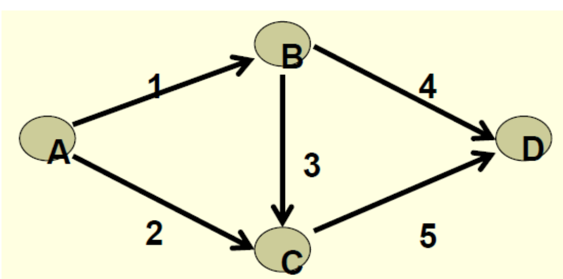
Problem 4-10.

Note to instructor: Since the book does not discuss the Stochastic Assignment Method, the following references are useful for background for students to be able to solve the problem:

- *Maher, M.J., 1992. SAM—a stochastic assignment model. Mathematics in Transport Planning and Control. Oxford University Press.*
- *Maher, M.J., Hughes, P.C., 1997. A probit-based stochastic user equilibrium assignment model. Transportation Research Part B 31, 341–345.*

The problem relies on an understanding of variance and covariance from probability and statistics, so some review of these terms may be advisable before assigning the problem.

Solve for flow levels in each arc in the following four-node, five-arc model using the Stochastic Assignment Method (SAM). Link average and variance values, covariance values and the required values from the standard normal table are given. Assume $Q_{AB} = 200, Q_{AC} = 150, Q_{AD} = 100$.



Link values:

J	$E(X_j)$	$V(X_j)$
1	3	0.25
2	4	1
3	2	0.64
4	3	0.36

5	2	0.16

Covariance values:

	Y_1	Y_2	Y_3	Y_4	Y_5
Y_1		0	0	0	0
Y_2	0		0	0	0
Y_3	0	0		0.25	0
Y_4	0	0	0.25		0.0575
Y_5	0	0	0	0.0575	

Forward pass: Scan and merge steps

1. Scan from node A

$E(Y_1) = E(T_A) + E(X_1) = 0 + 3 = 3$

$V(Y_1) = V(T_A) + V(X_1) = 0 + 0.25 = 0.25$

$E(Y_2) = E(T_A) + E(X_2) = 0 + 4 = 4$

$V(Y_2) = V(T_A) + V(X_2) = 0 + 1 = 1$

$Cov(Y_1, Y_2) = 0$

$L = \{1,2\}$

2. Merge at node B

$E(T_B) = E(Y_1) = 3$

$V(T_B) = V(Y_1) = 0.25$

$p_1 = 1$

$L = \{2\}$

3. Scan from node B

$E(Y_3) = E(T_B) + E(X_3) = 3 + 2 = 5$

$V(Y_3) = V(T_B) + V(X_3) = 0.25 + 0.64 = 0.89$

$E(Y_4) = E(T_B) + E(X_4) = 3 + 3 = 6$

$V(Y_4) = V(T_B) + V(X_4) = 0.25 + 0.36 = 0.61$

$Cov(Y_3, Y_4) = 0.25$

$Cov(Y_2, Y_3) = Cov(Y_2, Y_4) = 0$

$L = \{2,3,4\}$

4. Merge at node C

$a^2 = V(Y_2) + V(Y_3) - 2Cov(Y_2, Y_3) = 1 + 0.89 - 2 \times 0 = 1.89$

$a = \sqrt{1.89} = 1.37$

$$\alpha = \frac{E(Y_2) - E(Y_3)}{a} = \frac{4 - 5}{1.37} = -0.73$$

$p_3 = \Phi(-0.73) = 0.23$

$p_2 = 1 - p_3 = 0.77$

$\emptyset(\alpha) = 0.31$

$\mu_T = E(T_C) = p_2\mu_2 + p_3\mu_3 - a\emptyset(\alpha) = 0.77 \times 4 + 0.23 \times 5 - 1.37 \times 0.31 = 3.81$

$\sigma_T^2 = V(T_C) = p_2(\mu_2^2 + \sigma_2^2) + p_3(\mu_3^2 + \sigma_3^2) - (\mu_2 + \mu_3)a\emptyset(\alpha) - \mu_T^2 = 0.71$

$L = \{4\}$

5. Scan from node C

$E(Y_5) = E(T_C) + E(X_5) = 3.81 + 2 = 5.81$

$V(Y_5) = V(T_C) + V(X_5) = 0.71 + 0.16 = 0.87$

$Cov(Y_4, Y_5) = 0.0575$

$L = \{4,5\}$

6. Merge at node D

$a^2 = V(Y_4) + V(Y_5) - 2Cov(Y_4, Y_5) = 0.61 + 0.87 - 2 \times 0.0575 = 1.365$

$a = \sqrt{1.365} = 1.17$

$$\alpha = \frac{E(Y_4) - E(Y_5)}{a} = \frac{6 - 5.81}{1.17} = 0.16$$

$$p_5 = \Phi(0.16) = 0.56$$

$$p_4 = 1 - p_5 = 0.44$$

$$\emptyset(\alpha) = 0.39$$

$$\mu_T = E(T_D) = p_4\mu_4 + p_5\mu_5 - a\emptyset(\alpha) = 0.44 \times 6 + 0.56 \times 5.81 - 1.17 \times 0.39 = 5.44$$

$$\sigma_T^2 = V(T_D) = p_4(\mu_4^2 + \sigma_4^2) + p_5\left(\mu_5^2 + \sigma_5^2\right) - (\mu_4 + \mu_5)a\emptyset(\alpha) - \mu_T^2 = 0.52$$

$L = \{\text{Null}\}$

Backward pass: Assign flows

Node D: $Q_D = 100$, $0.44 \times 100 = 44$ on B→D, $0.56 \times 100 = 56$ on C→D

Node C: $56 + 150 = 206$, $0.23 \times 206 = 47.4$ on B→C, $0.77 \times 206 = 158.6$ on A→C

Node B: $44 + 47.4 + 200 = 291.4$ on A→B

Problem 4-11.

Assume the following utility function:

$$U_k = A_k - 0.10T_a - 0.07T_w - 0.08T_r - 0.02C$$

where: T_a is the access time (min),

T_w is the waiting time (min),

T_r is the riding or in-vehicle-travel time (min), and

C is the out-of-pocket cost (cents).

a. Apply the logit model to calculate the shares of the automobile mode ($A_k = 0$) and the transit mode ($A_k = -0.20$) for an O–D pair that has the following characteristics:

Mode	T_a	T_w	T_r	C
Auto	2	0	20	350
Transit	8	6	30	200

Solution.

The utility equation yields:

$$U_{auto} = -0.10 \times 2 - 0.07 \times 0 - 0.08 \times 20 - 0.02 \times 350 = -8.8$$

$$U_{tran} = -0.20 - 0.10 \times 8 - 0.07 \times 6 - 0.08 \times 30 - 0.02 \times 200 = -7.82$$

Using the logit equation:

$$P_{auto} = \frac{e^{U_{auto}}}{e^{U_{auto}} + e^{U_{tran}}} = \frac{e^{-8.8}}{e^{-8.8} + e^{-7.82}} \approx 0.273$$

$$P_{tran} = \frac{e^{U_{tran}}}{e^{U_{auto}} + e^{U_{tran}}} = \frac{e^{-7.82}}{e^{-8.8} + e^{-7.82}} \approx 0.727$$

Therefore, the shares of automobile mode and transit mode are approximately 27.3% and 72.7%, respectively.

b. Estimate the patronage shift for both (transit and auto) transportation modes that would result if the transit fare were increased from $2.00 to $2.50.

Solution.

The change in utility for the transit mode from the fare increase is:

$$\Delta U_{tran} = -0.02 * 50 = -1$$

Then using the base shares and the utility changes (which for auto is zero) in a pivot-point calculation, we obtain:

$$P_{auto}' = \frac{P_{auto} e^{0}}{P_{auto} e^{0} + P_{tran} e^{\Delta U_{tran}}} = \frac{0.273}{0.273 e^{0} + 0.727 e^{-1}} \approx 0.505$$

$$P_{tran}' = \frac{P_{tran} e^{\Delta U_{tran}}}{P_{auto} e^{0} + P_{tran} e^{\Delta U_{tran}}} = \frac{0.727 e^{-1}}{0.273 e^{0} + 0.727 e^{-1}} \approx 0.495$$

The revised mode shares can also be calculated by recalculating the utilities and using the original logit equations:

$$U_{auto} = -0.10 \times 2 - 0.07 \times 0 - 0.08 \times 20 - 0.02 \times 350 = -8.8$$

$$U_{tran} = -0.20 - 0.10 \times 8 - 0.07 \times 6 - 0.08 \times 30 - 0.02 \times 250 = -8.82$$

Using the logit equation:

$$P_{auto} = \frac{e^{U_{auto}}}{e^{U_{auto}} + e^{U_{tran}}} = \frac{e^{-8.8}}{e^{-8.8} + e^{-8.82}} \approx 0.505$$

$$P_{tran} = \frac{e^{U_{tran}}}{e^{U_{auto}} + e^{U_{tran}}} = \frac{e^{-8.82}}{e^{-8.8} + e^{-8.82}} \approx 0.495$$

You can do the calculations either way; the results are the same. If the transit fare were increased from \$2.00 to \$2.50, the share of transit mode would drop from 72.7% to 49.5% and the share of automobile mode would increase from 27.3% to 50.5%.

Problem 4-12.

In an urban area where a logit model for mode choice has been estimated, the coefficient on out-of-pocket cost (measured in cents) is −0.013. In a corridor of that area, the current observed shares of three modes are: drive-alone 65%, carpool 21%, and transit 14%. A toll scheme is to be implemented in this corridor that will have the effect of increasing the drive-alone cost from \$3.00 to \$3.50. What is the predicted effect on the shares of all three modes as a result of this change?

Solution.

This is also an application of the pivot-point analysis, with a change in the utility for the drive-alone mode.

$$\Delta U_{DA} = -0.013 * 50 = -0.65$$

$$P_{DA}' = \frac{P_{DA} e^{\Delta U_{DA}}}{P_{DA} e^{\Delta U_{DA}} + P_T e^0 + P_C e^0} = \frac{0.65 e^{-0.65}}{0.65 e^{-0.65} + 0.21 + 0.14} \approx 0.5$$

$$P_C' = \frac{P_C e^0}{P_{DA} e^{\Delta U_{DA}} + P_T e^0 + P_C e^0} = \frac{0.21}{0.65 e^{-0.65} + 0.21 + 0.14} \approx 0.3$$

$$P_T' = \frac{P_T e^0}{P_{DA} e^{\Delta U_{DA}} + P_T e^0 + P_C e^0} = \frac{0.14}{0.65 e^{-0.65} + 0.21 + 0.14} \approx 0.2$$

Therefore, if the drive-alone cost were increased from \$3.00 to \$3.50, its share would drop from 65% to 50%. Correspondingly, the share of carpool would increase from 21% to 30% and the share of transit would increase from 14% to 20%.

Chapter 5 Background on Transportation Systems and Vehicle Design

Problem 5-1.

Based on available data on train and commodity movements, the total tons and ton-miles (tmi) of freight moved by U.S. railroads in 2011 for different commodity groups are estimated in the table below. During that year, it is estimated that the average railcar carried 62.9 tons of freight, and the average train had 39.1 cars besides the locomotive(s). Calculate total tons moved, total ton-miles moved, ton-miles per ton, total car-miles traveled, and total train-miles traveled. Note that the "other types" group captures all other commodities, including mixed freight, not captured in the top five most common commodities by tonnage.

Commodity	Million Tons	Billion Ton-miles
Coal	816	748
Chemicals	194	178
Grain	157	144
Food products	107	98
Non-metallic minerals	128	117
Other types	484	444

Solution.

Total tons and ton-miles are found by summing the appropriate columns in the table, resulting 1.89 billion tons and 1.73 trillion ton-miles, respectively. Ton-miles per ton is then:

$$\frac{1.73\times10^{12}\,tmi}{1.89\times10^{9}\,ton} = 917\,tmi/ton$$

Since the average car carries 62.9 tons, total car-miles is estimated by dividing ton-miles by tons per car, giving:

$$\frac{1.73\times10^{12}\,tmi}{62.9\,tons/car} = 2.75\times10^{10}\,car-miles$$

Since the average train had 39.1 cars, total train-miles can be found by dividing total car-miles by the number of cars per train, giving:

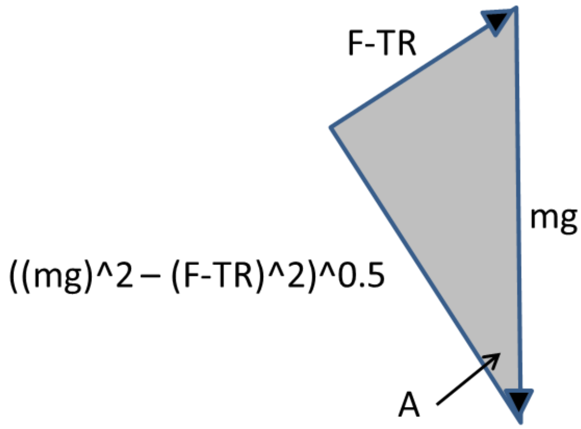
$$\frac{2.75\times10^{10}\,car-miles}{39.1\,cars/train} = 7.03\times10^{8}\,train-miles$$

Problem 5-2.

Use the balance of forces acting on a vehicle on a slope as shown in Fig. 5-1 and the definition of percent grade (i.e., ratio of vertical rise to horizontal run) to derive Eq.5-2.

Solution.

The equation is derived from a balance of forces diagram based on the force of gravity in one direction and tractive force of the vehicle in the other. Create a right triangle with the tractive force along one of the sides and the force of gravity along the hypotenuse, as shown in the diagram:



Notice that the diagram represents the two opposite forces just in balance, in other words F_{TR} just equaling the downward force of gravity. The length L of the other side can then be calculated using the Pythagorean theorem:

$$L = \sqrt{(mg^2) - (F_{TR})^2}$$

Next, by definition of congruent triangles, the angle A in the diagram is equal in size to the angle between the slope being climbed by the vehicle and level ground. Since, by definition, the percent gradient is the ratio of vertical rise to horizontal run, the formula is derived as follows:

$$GR_{\max} = 100 \times \frac{D_{vertical}}{D_{horizontal}} = 100 \times \left[\frac{F_{\mathrm{TR}}}{\sqrt{(mg)^2 - F_{\mathrm{TR}}^2}} \right]$$

This concludes the derivation.

Problem 5-3.

A high-efficiency passenger car has a frontal area of 2.4 m^2, a drag coefficient of 0.25, a rolling resistance value of 0.01, maximum tractive force at low speed of 2800 N, maximum power at high speed of 32 kW, and a curb weight of 1200 kg. Assume air density of 1.1 kg/m^3. Calculate: (a) power requirement at a cruising speed of 100 km/h, (b) maximum speed, and (c) maximum gradability.

Solution

Part (a): Convert given speed of 100 km/h to 27.8 m/s.

Using tractive power formula:

$$
\begin{aligned}
P_{\text{TR}} &= 0.5\rho A_F C_D V^3 + mgV\,C_o \\
&= 0.5(1.1)(2.4)(0.25)(27.8)^3 + (1200)(9.8)(27.8)(0.01) \\
&= 10{,}359\,\text{W} = 10.4\,\text{kW}
\end{aligned}
$$

Part (b): Solve using a numerical solver, such as Solver or Goal Seek in MS Excel. Use of the solver gives a maximum speed of 43.4 m/s, or 156.1 km/h. This is verified as follows:

$$
\begin{aligned}
P_{\text{TR}} &= 0.5\rho A_F C_D V^3 + mgV\,C_o \\
&= 0.5(1.1)(3.8)(0.42)(43.4)^3 + (2500)(9.8)(43.4)(0.011) \\
&= 32{,}000\,\text{W} = 32\,\text{kW}
\end{aligned}
$$

This calculation confirms that the tractive power requirement P_{TR} exactly balances the maximum available.

Part (c): Based on the maximum low-speed tractive force of 2800 N, the maximum gradability is as follows:

$$
\begin{aligned}
GR_{\max} &= 100 \times \left[\frac{F_{\text{TR}}}{\sqrt{(mg)^2 - F_{\text{TR}}^2}} \right] \\
&= 100 \times \left[\frac{2800}{\sqrt{(1200 \cdot 9.8)^2 - (2800)^2}} \right] \\
&= 100 \times \frac{2800}{11422} = 24.5\%
\end{aligned}
$$

Problem 5-4.

A large sport-utility vehicle (SUV) has a frontal area of 3.8 m^2, a drag coefficient of 0.42, a rolling resistance value of 0.011, maximum tractive force at low speeds of 6500 N, maximum power of 115 kW at high speed and in highest gear, and a curb weight of 2500 kg. Assume air density of 1.1 kg/m^3. Calculate (a) power requirement at a cruising speed of 100 km/h, (b) maximum speed, and (c) maximum gradability.

Solution

Part (a): Convert given speed of 100 km/h to 27.8 m/s.

Using tractive power formula:

$$P_{\text{TR}} = 0.5\rho A_F C_D V^3 + mgV C_o$$
$$= \; 0.5(1.1)(3.8)(0.42)(27.8)^3 + (2500)(9.8)(27.8)(0.011)$$
$$= 26{,}300\,\text{W} = 26.3\,\text{kW}$$

Part (b): Solve using a numerical solver, such as Solver or Goal Seek in MS Excel. Use of the solver gives a maximum speed of 48.8 m/s, or 175.6 km/h. This is verified as follows:

$$P_{\text{TR}} = 0.5\rho A_F C_D V^3 + mgV C_o$$
$$= \; 0.5(1.1)(3.8)(0.42)(48.8)^3 + (2500)(9.8)(48.8)(0.011)$$
$$= 115{,}000\,\text{W} = 115\,\text{kW}$$

This calculation confirms that the tractive power requirement P_{TR} exactly balances the maximum available.

Part (c): Based on the maximum low-speed tractive force of 6500 N, the maximum gradability is as follows:

$$GR_{\max} = 100 \times \left[\frac{F_{\text{TR}}}{\sqrt{(mg)^2 - F_{\text{TR}}^2}} \right]$$
$$= 100 \times \left[\frac{6500}{\sqrt{(2500 \cdot 9.8)^2 - (6500)^2}} \right]$$
$$= 100 \times \frac{6500}{23622} = 27.5\%$$

Problem 5-5.

According to the EPA, a representative model of the Toyota Prius has a highway fuel economy value of 52 mpg. What is the difference in mpg between this value and the delivered mpg if the vehicle cruises at 100 km/h and requires the calculated tractive effort based on C_D = 0.29, C_o = 0.1, frontal area 2.4 m^2, and curb weight of 1383 kg? Assume the drivetrain efficiency from gasoline in the tank to tractive effort is 35%, air density of 1.1 kg/m^3, and that gasoline has an energy content of 115,400 Btu/gal.

Solution

Part (a): Assuming an air density of 1.1 kg/m^3, power requirement is solved using the equation for tractive power as a function of velocity V = 100 km/h = 27.8 m/s:

$$P_{\text{TR}} = 0.5\rho A_F C_D V^3 + mgV C_o$$
$$= \; 0.5(1.1)(2.4)(0.29)(27.8)^3 + (1383)(9.8)(27.8)(0.1)$$
$$= 11{,}992\,\text{W} = 11.99\,\text{kW}$$

This power requirement is converted to a fuel economy value by arbitrarily assuming that the vehicle will travel 52 mi, which is equivalent to 83.2 km. This distance takes 0.83 hour to travel at 100 km/h. Therefore energy output and energy input required are calculated as follows:

$$E_{out} = (11.99)(0.83h) = 9.98kWh$$

$$E_{in} = \frac{9.98}{0.35} = 28.5kWh = 102.6MJ$$

The energy content of 1 gal of 115,400 Btu is equivalent to 121.7 MJ. Therefore the fuel economy in mpg and difference in mpg can be calculated as follows:

$$F_{consumed} = \frac{102.6MJ}{121.7MJ/gal} = 0.843gal.gasoline$$

$$MPG = \frac{52mi}{0.843gal} = 61.7$$

$$\Delta MPG = 61.7 - 52 = 9.7mpg$$

Problem 5-6.

A current model high-efficiency midsize car has a frontal area of 2.6 m^2, a drag coefficient of 0.25, a rolling resistance value of 0.011, maximum tractive force at low speeds of 4000 N, maximum power of 100 kW at high speed and in highest gear, and a curb weight of 1300 kg. Assume air density of 1 kg/m^3. Calculate (a) power requirement at a cruising speed of 100 km/h, (b) maximum speed, and (c) maximum gradability.

Solution:

Part (a): Assuming an air density of 1 kg/m^3, power requirement is solved using the equation for tractive power as a function of velocity V = 100 km/h = 27.8 m/s:

$$P_{TR} = 0.5\rho A_F C_D V^3 + mgV\,C_o$$
$$= 0.5(1)(2.6)(0.25)(27.8)^3 + (1300)(9.8)(27.8)(0.011)$$
$$= 10{,}900\,\text{W} = 10.9\,\text{kW}$$

Part (b): Since the tractive power equation cannot be solved in closed form for V_{max} as a function of $P_{TR\text{-}max}$, trial and error or the solver function in a spreadsheet can be used to find the value of V_{max}. The solver function is set up to find the value of V such that the difference between the right-hand side of the equation and P_{TR} = 100 kW is zero. To three significant digits, the value is V_{max} = 65.4 m/s = 147.1 mi/h = 235.4 km/h. This value can be verified as follows:

$$0.5\rho A_F C_D V^3 + mgV\,C_o$$
$$= 0.5(1)(2.6)(0.25)(65.4)^3 + (1300)(9.8)(65.4)(0.011)$$
$$= 90{,}837 + 9163 = 100{,}000W = 100\,kW$$

Part (c): Based on the maximum low-speed tractive force of 4000 N, the maximum gradability is as follows:

$$GR_{max} = 100 \times \left[\frac{F_{TR}}{\sqrt{(mg)^2 - F_{TR}^2}} \right]$$

$$= 100 \times \left[\frac{4000}{\sqrt{(1300 \cdot 9.8)^2 - (4000)^2}} \right]$$

$$= 100 \times \frac{4000}{12096} = 33.1\%$$

Problem 5-7.

As shown in Fig.5-2, the tendency of power output is to decrease with increasing speed near the maximum speed of the vehicle. Therefore, the stated maximum speed of the vehicle in Problem 5-6 would likely exceed the actual maximum power available in a real-world situation. Suppose the vehicle that is the same in terms of all other specifications achieves maximum power of 40 kW at high speed and in highest gear (40% of the absolute maximum power value in 5-6). What is the new value of maximum speed?

Solution:

Since the tractive power equation cannot be solved in closed form for V_{max} as a function of $P_{TR\text{-}max}$, trial and error or the solver or "goal seek" function in a spreadsheet can be used to find the value of V_{max}. The solver or goal seek function is set up to find the value of V such that the difference between the right-hand side of the equation and P_{TR} = 29.2 kW is zero. To three significant digits, the value is V_{max} = 42.5 m/s = 153.1 km/h. This value can be verified as follows:

$$0.5 \rho A_F C_D V^3 + mgV C_o$$

$$= 0.5(1)(2.5)(0.25)(42.5)^3 + (1237)(9.8)(42.5)(0.1)$$

$$= 24044W + 5156W = 29200W = 29.2\,kW$$

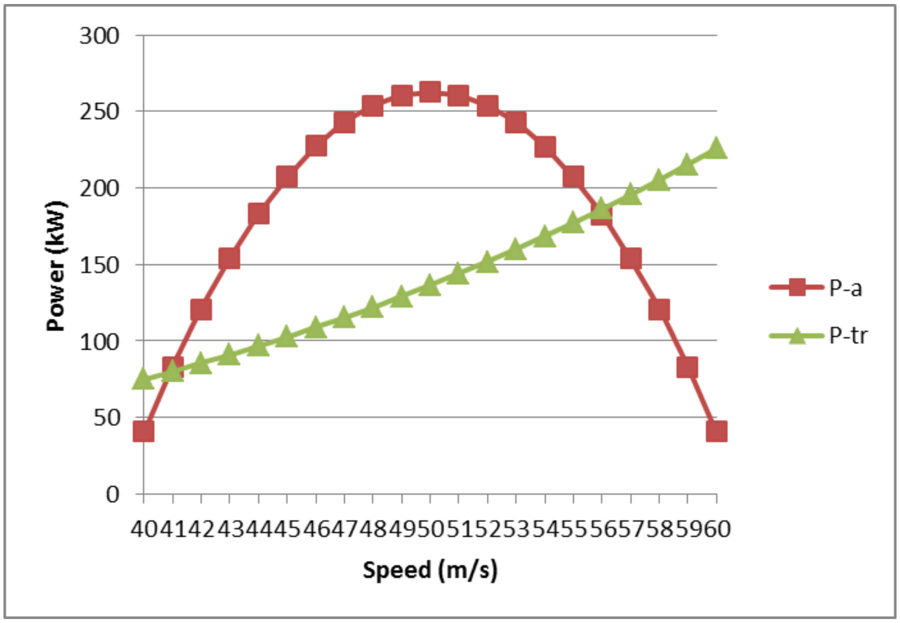
Problem 5-8.

Figure 5-2 shows that in the vicinity of an automobile's maximum speed, power available P_a decreases with increasing speed even as total tractive power required increases. In this problem you will solve for maximum speed taking this factor into account. A large sport-utility vehicle (SUV) has a frontal area of 3.8 m^2, a drag coefficient of 0.45, a rolling resistance value of 0.011, and a curb weight of 3500 kg. Power available P_a in the highest gear measured in kW is a function of velocity v in m/s as shown in the function below. (A) Draw a figure of P_a and P_{TR} over the relevant range. (B) What is the predicted maximum speed in both m/s and mi/h? Use a numerical solver to compute, and then verify the value by substituting back into the appropriate equations.

$$P_a = \left(-6.53 \times 10^{-7}\right)v^3 - 2.221v^2 + 222.1v - 5289.6$$

Solution:

Part (a): As an illustration, the figure below plots *P-a* and *P-tr* on the same axes for values from 40 to 60 m/s. Note that the function for P_{TR} gives results in watts, which must therefore be converted to kW.



Part (b): Using a solver and setting the two equations equal to each other, the power available and power required are equal at *v* = 55.9 m/s, with the resulting power value of *P* = 185.4 kW for both functions. Converting to standard units gives 55.9 m/s × 2.25 = 125.8 mi/h. Note that the functions also intersect at approximately 41 m/s, but this is not the correct answer since the question asks for the maximum speed, which occurs at 55.9 m/s. Plugging 55.9 m/s in to the equation for *P-tr*:

$$P_{TR} = 0.5\rho A_F C_D V^3 + mgV C_o$$
$$= \; 0.5(1.1)(3.8)(0.45)(55.9)^3 + (3500)(9.8)(55.9)(0.01)$$
$$= 185{,}400W = 185.4kW$$

Plugging in to the equation for *P-a*:

$$P_a = \left(-6.53\times10^{-7}\right)v^3 - 2.221v^2 + 222.1v - 5289.6$$
$$= \left(-6.53\times10^{-7}\right)(55.9)^3 - 2.221(55.9)^2 + 222.1(55.9) - 5289.6$$
$$= 185.4kW$$

Problem 5-9.

This problem builds on Problem 5-3. Suppose that the vehicle in 5-3 cruises at 100 km/h or 62.5 mi/h continuously for 1 hour, and that the overall efficiency of conversion of energy from the fuel to tractive power is 24%. A gallon of gasoline has a net energy content of 115,400 Btu. What is the delivered fuel economy in miles per gallon?

Solution.

Note that this problem mixes metric and standard units, since the reading does not provide a tractive power equation in standard units. Since the tractive power requirement is previously calculated as 9.7 kW, the energy in Btu is the following:

$$(9.7kW)(1h) = 9.7kWh$$

$$(9.7kWh)\left(\frac{3.6MJ}{1kWh}\right) = 34.97MJ$$

$$(34.97MJ)\left(\frac{1000Btu}{1.055MJ}\right) = 33{,}150Btu$$

Next, we use the efficiency of 24%, distance traveled of 62.5 mi, and energy content per gallon of gasoline to calculate mpg:

$$33{,}150Btu\left(\frac{1}{0.24}\right) = 138{,}100Btu$$

$$138{,}100Btu\left(\frac{1gal}{115{,}400Btu}\right) = 1.2gal$$

$$\frac{62.5mi}{1.2gal} = 52.2mpg$$

Problem 5-10.

A current model of the Honda Insight has a frontal area of 2.5 m^2, a drag coefficient of 0.25, a rolling resistance value of 0.01, maximum drivetrain force at low speeds of 3200 N, maximum power of 29.2 kW at high speed and in highest gear (40% of the published maximum power value of 73 kW), and a curb weight of 1237 kg. Assume air density of 1 kg/m^3. Calculate:

a. Power requirement at a cruising speed of 100 km/h
b. Maximum speed
c. Maximum gradability

Solution:

Part (a): Assuming an air density of 1 kg/m^3, power requirement is solved using the equation for tractive power as a function of velocity V = 100 km/h = 27.8 m/s:

$$\begin{aligned} P_{\mathrm{TR}} &= 0.5\rho A_F C_D V^3 + mgV C_o \\ &= 0.5(1)(2.5)(0.25)(27.8)^3 + (1237)(9.8)(27.8)(0.1) \\ &= 10065\,\mathrm{W} = 10.1\mathrm{kW} \end{aligned}$$

Part (b): Since the tractive power equation cannot be solved in closed form for $V_{\max}$ as a function of $P_{\mathrm{TR\text{-}max}}$, trial and error or the solver or "goal seek" function in a spreadsheet can be used to find the value of $V_{\max}$. The solver or goal seek function is set up to find the value of V such that the difference between the right-hand side of the equation and P_{TR} = 29.2 kW is zero. To three significant digits, the value is $V_{\max}$ = 42.5 m/s = 153.1 km/h. This value can be verified as follows:

$$\begin{aligned} &0.5\rho A_F C_D V^3 + mgV C_o \\ &= 0.5(1)(2.5)(0.25)(42.5)^3 + (1237)(9.8)(42.5)(0.1) \\ &= 24044W + 5156W = 29200W = 29.2\,kW \end{aligned}$$

Part (c): Based on the maximum low-speed tractive force of 3200 N, the maximum gradability is as follows:

$$GR_{\max} = 100 \times \left[\frac{F_{\mathrm{TR}}}{\sqrt{(mg)^2 - F_{\mathrm{TR}}^2}} \right]$$

$$= 100 \times \left[\frac{3200}{\sqrt{(1237 \cdot 9.8)^2 - (3200)^2}} \right]$$

$$= 100 \times \frac{3200}{11693} = 27.4\%$$

Chapter 6 Physical Design of Transportation Facilities

Problem 6.1.

Determine the minimum stopping sight distance on a −2.5% grade at a design speed of 90 km/h.

<u>Solution.</u>

Total required stopping sight distance:

$$s = d_r + d_b$$

Reaction distance:

$$d_r = vt_r = (90\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)(2.5\text{s}) = 62.5\text{m}$$

Braking distance:

$f = 0.30$ (Table 3-3)

$G = -0.025$ (given)

$$d_b = \frac{v^2}{2g(f \pm G)} = \frac{\left[(90\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)\right]^2}{2(9.8\text{m/s}^2)(0.30 - 0.025)} = 116.0\text{m}$$

Total sight distance:

$$s = d_r + d_b = 62.5 + 116.0 = 178.5\text{ m}$$

Problem 6.2.

Determine the minimum stopping sight distance on a +1.5% grade at a design speed of 100 km/h.

<u>Solution.</u>

Total required stopping sight distance:

$$s = d_r + d_b$$

Reaction distance:

$$d_r = vt_r = (100\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)(2.5\text{s}) = 69.4\text{m}$$

Braking distance:

f = 0.29 (Table 3-3)

G = + 0.015 (given)

$$d_b = \frac{v^2}{2g(f \pm G)} = \frac{\left[(100\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)\right]^2}{2(9.8\text{m/s}^2)(0.29+0.015)} = 129.1\text{m}$$

Total sight distance:

$$s = d_r + d_b = 69.4 + 129.1 = 198.5\text{ m}$$

Problem 6.3.

Determine the minimum stopping sight distance on a −4.0% grade at a design speed of 70 km/h.

Solution.

Total required stopping sight distance:

$$s = d_r + d_b$$

Reaction distance:

$$d_r = vt_r = (70\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)(2.5\text{s}) = 48.6\text{m}$$

Braking distance:

f = 0.31 (Table 3-3)

G = – 0.04 (given)

$$d_b = \frac{v^2}{2g(f \pm G)} = \frac{\left[(70\text{km/h})\left(\frac{1000\text{m/km}}{3600\text{s/h}}\right)\right]^2}{2(9.8\text{m/s}^2)(0.31-0.04)} = 71.4\text{m}$$

Total sight distance:

$$s = d_r + d_b = 48.6 + 71.4 = 120.0 \text{ m}$$

Problem 6.4.

Design standards link vehicle characteristics, human characteristics, and the characteristics of the transportation facility. What features of human and vehicle characteristics are important in the derivation of design standards?

Solution.

Human characteristics that are important in the derivation of design standards include visual ability, ability to hear, reaction times, gap acceptance behavior, steering behavior, and the subjective sense of comfort. Vehicular characteristics that are important in the derivation of design standards include physical dimensions (length, width, height, and wheelbase), weight (gross weight and wheel loads), acceleration and deceleration characteristics, maximum speed, and (for aircraft only) lift.

Problem 6.5.

List and briefly describe at least five transportation facility characteristics typically specified by design standards**.**

Solution.

Any five of the following:

Minimum radius of horizontal curve. For a given design speed, minimum curve radius is limited by maximum allowable side friction, which is usually based on a comfort standard; maximum superelevation rate (or banking) for the curve; and the necessity to maintain stopping sight distance.

Maximum rate of superelevation. For highways, maximum superelevation rate is limited by side friction and by presence of roadside features such as driveways. For railways, it is limited by the need to limit imbalances in the loads on the rails.

Maximum grade. Maximum upgrades are limited by vehicle power/weight ratios and vehicle traction. Maximum downgrades are also limited by stopping distances and sight distances.

Minimum grades for some types of highway are limited by the need to provide drainage.

Minimum cross-slopes for highways, runways, and taxiways are limited by the need to provide drainage.

Minimum length of vertical curve. For highways minimum length of vertical curve is limited by stopping or passing sight distance requirements, vertical acceleration, and appearance standards. For railways, minimum length of vertical curve is also limited by the need to prevent jerk on couplings in sag vertical curves. For runways and taxiways, minimum length of vertical curve is limited by sight distance requirements.

Edge radii in roadway and taxiway intersections are limited by vehicle turning radii. These, in turn, are related to vehicle wheelbase dimensions.

Minimum intersection setbacks (minimum distances to obstructions to vision) are limited by stopping sight distance and driver gap-acceptance behavior.

Freeway ramp junction details are limited by gap-acceptance behavior, steering behavior in entering or exiting lanes, and vehicle acceleration and deceleration capabilities.

Horizontal and vertical clearances are limited by vehicle dimensions and in the case of horizontal clearances for highways, by the need to provide clear recovery zones for vehicles that run off the road.

Problem 6.6.

Four basic elements of facility plans document the geometry of linear transportation facilities such as highways and railways. List and briefly describe these four elements.

Solution.

1. The *plan view* (or simply "plan"). This is a drawing of the facility as it would look to an observer directly above it.

2. The *profile*. This drawing has elevation as its vertical axis, and horizontal distance, as measured along the centerline of the facility (or other recognized reference line), as its horizontal axis.

3. The *geometric cross-section*. This view has elevation as its vertical axis and horizontal distance, measured perpendicular to the centerline, as its horizontal axis.

4. The *superelevation diagram*. This applies to curved facilities, such as highways or railways, only. It consists of a graph with roadway or railway cross-slope (vertical axis) versus horizontal distance (horizontal axis). The cross-slope is measured relative to the centerline or some other axis of rotation for the facility. Alternatively, the diagram may show the elevation of the edge of pavement on the vertical axis.

Problem 6.7.

Spreadsheet. Use a spreadsheet to construct a table of stopping sight distances for design speeds ranging from 30 km/h to 120 km/h in increments of 10 km/h and grades ranging from −6% to +6% in 2% increments.

Solution:

Worksheet

km/h -> m/s factor =	0.277778
Acceleration of gravity =	9.8
Reaction time =	2.5

		Grade						
Speed, km/h	f	-0.06	-0.04	-0.02	0.00	0.02	0.04	0.06
30.00	0.40	31.25	30.68	30.16	29.69	29.27	28.89	28.54
40.00	0.38	47.46	46.30	45.27	44.35	43.52	42.77	42.09
50.00	0.35	68.66	66.47	64.55	62.84	61.32	59.96	58.73
60.00	0.33	94.16	90.54	87.38	84.61	82.16	79.97	78.01
70.00	0.31	125.77	120.06	115.13	110.84	107.07	103.73	100.75
80.00	0.30	160.54	152.46	145.54	139.54	134.29	129.66	125.54
90.00	0.30	195.37	185.15	176.38	168.79	162.15	156.29	151.08
100.00	0.29	240.61	226.91	215.25	205.19	196.44	188.74	181.92
110.00	0.28	292.91	274.87	259.60	246.51	235.17	225.25	216.49
120.00	0.28	341.01	319.54	301.37	285.80	272.30	260.49	250.07

Notes:

Cell names:

D1	fac
D2	grav
D3	t

Column names for table:

Speed column	v
F column	f
Grade columns	G

Formula for cells in table is as follows:

v × fac × *t* + ((*v* × fac) ^ 2/((2 × grav) × (*f* + *G*)))

Chapter 7 Overview of Passenger Transportation

Problem 7-1.

Suppose that the relationship between speed and density for a particular highway facility is: $u = 6\sqrt{120 - k}$, where the speed, *u*, is in mi/h and the density, *k*, is in vehicles/m (veh/mi). (a) What is the free-flow speed for this facility? (b) What is the jam density? (c) What is the capacity ($q_{\max}$) for the facility? (d) At what speed is the maximum flow achieved?

Solution.

Part (a): Free-flow speed is at *k* = 0. Therefore, $u_f = 6*\sqrt{120}$ = 65.7 mi/h.

Part (b): Jam density occurs when *u* = 0. Therefore, $6\sqrt{120-k_j} = 0 \quad \Rightarrow \quad k_j = 120$ veh/mi.

Part (c): Flow on this facility is $q = uk = 6k\sqrt{120-k}$. To maximize flow, we differentiate with respect to k:

$$\frac{dq}{dk} = 6\sqrt{120-k} - 6\frac{1}{2}k(120-k)^{-\frac{1}{2}} = 0$$
$$\Rightarrow \quad k = 80\, veh/mile$$

At that density, q_{max} is $q_{\max} = 6*80\sqrt{120-80} = 3036\, veh/hour$

Part (d): At a density of *k* = 80 veh/mi, the speed is $u = 6\sqrt{120-80} = 38\, miles/hour$

Problem 7-2.

Along a two-lane rural highway where there is no opportunity to pass because of curves and short sight distances, the speed-density relationship can be represented by a Greenshields model with u_f = 62 mi/h and k_j = 100 veh/mi. Traffic is moving at an average speed of 53 mi/h when a piece of farm equipment pulls onto the road and travels at 20 mi/h.

Part (a): What is the speed of the shock wave at the end of the platoon of vehicles that builds up behind the farm implement as it travels?

Solution.

We know that u_f = 62 mi/h and *kj* = 100 veh/mi, so the relationship between speed and density can be expressed as *u* = 62 − 0.62 *k* (using the Greenshields model).

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}, \text{ where } \begin{array}{l} u_1 = 53mph,\ k_1 = \frac{62-u_1}{0.62} = 14.5\, veh/mile, q_1 = u_1 k_1 = 768.5\, veh/hr, \\ u_2 = 20mph,\ k_2 = \frac{62-u_2}{0.62} = 67.7\, veh/mile, q_2 = u_2 k_2 = 1354\, veh/hr \end{array}$$

Therefore, $u_w = \frac{1354 - 768.5}{67.7 - 14.5} = 11\, mile/hr$

Part (b): After 5 minutes on the road, the farm implement turns off and leaves the road. At that time, how long (in miles) is the platoon that has built up behind it on the road?

Solution.

$$L = (u_2 - u_w)t = (20-11) * \frac{5}{60} = 0.75 mile$$

Part (c): At the implied density within the platoon, approximately how many vehicles are backed up in the platoon?

Solution.

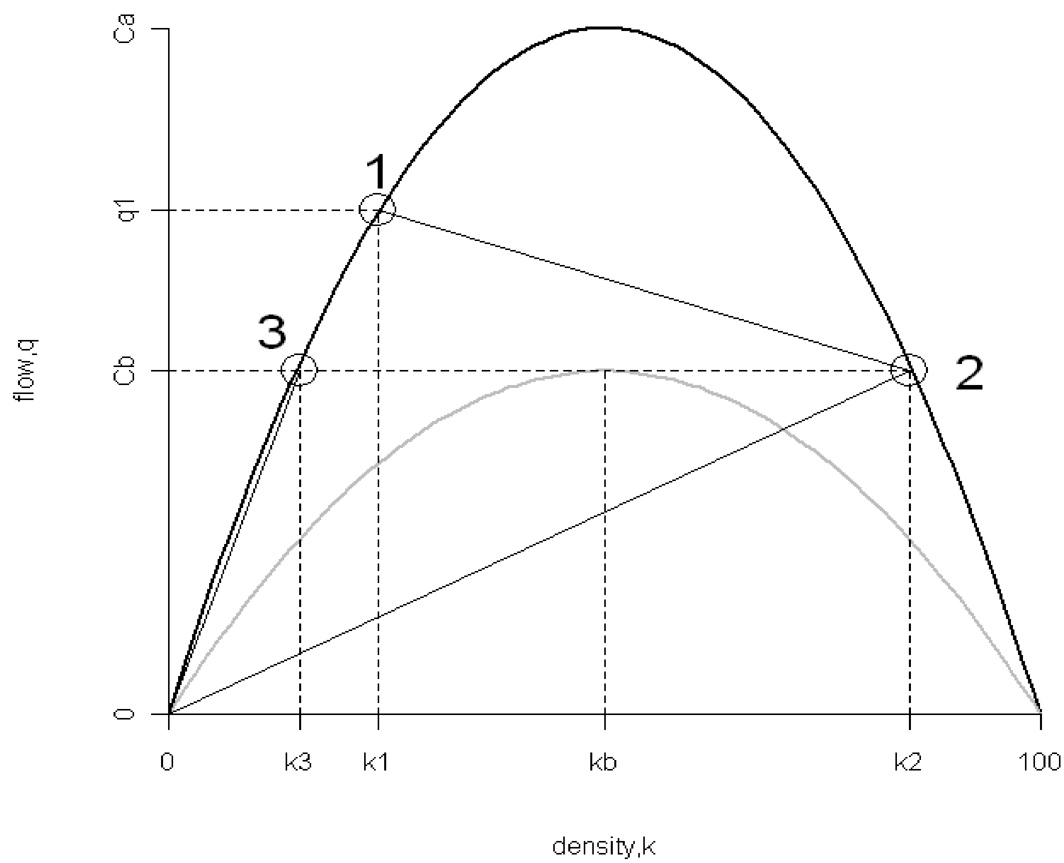
Density within the platoon is k_2 = 67.7 veh/mi [as computed in part (a)]. Therefore, the number of vehicles in the platoon can be estimated by $L \times k_2 = 0.75 \times 67.7 = 51$ veh.

Problem 7-3.

A construction zone on a freeway creates a bottleneck section where only one lane out of two is open. You may assume that traffic operates according to a Greenshields speed-density relationship, with u_f = 68 mi/h and k_j = 100 veh/lane/mi. Approaching the construction zone, traffic flow is 2500 veh/h (or 1250 veh/lane/h). Inside the bottleneck zone, assume that the same speed-density relationship exists, but there is only one lane available.

a. What are the speed and density inside the queue that is building up in front of the construction zone?
b. What is the speed of the wave representing the back of the queue?
c. What is the speed of the traffic exiting the construction zone and resuming travel on the two-lane facility?

Diagram to accompany solution of Problem 7-3:



Quick answers: (a) in queue, $u = 10$ mi/h, $k = 85$ veh/mi; (b) wave speed $u - w = -6.5$ mi/h (=−13 mi/h also accepted with no points deducted); (c) $u = 57$ mi/h.

Detailed solution:

a. The normal flow function for the two-lane facility based on density can be written as below:

$$q_a = 2*u*k = 2k(68 - 0.68k).$$

Note that k is measured in veh/lane/mi. This q-k relationship is plotted as the upper (darker) curve in the figure above.

Flow-density in the bottleneck (one lane) can be written as $q_b = uk = k(68 - 0.68k)$. This is plotted as the "inner" q-k relationship in the figure (the gray curve). The capacity of the bottleneck is as below:

$$c_b = \frac{c_a}{2} = \frac{k_j u_f}{4} = \frac{100*68}{4} = 1700 veh/hr \text{ and } k_b = 50 \text{veh / mile}.$$

The flow approaching the bottleneck (at point 1) is $q_1 = 2500 \text{veh / hr}$ at density k_1 and with speed u_1. k_1 can be solved from $q_1 = 2k_1(68 - 0.68k_1) = 2500$. This means k_1 = 24 or 76. Since k_1 should be smaller than k_b (50 veh/lane/mi) (density in the bottleneck at capacity), we have $k_1 = 24$ veh/lane/mile. Therefore, speed $u_1 = 68 - 0.68*24 = 51\text{mph}$.

The flow through the bottleneck is constrained by its capacity $C_b = C_a / 2$. Thus, upstream from the bottleneck (point 2), there is a transition (point 1→point 2), reducing the flow to c_b at a higher density k_2 and a lower speed u_2. We have $q_2 = 2k_2(68 - 0.68k_2) = c_b = 1700$. Thus, k_2 =15 or 85. The density inside the queue k_2 should be higher than k_1 (24 veh/lane/mi), therefore, k_2 = 85 veh/lane/mi. (The point at k = 15 veh/lane/mi is the corresponding k for point 3 which will be used in part (c). At k_2 = 85, $u_2 = 68 - 0.68*k_2 = 10\text{mph}$. Thus, the speed and density inside the queue building up before the bottleneck is u_2 = 10 mph at density k_2 = 85 veh/lane/mi.

b. What is the speed of the wave representing the back of the queue?

In order to calculate the speed of the back of the queue, we need know the flow and density of both the approaching traffic and the queue. These have been obtained in part (a).

The problem can be solved either on a two-lane or one-lane basis. In this case, we will solve on a one-lane basis, so flow values q must be divided in half. For the approaching traffic, q_1 =1250 veh/lane/h and.k_1 = 24 veh/lane/mi. Within the queue, q_2 = 850 veh/lane/h and k_2 = 85 veh/lane/mi. Therefore, the speed of the back of the queue is given by the slope of the line from point 1 to point 2:

$u_w = \frac{q_2 - q_1}{k_2 - k_1} = \frac{850 - 1250}{85 - 24} = -6.5mph$. The negative speed indicates that the wave is moving right to left (i.e., the queue is growing over time).

c. What is the speed of the traffic exiting the construction zone and resuming travel on the two-lane facility?

At the exit from the bottleneck (point 3), the volume q_3 is the same as c_b, but there is a reduction in density (from k_b to k_3). Using $q_3 = 2k_3(68 - 0.68k_3) = c_b = 1700$, we find that k_3 = 15 veh/lane/mi.

The speed of the traffic is $u_3 = \frac{c_b}{2k_3} = \frac{1700}{2*15} = 56.7mph$.

Problem 7-4.

Below are data in standard units for U.S. cars and light trucks for the period 1970 to 2000. Use Divisia analysis to create a table and a graph for the period 1970 to 2005, showing four curves: (1) actual fuel consumption, (2) trended fuel consumption, and the contribution of (3) energy intensity, and (4) structural changes to the difference between actual and trended fuel consumption.

Year	Car		Light Truck	
	bil.vmt	tril.btu	bil.vmt	bil.gal
1970	917	7836	123	1433
1975	1034	8537	201	2287
1980	1112	8080	291	2744
1985	1247	8262	391	3171
1990	1408	8019	575	4116
1995	1438	7866	790	5275
2000	1600	8445	923	6098

Solution.

(Note: Refer to discussion of Divisia analysis in Chap.3: Systems Tools)

First, combine above activity levels into total vmt and energy use, as shown in table below. From the combined energy and vmt in the year 1970, an overall average intensity of 8912 Btu/vmt is calculated, which is used to calculate the rest of the energy values in the trended column at the right in the table below:

Year	Activity	Energy	Trended
	bil.vmt	tril.btu	tril.btu
1970	1040	9269	9269
1975	1235	10824	11007
1980	1403	10824	12504
1985	1638	11433	14599
1990	1983	12135	17673
1995	2228	13141	19857
2000	2523	14543	22486

Next, create a table that breaks out intensity and modal share for the passenger car (PC) and light truck (LT) categories. Intensity is measured in Btu/vmt.

Year	vmt Share		Intensity [MBtu/vmt]	
	Car	LT	Car	LT
1970	88.2%	11.8%	8.55	11.65
1975	83.7%	16.3%	8.26	11.38
1980	79.3%	20.7%	7.27	9.43
1985	76.1%	23.9%	6.63	8.11
1990	71.0%	29.0%	5.70	7.16
1995	64.5%	35.5%	5.47	6.68
2000	63.4%	36.6%	5.28	6.61

Next, create a table of the elements needed for the decomposition for PC as below:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$	Terms	

1970	8.5	0.882					Intensity	Structure
1975	8.3	0.837	−0.3	-0.0445	16.8	1.7190	(0.2484)	(0.3737)
1980	7.3	0.793	−1.0	−0.0447	15.5	1.6298	(0.8068)	(0.3466)
1985	6.6	0.761	−0.6	−0.0313	13.9	1.5539	(0.4978)	(0.2174)
1990	5.7	0.710	−0.9	−0.0513	12.3	1.4713	(0.6843)	(0.3158)
1995	5.5	0.645	−0.2	−0.0646	11.2	1.3555	(0.1526)	(0.3607)
2000	5.3	0.634	−0.2	−0.0113	10.7	1.2796	(0.1228)	(0.0605)

A similar table for LT:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$	Terms	
1970	11.7	0.118					Intensity	Structure
1975	11.4	0.163	−0.3	0.0445	23.0	0.2810	(0.0383)	0.5122
1980	9.4	0.207	−1.9	0.0447	20.8	0.3702	(0.3606)	0.4646
1985	8.1	0.239	−1.3	0.0313	17.5	0.4461	(0.2943)	0.2744

1990	7.2	0.290	−1.0	0.0513	15.3	0.5287	(0.2516)	0.3913
1995	6.7	0.355	−0.5	0.0646	13.8	0.6445	(0.1550)	0.4470
2000	6.6	0.366	−0.1	0.0113	13.3	0.7204	(0.0254)	0.0748

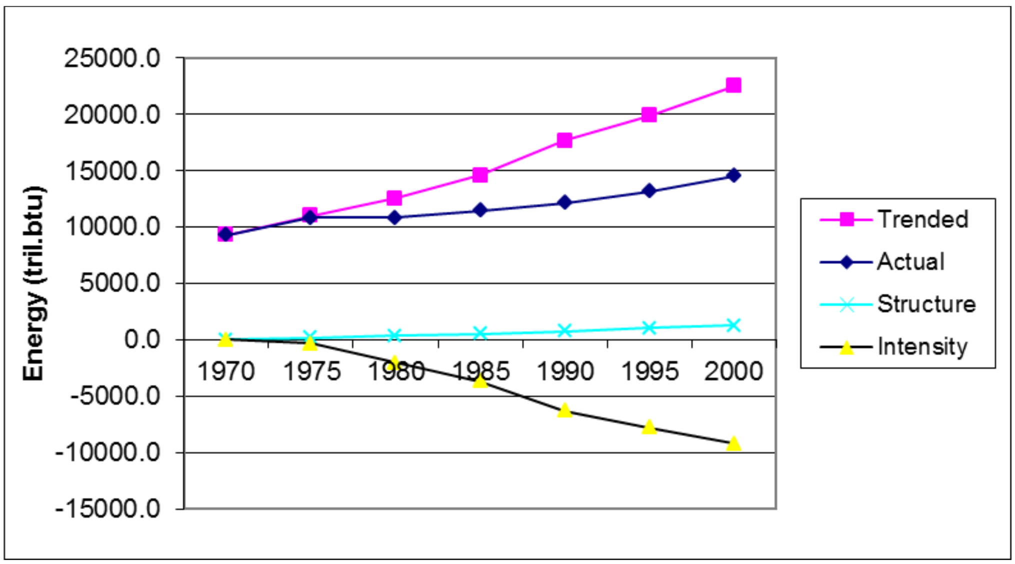
In order to graph the effects of "intensity" and "structure," we need to calculate a cumulative value for each year. For 1975, the value is simply the sum of the PC and LT values. For subsequent years, it is the value for the year being calculated plus the sum of all previous years. It is convenient to have two columns for each effect, one for the current year (i.e., the sum of the values for the current year) and one for the cumulative values:

	Sum of values (PC + LT)		Cumulative (cols. 1 and 2)	
	Intensity	Structure	Intensity	Structure
1975	–0.243	0.117	–0.243	0.117
1980	–1.012	0.102	–1.255	0.220
1985	–0.697	0.049	–1.952	0.269
1990	–0.797	0.065	–2.750	0.334
1995	–0.276	0.074	–3.025	0.407
2000	–0.122	0.012	–3.147	0.420

The cumulative change in intensity and structure can be multiplied by the activity (vmt) in a given year to give the impact on fuel consumption, in trillions of Btu. This data series then provides the input for the figure that you are asked to draw, below. The "trended" data are the activity level multiplied by 1970 overall intensity (already shown in the table above). Next are the intensity and structure terms, followed by the sum of trended, intensity and structure ("T + I + S"). Note that the rightmost column equals the actual value, confirming that the analysis is correct.

	Actual	Trended	Intensity	Structure	$T + I + S$
1970	9269.0	9269.0	0	0	9269.0
1975	10824.0	11006.9	−354.0	171.0	10824.0
1980	10824.0	12504.2	−2040.1	359.9	10824.0
1985	11433.0	14598.7	−3679.3	513.7	11433.0
1990	12135.0	17673.5	−6310.1	771.7	12135.0
1995	13141.0	19857.1	−7775.2	1059.2	13141.0
2000	14543.0	22486.2	−9178.7	1235.4	14543.0

Figure to accompany table:



Problem 7-5.

A traveler (we'll call her Sarah) commutes to work in a large city. She has a choice between using the bus and driving. If she uses the bus, she can walk 0.25 mi from her apartment to the bus stop (taking 5 minutes), wait an average of 5 minutes for the bus to arrive, and then travel downtown. Her trip requires one transfer, and requires a total of 40 minutes (including another 5 minutes of waiting at the transfer point, to get the second bus). When she reaches downtown, she has a 6-minute walk to her workplace. In the afternoon, she retraces the path in the other direction, requiring an equal amount of time. The bus fare (including transfer) is $2.00 each direction.

On the other hand, Sarah can also choose to drive to work. The distance is 10 mi and the average cost of owning and operating a vehicle can be estimated at $0.50 per mile. In addition, she would have to pay $4.00 per day for parking downtown. If she drives to work, the trip takes her 20 minutes (including the walk time between the parking garage and her workplace) in each direction.

a. It is observed that Sarah chooses to drive to work on a regular basis. What does this imply about her "value of time?"
b. How does this relate to the conclusion reached in the 2009 Urban Mobility Report from the Texas Transportation Institute that Americans incur about 4.2 billion hours of delay due to congestion each year, and that delay represents a "cost" of approximately $80 billion (for purposes of this problem, you can ignore the component of this $80 billion figure generated by wasted fuel consumption in congestion).

Solution.

Part (a): Sarah's choice to drive to work suggests that the cost (money and time) of making the trip by bus is higher than that by driving.

Travel time by bus: two-way travel time = 2 × (time from apartment to bus stop + waiting time + time on bus + time from downtown to workplace) = 2 × (5 + 5 + 40 + 6) = 112 minutes. Two-way bus fare = 2 × $2 = $4

Travel time by car: two-way travel time = 2 × (driving + time from parking to workplace)= 2 × 20 = 40 minutes. Travel cost = 2 × (0.5 × 10) + 4 = $14.

If Sarah values time by x $/min, then

$$4 + 112 \times x > 14 + 40 \times x$$

$$72x > 10$$

$x > \$0.139$/min (equivalent to ~$8.33/h)

If Sarah values time at more than $8.33 per hour, the total cost for the trip by bus will be greater than the cost by car. So if she decides to drive to work, she must value her time as being worth at least $8.33 per hour.

Part (b): Using the figures from the Urban Mobility Report, we can infer that they are using an average cost of delay per hour: $80 billion/4.2 billion hours = $19/h (approximately). From part (a), we know that Sarah has a value of time that is at least $8.33 per hour. The value from the Urban Mobility Report is substantially above $8.33 per hour, so our finding regarding Sarah's behavior is quite consistent with that report. If her value of time were about $19/h, she would certainly choose to drive to work. Alternatively, the value of wasted fuel might be $30 B /year, in which case lost time is worth $50 B/year, hours are still 4.2 B/year, and the value of time is $11/90/h, which is still more than $8.33. You are not required to mention lost fuel, so either answer is OK.

Problem 7-6.

A traveler (we'll call her Nora) commutes to work in a large city. She has a choice between using the bus and driving. If she uses the bus, she can walk 0.2 mi from her apartment to the bus stop (taking 4 minutes), wait an average of 5 minutes for the bus to arrive, and then travel downtown. The bus trip to downtown requires a total of 38 minutes. When she reaches downtown, she has a 3-minute walk to her workplace. In the afternoon, she retraces the path in the other direction, requiring an equal amount of time. The bus fare is $2.00 each direction.

On the other hand, Nora can also choose to drive to work. The distance is 11 mi and the average cost of owning and operating a vehicle can be estimated at $0.50 per mile. In addition, she would have to pay $4.00 per day for parking downtown. If she drives to work, the trip takes her 26 minutes (including the walk time between the parking garage and her workplace) in each direction.

c. It is observed that Nora chooses to drive to work on a regular basis. What does this imply about her "value of time?"

d. How does this relate to the conclusion reached in the 2009 Urban Mobility Report from the Texas Transportation Institute that Americans incur about 4.2 billion hours of delay due to congestion each year, and that delay represents a "cost" of approximately $80 billion?

<u>Solution.</u>

Part (a): Nora's choice to drive to work suggests that the cost (money and time) of making the trip by bus is higher than that by driving.

Travel time by bus: two-way travel time = 2 × (time from apartment to bus stop + waiting time + time on bus + time from downtown to workplace) = 2 × (4 + 5 + 38 + 3) = 100 minutes. The two-way bus fare is 2 × $2 = $4.

Travel time by car: two-way travel time = 2 × (driving + time from parking to workplace)= 2 × 26 = 52 minutes. Additional cost = 2 × ($0.50 × 11) + 4 = $15. If Nora values time by *x* $/min, then

$$4 + 100 \times x > 15 + 52 \times x$$

$$48x > 11$$

$$x > \$0.229/\text{min (equivalent to} \sim\$13.75/\text{h)}$$

If Nora values time at more than $13.75/h, the total cost for the trip by bus will be greater than the cost by car. So if she decides to drive to work, she must value her time as being worth at least $13.75/h.

Part (b): Using the figures from the Urban Mobility Report, we can infer that they are using an average cost of delay per hour: $80 billion/4.2 billion hours = $19/h (approximately). From part (a), we know that Nora has a value of time that is at least $13.75/h. The value from the Urban Mobility Report is substantially above $13.75/h, so our finding regarding Nora's behavior is quite consistent with that report. If her value of time were about $19/h, she would certainly choose to drive to work. Note that the reduction of the total cost of congestion by the value of wasted fuel might change the outcome. If this fuel were worth \$22 billion, the remaining value of congestion would be \$58 billion, at which point the value of time (= \$58B/4.2B h) would be approximately \$13.75/h.

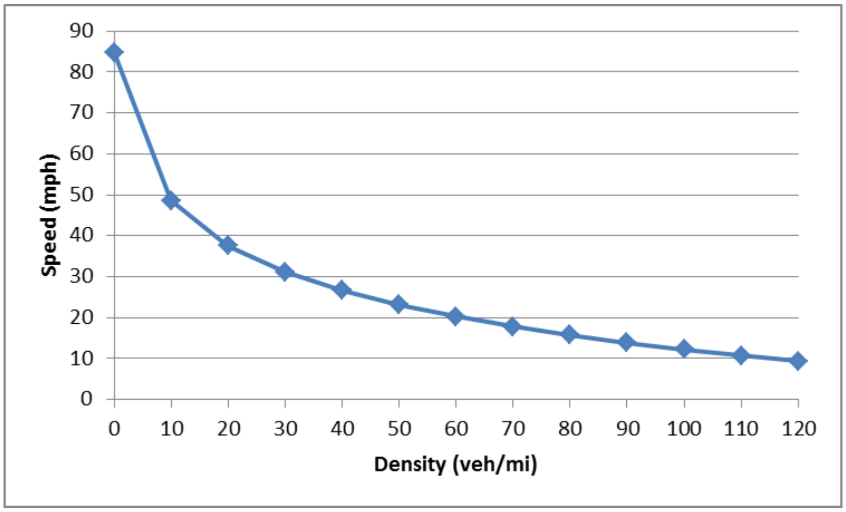
Problem 7-7.

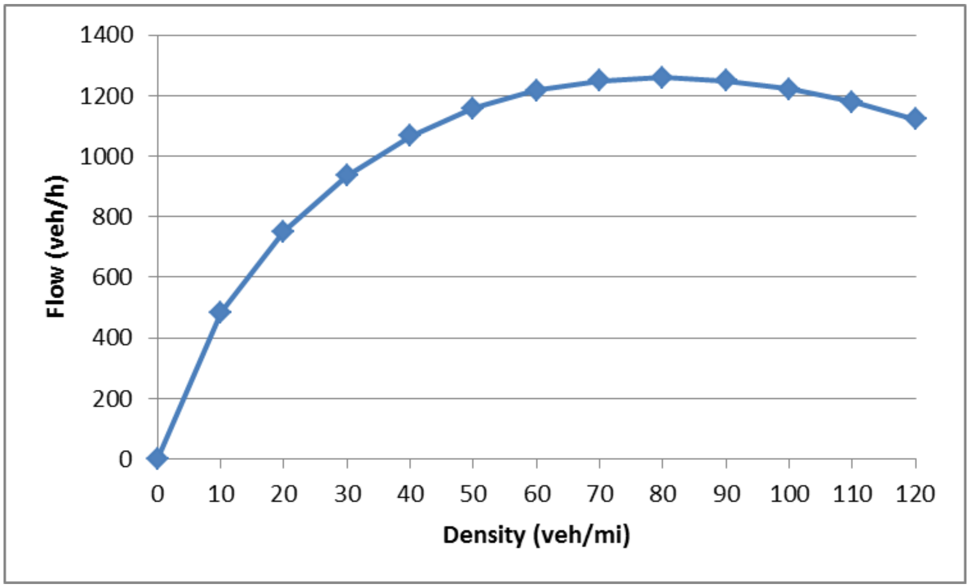
The Greenberg function is a speed-density relationship due to Greenberg, where he fit a statistical equation (u = 84.844 – 15.77 ln k) to the speed-density data taken from the Merritt Parkway. In this sense, the Greenberg model is like the Greenshields model except that the relationship is logarithmic rather than linear. (a) The Greenberg model makes unrealistic predictions for speed over certain ranges of density. For what values of k does the model make such predictions? (b) With the answer to part (a) in mind, plot speed in one figure and flow in another as a function of density. (c) The Greenberg model fitted to the Merritt Parkway implies a capacity of that facility. What is it (in veh/h)?

Solution.

Part (a): The Greenberg model breaks down at both very low and very high values of k. For instance, at k = 0, the model predicts infinitely large speed, and similarly for values k = 1, 2, 3 the corresponding values are u = 84 mi/h, 74 mi/h, and 68 mi/h, respectively. At high values above 120 veh/mi, the model continues to predict small but positive values for u even though loading the roadway to such a degree becomes physically impossible.

Part (b): Figures for speed versus density and flow versus density are shown below. Note that to avoid range problems for k = 0 data point in plotting u versus k, a value of k = 1 is actually used.





Part (c): If speed is given by $u = 84.844 - 15.77 \ln k$, then flow is as below:

$$q = uk = 84.844k - 15.77\, k \ln k$$

To maximize q, set $dq/dk = 0$, so we have $84.844 - 15.77 \ln k - 15.77 = 0$.

Solving for k we obtain $k = 80$ veh/mi.

The capacity q_{max} is then $q_{max} = 84.844 \times 80 - 15.77 \times 80 \times \ln 80 = 1259$ veh/h.

Problem 7-8.

Two lanes approaching a construction bottleneck on a highway must narrow down to one lane. The relationship between speed and density per lane for flow both approaching and within the bottleneck can be modeled using a Greenshields model, with free-flow speed of 120 km/h and jam density of 70 veh/lane/km. Upstream from the queue that forms to enter the bottleneck, traffic approaches with a flow rate of 2700 veh/h. A shock wave forms at the back of the queue as drivers slow down from the upstream speed. What is the speed of the shock wave?

Solution.

Calculations for one lane in bottleneck:

$$q_{max} = \frac{k_j u_f}{4} = \frac{(120)(70)}{4} = 2100 v/h$$

Calculations for two-lane queue waiting for bottleneck:

$$q_2 = \frac{2100}{2} = 1050 veh/lane/hr$$
$$q_2 = uk = 120k - 1.714k^2$$
$$1.714k^2 - 120k + 1050 = 0$$
$$k = 59.8, 10.3$$
$$k = \sim 60 veh/lane/km$$

Calculations upstream from queue:

$$q_1 = \frac{2700}{2} = 1350 veh/lane/hr$$
$$q_2 = uk = 120k - 1.714k^2$$
$$1.714k^2 - 120k + 1350 = 0$$
$$k = 55.9, 14.1$$
$$k = \sim 14 veh/lane/km$$

Speed of shock wave:

$$u_w = \frac{\Delta q}{\Delta k} = \frac{(1050 - 1350)}{60 - 14} = \frac{-300}{46} = -6.5 km/h$$

Problem 7-9.

Note to instructor: Background on the gravity model may be necessary to support the posting of this problem.

A gravity model is a tool used to allocate trips generated at multiple sources in a network to multiple destinations, based on the size of supply and demand, and relative cost of travel between source and destination. Let T_{ij} be trips from i to j, P_i the number of trips generated at i, A_j the overall attractiveness of j, F_{ij} the friction factor from i to j, K_{ij} the socioeconomic factor for combination ij, and W_{ij} the impedance for combination ij. Then, in a simple version of the gravity model F_{ij} is calculated from W_{ij} and T_{ij} from all other factors as follows:

$$F_{ij} = \frac{1}{W_{ij}^c}$$
$$T_{ij} = P_i \frac{A_j F_{ij} K_{ij}}{\sum_m A_m F_{im} K_{im}}$$

Use a gravity model to create a 6×6 origin-destination (OD) flow matrix based on the following information. The flows generated by origin I, attractiveness values for each destination J, and impedance values W_{IJ} are given in the table below. Note that we are not using the socioeconomic adjustment factor in this model (i.e., K_{IJ} = 1 throughout) and the friction factor F_{IJ} is calculated using the above model with c =

2 throughout. Note also that in this example the impedance values are symmetric, i.e., $W_{IJ} = W_{JI}$ for all OD pairs; in the grid shown, the origins *I* are arranged in rows and the destinations *J* are in columns. If it is useful, the data can be copied and pasted from the soft copy of the assignment into a spreadsheet.

Number	Product	Attract.	W-IJ Impedance Table					
0	at *I*	at *J*	1	2	3	4	5	6
1	1500	4	5	10	15	10	10	18
2	700	8	10	5	12	16	13	16
3	400	5	15	12	5	19	12	10
4	500	6	10	16	10	5	12	20
5	3100	1	10	13	12	12	5	14
6	3600	1	18	16	10	20	14	5
Sum	9800							

<u>Solution.</u>

Based on the equation, impedances *W* are converted to friction factors *F* as follows:

I\J	1	2	3	4	5	6
1	0.0400	0.0100	0.0044	0.0100	0.0100	0.0031
2	0.0100	0.0400	0.0069	0.0039	0.0059	0.0039
3	0.0044	0.0069	0.0400	0.0028	0.0069	0.0100
4	0.0100	0.0039	0.0100	0.0400	0.0069	0.0025
5	0.0100	0.0059	0.0069	0.0069	0.0400	0.0051
6	0.0031	0.0039	0.0100	0.0025	0.0051	0.0400

Then for each origin 1, 3, 5, 2, 4, 6 the following tables result based on given levels of outgoing flow, the attraction *A* of destination *J*, the value of *F*, and the *K* factor of *K* = 1 throughout:

Table for *I* = 1 Tot flow = 1500

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0400	1	0.1600	0.477	716
2	8	0.0100	1	0.0800	0.239	358
3	5	0.0044	1	0.0222	0.066	99
4	6	0.0100	1	0.0600	0.179	268
5	1	0.0100	1	0.0100	0.030	45
6	1	0.0031	1	0.0031	0.009	14
Ssum				0.3353		1,500

Table for *I* = 3 Tot flow = 400

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0044	1	0.0178	0.051	20
2	8	0.0069	1	0.0556	0.159	63
3	5	0.0400	1	0.2000	0.571	228

4	6	0.0100	1	0.0600	0.171	69
5	1	0.0069	1	0.0069	0.020	8
6	1	0.0100	1	0.0100	0.029	11
Sum				0.350278	1.000	400

Table for *I* = 5 Tot flow = 3100

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0100	1	0.0400	0.192	594
2	8	0.0059	1	0.0473	0.227	703
3	5	0.0069	1	0.0347	0.166	515
4	6	0.0069	1	0.0417	0.200	619
5	1	0.0400	1	0.0400	0.192	594
6	1	0.0051	1	0.0051	0.024	76
Sum				0.208828	1.000	3,100

Table for *I* = 2 Tot flow = 700

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0100	1	0.0400	0.093	65
2	8	0.0400	1	0.3200	0.748	523
3	5	0.0069	1	0.0347	0.081	57
4	6	0.0039	1	0.0234	0.055	38
5	1	0.0059	1	0.0059	0.014	10
6	1	0.0039	1	0.0039	0.009	6
Sum				0.4280		700

Table for *I* = 4 Tot flow = 500

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0100	1	0.0400	0.120	60
2	8	0.0039	1	0.0313	0.093	47
3	5	0.0028	1	0.0139	0.041	21
4	6	0.0400	1	0.2400	0.717	359
5	1		1		0.021	

		0.0069		0.0069		10
6	1	0.0025	1	0.0025	0.007	4
Sum				0.334545	1.000	500

Table for *I* = 6 Tot flow = 3600

J	A	F	K	AFK	P-ij	Q-ij
1	4	0.0031	1	0.0123	0.080	289
2	8	0.0039	1	0.0313	0.203	732
3	5	0.0100	1	0.0500	0.325	1171
4	6	0.0025	1	0.0150	0.098	351
5	1	0.0051	1	0.0051	0.033	120
6	1	0.0400	1	0.0400	0.260	937
Sum				0.153698	1.000	3600

Finally, the flow values from the rightmost column can be compiled into an OD matrix, in which the total sum of flows equals the total number of trips 9800:

I\J	1	2	3	4	5	6	**Trips**
1							

	716	358	99	268	45	14	1500
2	65	523	57	38	10	6	700
3	20	63	228	69	8	11	400
4	60	47	21	359	10	4	500
5	594	703	515	619	594	76	3100
6	289	732	1171	351	120	937	3600
A*j	1744	2426	2092	1704	786	1048	9800

Problem 7-10.

Note to instructor: This problem considers congestion pricing, which is a technique for counteracting the negative effects of congestion in a transportation network. The goal of congestion pricing is for the number of travelers Q that choose to join the network to pay the marginal cost of adding one more traveler at that level of demand so that the price corresponding to inverse demand P(Q) is equal to marginal cost as given below:

$$P(Q) = MC(Q)$$

Since the cost perceived by the traveler before any price intervention is the average cost AC(Q) and generally AC < MC in these situations, the goal of the calculation is to find the additional charge required to achieve an equilibrium at the desired value of Q.

A 5-mi long section of road has the following relationship between the number of vehicles Q occupying the road (on a per lane basis) and the total cost incurred by all the vehicles TC:

$$TC = 3.93Q,\ \ 0 \le \mathrm{Q} \le 164$$

$$TC = \left(3.89 \times 10^{-5}\right)Q^3 + (0.002443)Q^2 + 0.0002388Q + 404.18,\ \ Q \ge 164 vehicle$$

Thus at Q = 164 vehicles, the two functions are approximately equal to the nearest \$0.01. In addition, the road has the following relationship between the number of vehicles per lane and the price P paid by each individual vehicle (inverse demand curve):

$$P = 25 - 0.0625Q$$

Questions.

a. Derive the function for average cost per vehicle AC and solve for the number of vehicles where the value of AC is equal to the price P (equilibrium flow).
b. Derive the function for marginal cost per vehicle MC and solve for the number of vehicles where the value of MC is equal to the price P (socially optimal level of flow).
c. Calculate the congestion price that must be added to each vehicle to move from equilibrium where $AC = P$ to one where $MC = P$.
d. Suppose the relationship between speed and density per mile on a per lane basis for the road is $u = 83.1 - 0.722k$. What is average speed of vehicles when $AC = P$ and when $MC = P$? Note that the given speed-density relationship does not hold at low values of Q, however in the range around the equilibrium values, it does hold.
e. Draw a graph for the range $0 < Q < 400$ vehicles of (1) the inverse demand curve, (2) the AC curve, (3) the MC curve, and (4) the cost experienced by the vehicles after paying the direct cost and the congestion price (hint: to the AC curve add the congestion price from part (c) throughout. Suggestion: although in general I allow either hand-drawn or computer-printed graphs for most problems, for this one I recommend a computer-printed graph since otherwise it may be difficult to draw the graph accurately.

Solution.

Part (a): For $Q \geq 164$

$$TC = \left(3.89\times10^{-5}\right)Q^3 + \left(0.002443\right)Q^2 + 0.0002388Q + 404.18$$

$$AC = \left(3.89\times10^{-5}\right)Q^2 + \left(0.002443\right)Q + 0.0002388 + \frac{404.18}{Q}$$

$$(3.89\times10^{-5})Q^2 + 0.002443Q + 0.0002388 + \frac{404.18}{Q} = 25 - 0.0625Q$$

$$\Rightarrow Q = 308$$

For $0 \leq Q \leq 164$, $TC = 3.93Q$. So $AC = 3.93 = 25 - 0.0625Q$

We get Q = 337, which is greater than 164. So there is no feasible solution for $0 \leq Q \leq 164$. The conclusion therefore is that Q=308.

Part (b): For $Q \geq 164$

$$MC = 0.0001167Q^2 + 0.004886Q + 0.0002388$$

$0.0001167Q^2 + 0.004886Q + 0.0002388 = 25 - 0.0625Q$

$\Rightarrow Q = 257$

For $0 \leq Q \leq 164$, $TC = 3.93Q$ So $MC = 3.93 = 25 - 0.0625Q$

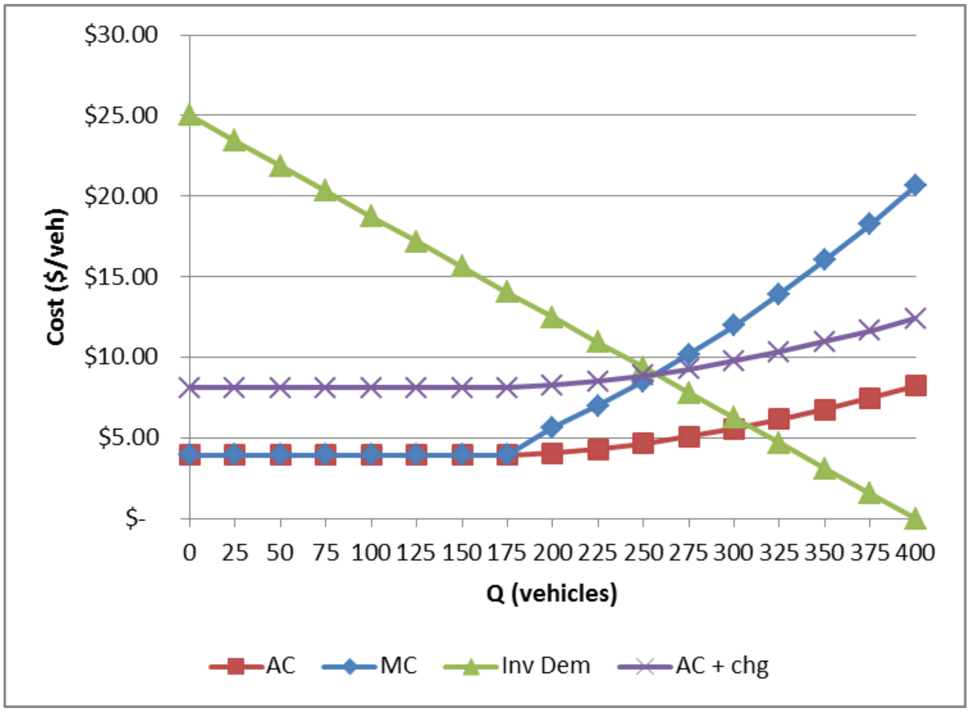
We get Q=337, which is greater than 164. So there is no feasible solution for $0 \leq Q \leq 164$.

Conclusion: Q = 257

Part (c): From (a) and (b), when Q = 257, AC = \$4.77 and MC = \$8.97. So the congestion price is \$(8.97-4.77) = \$4.20.

Part (d): When Q = 308, $k = \frac{308}{5} = 61.6, u = 83.1 - 0.722k = 38.6$ and when Q = 257, $k = \frac{257}{5} = 51.4, u = 83.1 - 0.722k = 45.9$.

Part (e): The resulting graph is the following:



Problem 7-11.

Along a two-lane rural highway where there is no opportunity to pass because of curves and short sight distances, the speed-density relationship can be represented by a Greenshields model with u_f = 62 mi/h and k_j = 120 veh/mi. Traffic is moving at an average speed of 52 mi/h when a piece of farm equipment pulls onto the road and travels at 18 mi/h.

a. What is the speed of the shock wave at the end of the platoon of vehicles that builds up behind the farm implement as it travels?
b. After 3 minutes on the road, the farm implement turns off and leaves the road. At that time, how long (in miles) is the platoon that has built up behind it on the road?
c. At the implied density within the platoon, approximately how many vehicles are backed up in the platoon?
d. When the farm implement has left the road, the platoon disperses from the front at density and speed conditions that represent maximum flow rate for this roadway. What is the speed of the wave representing this dispersal at the front of the platoon?

Solution.

Part (a): We know that $u_f = 62$ mi/h and $k_j = 120$ veh/mi, so the relationship between speed and density can be expressed as $u = 62 - 0.5167\,k$ (using the Greenshields model).

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}, \text{ where } \begin{aligned} &u_1 = 52\,mph,\ k_1 = \frac{62 - u_1}{0.5167} = 19.4\,veh/mile, \quad q_1 = u_1 k_1 = 1009\,veh/hr \\ &u_2 = 18\,mph,\ k_2 = \frac{62 - u_2}{0.5167} = 85.2\,veh/mile, \quad q_2 = u_2 k_2 = 1534\,veh/hr \end{aligned}$$

Therefore, $u_w = \dfrac{1534 - 1009}{85.2 - 19.4} = 8\,mile/hr$.

Part (b): $L = (u_2 - u_w)t = (18 - 8) * \dfrac{3}{60} = 0.5\,mile$

Part (c): Density within the platoon is $k_2 = 85.2$ veh/mi [as computed in part (a)]. Therefore, the number of vehicles in the platoon can be estimated by $L \times k_2 = 0.5 \times 85.2 \approx 43$ veh.

Part (d): Because the speed-density relationship for this facility is a Greenshields model, the maximum flow rate is $\dfrac{u_f k_j}{4} = \dfrac{62(120)}{4} = 1860\,veh/hour$ at a density of $\dfrac{k_j}{2} = \dfrac{120}{2} = 60\,veh/mile$. Thus, the wavefront at the leading edge of the platoon is moving at the following speed:

$$u_w = \frac{1860 - 1534}{60 - 85.2} = -12.9\,mile/hr.$$

Problem 7-13.

A construction zone on a freeway creates a bottleneck section where only one lane out of two is open. For the two-lane section, the "normal" flow-density relationship is as below:

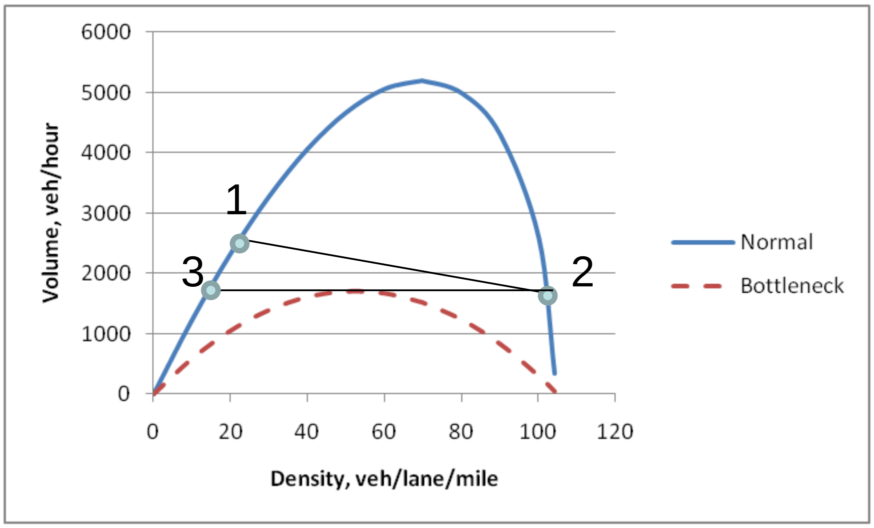
$$q = 11.6k\sqrt{125 - 1.2k}$$

where q is the total flow (veh/h) across two lanes and k is the density in each lane (veh/lane/mi). Approaching the construction zone, traffic flow is 2500 veh/h (or 1250 veh/lane/h). Inside the single-lane bottleneck zone, assume that the flow-density relationship is as follows:

$$q = 65k - 0.62k^2$$

d. What are the speed and density inside the queue that is building up in front of the construction zone?

<u>Solution.</u>
Flow-density diagram for the problem is shown below:



The normal flow-density function for the two-lane facility is plotted as the upper (solid) curve in the figure above. Flow-density in the bottleneck (one lane) is plotted as the "inner" q-k relationship in the figure (the dashed curve). The capacity of the bottleneck can be determined by setting the derivative of flow to zero:

$$\frac{dq}{dk} = 65 - 1.24k = 0 \quad \Rightarrow \quad k = 52.4\, veh/lane/mile$$

At this density, the maximum flow is $q = 65(52.4) - 0.62(52.4)^2 = 1704 \quad veh/hr$.

The flow through the queue in front of the bottleneck is constrained by the bottleneck capacity, so point 2 is at a flow of q = 1704 veh/h. We can solve from the flow-density relationship to find the density inside the queue (point 2) as k_2 = 102.45 veh/lane/mi. At a flow rate of 1704 veh/h (or 852 veh/lane/h), the speed is $u_2 = \frac{852}{102.45} = 8.3$ mph.

e. What is the speed of the wave representing the back of the queue? (10 points)

Solution.

The flow approaching the bottleneck (at point 1) is $q_1 = 2500 \text{ veh/hr}$ at density k_1 and with speed u_1. k_1 can be solved from the flow equation, yielding k_1 = 21.65 veh/lane/mi.

Therefore, the speed of the back of the queue is given by the slope of the line from point 1 to point 2: $u_w = \frac{(q_2 - q_1)/2}{k_2 - k_1} = \frac{(1704 - 2500)/2}{102.45 - 21.65} = -4.9\, mph$. Note that the change in volume must be on a per lane basis, to be consistent with the density on a per lane basis. This is the reason for dividing the volume change by 2. The negative speed indicates that the wave is moving back up the roadway (i.e., the queue is growing over time).

f. What is the speed of the traffic exiting the construction zone and resuming travel on the two-lane facility?

Solution.

At the exit from the bottleneck (point 3), the volume q_3 is the same as through the bottleneck, 1704 veh/h, but there is a reduction in density (to k_3). Using the flow-density expression for the "normal" conditions on two lanes, we find that k_3 = 14.15 veh/lane/mi. The speed of the traffic is $u_3 = \frac{1704/2}{14.15} = 60.2\, mph$.

Problem 7-14.

Suppose that the relationship between speed and density for a particular highway facility is as follows:

$$u = 5.8\sqrt{125 - 1.2k}$$

where the speed, u, is in miles/hour and the density, k, is in vehicles/mile.

a. What is the free-flow speed for this facility?

Solution:

Free-flow speed is at $k = 0$. Therefore $u_f = 5.8 * \sqrt{125} = 64.8$ mi/h.

b. What is the jam density?

Solution:

Jam density occurs when $u = 0$. Therefore, $5.8\sqrt{125 - 1.2k_j} = 0 \quad \Rightarrow \quad k_j \approx 104$ veh/mile.

c. What is the capacity (q_{max}) for the facility?

Solution:

Flow on this facility is $q = uk = 5.8k\sqrt{125 - 1.2k}$. To maximize flow, we differentiate with respect to k:

$$\frac{dq}{dk} = 5.8\sqrt{125 - 1.2k} - 5.8\left(\frac{1.2}{2}\right)k(125 - 1.2k)^{-\frac{1}{2}} = 0$$

$$\Rightarrow \quad k \approx 69.4 \, veh/mile$$

At that density, q_{max} is as below:

$$q_{max} = 5.8 * k_{max}\sqrt{125 - 1.2k_{max}}$$

$$q_{max} = 5.8 * 69.4\sqrt{125 - 1.2(69.4)} = 2600 \, veh/hour$$

d. At what speed is the maximum flow achieved? (5 points)

Solution:

At a density of k_{max} = 69.4 veh/mi, the speed is as below:

$$u = 5.8\sqrt{125 - 1.2(69.4)} = 37.5 \, miles/hour$$

Problem 7-15.

The Bureau of Public Roads flow curve, or "BPR curve" for short, is a well-known model of the effect of traffic density on vehicle speed, similar to the speed/flow/density models presented in this chapter. The following equation defines the BPR curve as below:

$$T = T_0\left(1 + \alpha\left(\frac{v}{c}\right)^{\beta}\right)$$

where T = travel time (min), T_0 = free-flow travel time (min), v = volume (car/h), c = practical capacity (car/h), α and β = parameters to fit the curve to a particular roadway. Suppose we model a freeway over a distance of 30 mi, using the BPR curve. The capacity of the road is 2500 veh/lane/h, and assume three lanes of traffic in the inbound direction for the entire length of the road. For this highway, assume α = 1.5 and β = 4, and a free-flow travel time based on the vehicle being able to travel a constant 60 mi/h. Value lost time due to congestion at $12/h and assume one occupant per vehicle. Throughout this problem, ignore transition effects between peak and non-peak conditions.

a. Plot T as a function of v for traffic volume values from 0 to 3000 cars/lane/h.

b. Suppose 2000 cars per lane per hour travel in the morning rush hour for a 2-hour rush period each workday morning. What is their average speed?

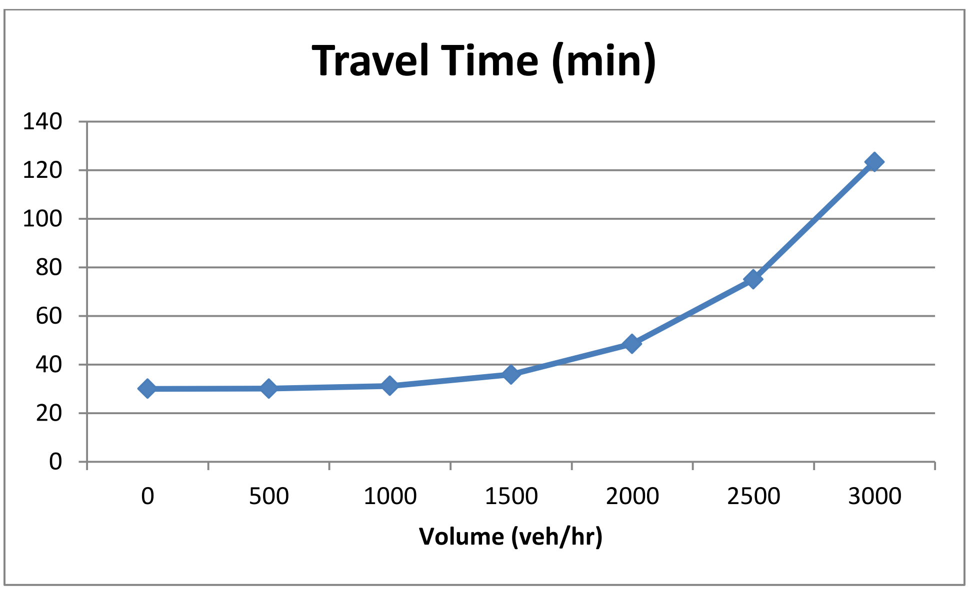
c. What is the value of time lost in one morning compared to the same situation where the same number of cars travel the length of road in time T_0?

Solution.

Part (a): Plot of T: Use 2000 veh/lane/h as an example and substitute appropriate values. Note that travel speed at 60 mi/h is 30 minutes.

$$T = T_0\left(1+\alpha\left(\frac{v}{c}\right)^{\beta}\right) = 30\left(1+1.5\left(\frac{2000}{2500}\right)^{4}\right) = 48.4\,\text{min}$$

Substituting other values gives the following curve:



Part (b): The travel time was already given above at 2000 veh/lane/h. The speed is therefore:

$$(48.4\min)\left(\frac{1h}{60\min}\right)=0.807h$$

$$\frac{30mi}{0.807h}=37.2mph$$

Answer: 37.2 mi/h.

Part (c): The total number of vehicles with 2000 veh/lane/h, three lanes, and 2 hours is 12,000 vehicles. Each vehicle travels for 48.4 minutes instead of 30 minutes, so the lost time is 18.4 minutes. The total value of lost time is as below:

$$(18.4\min/veh)\left(\frac{1h}{60\min}\right)(12{,}000veh)(\$12/h)=\$44{,}234.$$

Answer: $44,234.

Problem 7-16.

Inverse relationship between speed and density: *k* as a function of *u.* A four-lane expressway (two lanes in each direction) is found to have a relationship between vehicle density *k* in veh/km and speed *u* in km/h of the form $k = 71 - 0.64u$. The resulting curve thus crudely approximates that of Fig.7-6; although this observation is made for illustrative purposes only, you should not refer to this figure for solving the problem. Suppose we model a length of highway 12-km long, and set the value of lost time due to congestion at $12/h. Throughout this problem, ignore transition effects between peak and non-peak conditions.

a. Use calculus to calculate the velocity that maximizes flow in km/h, the vehicle density in veh/km at that velocity, and the maximum flow per lane per hour.
b. Suppose cars are traveling at 88 km/h over the length of freeway. How many vehicles are occupying the entire length in one direction at any given time? Recall that there are two lanes.
c. Consider the number of vehicles calculated in part (b). Suppose this group of vehicles is subject to congestion such that they must travel at the velocity that maximizes flow. What is the total value of lost time incurred by these vehicles from traveling over the 12-km length of freeway compared to the original conditions?
d. Now suppose this same group experiences heavy congestion such that they travel at 10 km/h. If they encounter 15 such days in a year, what is the total value of lost time incurred compared to the case where they are able to travel at the original speed?

<u>Solution.</u>

Note to instructor: This problem can also be solved using the Greenshields model relationship, but this solution is not presented here.

Part (a): Take derivative of q and solve for $u_{\max}$ and $q_{\max}$:

$$k = 71 - 0.64u$$
$$q = uk = 71u - 0.64u^2$$
$$dq/du = 71 - 1.28u$$
$$71 - 1.28u = 0$$
$$u_{max} = \frac{71}{1.28} = 55.5 km/h$$
$$k_{max} = 71 - 0.64u_{max} = 71 - 0.64(55.5) = 35.5 veh/km$$
$$q_{max} = k_{max}u_{max} = (35.5)(55.5) = 1969 veh/hr$$

Part (b): The density at 88 km/h is:

$$k = 71 - 0.64u = 71 - 0.64(88) = 14.7 veh/km$$

Therefore, the total occupancy of 12 km is:

$$(2 lanes)(14.7 veh/km)(12 km) = 352.3 veh$$

Answer: ~352 vehicles

Part (c): The value of lost time is calculated by calculating the duration at 88 km/h and 55.5 km/h, and then the total lost time for the group of cars:

$$(12 km)\left(\frac{1h}{88 km}\right) = 0.136h$$
$$(12 km)\left(\frac{1h}{55.5 km}\right) = 0.216h$$
$$0.216h - 0.136h = 0.08h$$
$$(0.08h/veh)(352 veh)(\$12/h) = \$338$$

Answer: $338 lost

Part (d): The total loss is based on the time required at 10 km/h compared to the free-flow speed:

$$(12 km)\left(\frac{1h}{10 km}\right) = 1.2h$$
$$1.2h - 0.136h = 1.064h$$
$$(1.064h/veh)(352 veh)(\$12/h)(15 days) = \$67{,}453$$

Answer: $67,453 lost.

Problem 7-17.

Note to instructor: See discussion of congestion pricing at the beginning of exercise 7-10.

A section of highway has the following characteristics relating speed, *u* (mi/h), flow, *q* (veh/h), and motorist average cost, *p* (cents/mi):

$$u = 60 - \frac{q}{120}$$

$$p = 2.5\left(3 + \frac{200}{u}\right)$$

$$q = \frac{38{,}000}{p} \qquad \text{(demand function)}$$

a. On a plot of price (cents/mi) versus flow (veh/h), show the average cost, the marginal social cost, and the inverse demand function.

Solution.

The inverse demand function is $p = \frac{38,000}{q}$

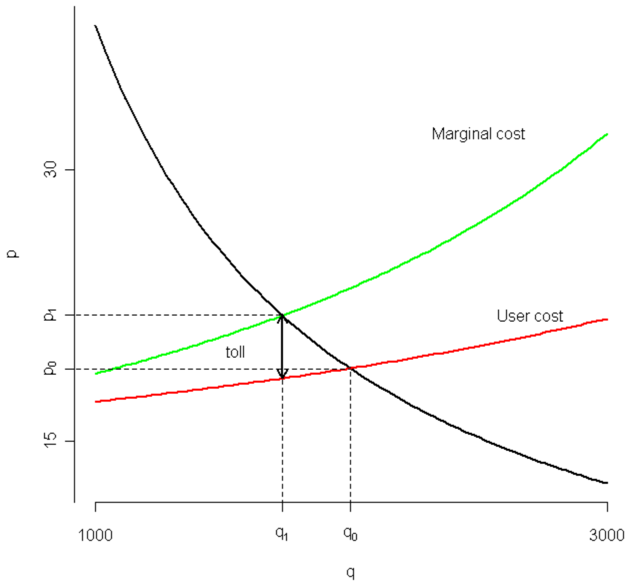
The average travel cost, *p*, can be written in terms of flow, *q*:

$$AC: \square p = 2.5(3 + \frac{200}{u}) = 2.5(3 + \frac{200}{60 - \frac{q}{120}}) = 7.5 + \frac{60000}{7200 - q}$$

The marginal social cost is constructed from the average cost function:

$$MC: \frac{dTC}{dq} = \frac{d(AC * q)}{dq} = \frac{7.5q + \frac{60000q}{7200 - q}}{dq} = 7.5 + \frac{60000}{7200 - q} + \frac{60000q}{(7200 - q)^2}$$

These functions are plotted as shown below.



b. In the absence of road pricing, what is the equilibrium flow and motorist cost that we should expect on this highway section?

Solution.

In the absence of road pricing, the equilibrium flow is defined by the intersection of the inverse demand curve and (average) user cost curve.

$$7.5 + \frac{60000}{7200 - q_0} = \frac{38000}{q_0}$$

$$7.5 q_0^{\ 2} - 152000 q_0 + 273600000 = 0$$

$$\Rightarrow \quad q_0 = 1998\, veh / hr$$

$$\Rightarrow \quad p_0 = \frac{38000}{1998} = 19\, cents / mile$$

c. What is the “economically efficient” flow if all costs were internalized?

Solution.

If all costs were internalized, the flow would be at the intersection point of the inverse demand curve and the marginal cost curve.

$$7.5+\frac{60000}{7200-q_1}+\frac{60000q_1}{(7200-q_1)^2}=\frac{38000}{q_1}$$
$$7.5q_1^{\,3}-146000q_1^{\,2}+1368000000q_1-1969920000000=0$$

$$\Rightarrow \quad q_1=1732\ veh/hr$$
$$\Rightarrow \quad p_1=\frac{38000}{1732}=22\ cents/mile$$

d. What toll should be charged to achieve this equilibrium?

Solution.

The toll that needs to be charged in order to achieve "economically efficient" flow is the difference between average and marginal cost at the efficient equilibrium flow (the height of the arrow shown in the figure).

When $q = q_1 = 1732$, the average user cost is $7.5+\frac{60000}{7200-1732}=18.5cents/mile$. Thus, the toll should be $p_1-18.5=22-18.5=3.5cents/mile$.

e. Suppose the tax of \$0.035/mi is charged on a roadway 44.5 mi in length, over which each vehicle emits 48.5 lb of CO_2 on average when they travel from one end to the other. (Emissions would actually change under less congested conditions but you can ignore this effect.) The drivers who previously drove the roadway and switched away now take public transportation, and the same trip on average emits 11.5 lb of CO_2 per traveler when the emissions from the public transportation vehicle are distributed across the riders. However, the provision of public transportation costs the government \$13.78 per rider above and beyond what the rider pays in fares. The tax collected can be applied toward this cost, but as you can see it does not cover the entire cost. The government is willing to support the public transportation operation by paying the difference as a subsidy provided the net cost per ton (i.e., 2000 lb) of CO_2 is \$100/ton or less (a threshold advocated by noted climate scientist James Hansen among others). Calculate the total CO_2 saved and the total cost of the subsidy per hour, and state whether or not the cost per ton meets the government's condition.

Solution.

From the previous problem, there are 1732 veh/h taking the toll road and paying the toll. Since the distance is 44.5 mi and the toll costs \$0.035/mi, the total revenue is:

$$Rvnue=(1732)(44.5)(\$0.035)=\$2{,}698$$

The number of riders is 1998 – 1732 = 266. The total funding requirement and shortfall after using the toll road revenue is:

$$Cost=(\$13.78)(266)=\$3{,}665$$
$$Shortfall=\$3{,}665-\$2{,}698=\$967$$

The CO_2 savings per rider switched is 48.5 – 11.5 = 37 lb CO_2. Calculating total CO_2 savings, dividing by total cost, and converting to cost per ton reduced:

$$(266)(37) = 9842 lbs$$

$$\frac{\$967}{9842 lbs} = \$0.0983 / lbCO_2$$

$$\$0.0983\left(\frac{2000 lbs}{ton}\right) = \$197 / ton$$

Thus the total savings is 9842 lb CO_2 and the cost is \$967. Since the figure of \$197/ton is greater than the \$100 maximum, the requirement is not met.

Problem 7-18.

Throughout this problem, consider a freeway during morning rush hour with three lanes in the inbound direction, a length of 20 mi, and time valued at \$14/h.

a. Under free-flow conditions, vehicles travel at v = 65 mi/h and a density of k = 25 veh/mi (approximately the right spacing to give the ”two-second separation” recommended by highway safety experts). What is the total number of vehicles that can fit on the roadway at this spacing (3 lanes × 20 mi)?
b. Now consider the total number of vehicles from part (a) distributed over three lanes and 20 mi to be a “cohort.” As vehicle speeds drop below approximately 65 mi/h, the speed of vehicles u as a function of density k adopts the following form: $u = 80 – 0.571k$. What is the value u_{max} that maximizes throughput q? At this value, what is q_{max} and how many minutes does it take for a vehicle in the cohort from part (a) take to cover the 20 mi, and what is the total dollar value of time lost for the cohort compared to if they had been able to travel at 65 mi/h free-flow conditions?
c. Now suppose that the cohort encounters severe congestion so that speed is reduced to 10 mi/h. What is the value of q, and what is the new total value of time lost compared to if they had been able to travel at 65 mi/h free-flow conditions?
d. True or False. The economic impact from congestion in going from 65 mi/h to u_{max} is modest compared to going from 65 mi/h to 10 mi/h? No explanation needed, just state T or F.

Solution:

Part (a): Multiplying density, length, and number of lanes gives:

$$N_{vehicle} = (25v / m)(20mi)(3la) = 1500veh$$

Part (b): This part could be solved using calculus, but since the Greenshields model applies, we will use the rules associated with it. According to this model:

$$u_{max} = \frac{u_f}{2} = \frac{80}{2} = 40mph$$
$$k_j = 140veh/mi$$
$$k_{max} = \frac{k_j}{2} = \frac{140}{2} = 70veh/mi$$
$$q_{max} = u_{max}k_{max} = 40 \cdot 70 = 2800veh/h$$

Since the roadway is 20 mi long, it takes 20/40 = 0.5 hour = 30 minutes to cover the distance. At 65 mi/h, the time required is:

$$t = \frac{20mi}{65mph} \cdot \frac{60m}{1h} = 18.5m$$

Therefore, the time lost per vehicle is 30 – 18.5 = 11.5 minutes. For 1500 vehicles, converting from minutes to hours and valuing time at $14/h, this results in $4025 lost due to congestion.

Part (c): To calculate *q*, it is necessary to rearrange the speed-density equation to calculate *k* as a function of *u*:

$$u = 80 - 0.571k$$
$$k = 140.1 - 1.751u$$
$$k(u = 10mph) = 140.1 - 1.751(10)$$
$$= 122.6veh/mi$$

Therefore, *q* = (10 mi/h)(122.6 veh/mi) = 1226 veh/h. The time required to travel 20 mi is 2 hours or 120 minutes, so the lost time is 120 – 18.5 = 101.5 minutes or 1.69 hours. At $14/h for 1500 vehicles, the value is $35,525.

Part (d): True. (Although not required as part of the answer, the comparison shows the relatively high impact of severe congestion.)

Problem 7-19.

A metropolitan region has 145 mi of limited-access highways and an area of 412 mi^2. The transportation authorities are considering an ITS system with three parts: (1) central control system, (2) area substations, and (3) roadside sensing and information provision systems. The central control system costs $200 million, the area substations cost $6 million and have a maximum area coverage of 10 mi^2, and roadside systems cost $2 million per mile. The labor force in the metropolitan area is 2.2 million people and 80% commute in some form of transportation that uses the highway system (car, carpool, vanpool, buses that use the highways). The average lost time in traffic is 62 hours per year per highway commuter, and a consultant study concludes that the ITS system would reduce time lost by 5 hours per highway commuter per year. The value of time is $12/h for time lost in congestion.

Use the benefit/cost or B/C ratio to evaluate the investment. The metro region adopts a discount rate of 6% and an investment lifetime of 15 years. Include the benefit to the commuters in the benefit part of the ratio, and the cost to the metro region in the cost part.

a. Calculate the annual benefit from reduced lost time in congestion.
b. Calculate the annualized cost by discounting the upfront cost of the system over the lifetime of the project.
c. Calculate the B/C ratio. What do you conclude about this project?
d. Short discussion: Give at least one example of an element left out of the benefit side of the B/C ratio by this limited analysis. Give at least one example of an element left out of the cost side.

Solution:

Part (a): Annual benefit:

$$(2.2M)(80\%)(5h/y)(\$15/h) = \$105.6M$$

Part (b): Annual cost. Note that the calculation assumes that since there are 412 mi^2 and 412 > 410, 42 rather than 41 substations are required.

$$CapCost = \$200M + \$6M \cdot 42 + \$2M \cdot 145 = \$742M$$

$$ACC = (\$742M)(A/P, 6\%, 15) = \$76.4M$$

Part (c): B/C ratio = 105.6/76.4 = 1.38. Therefore, the investment is viable.

Part (d): As examples, the benefit side leaves out the value of fuel saved. The cost side leaves out the ongoing maintenance cost beyond the annual capital cost.

Chapter 8 Public Transportation and Multimodal Solutions

Problem 8-1.

An urban arterial street with width of 15 ft is observed to have vehicle velocity *u* as a function of density *k* in the form of $u = 55 – 0.381\ k$ for values of *u* between 0 and 45 mi/h. Private vehicle traffic travels at an average speed of 17 mi/h. The average occupancy of the vehicles is $\alpha = 1.2$ persons/vehicle, and the length is 16 ft, which is typical of compact to midsized vehicles. What is the average shadow per vehicle, the time-area factor, and the per person time-area factor? Suppose 40-ft buses with average $\alpha = 22$ persons/vehicle and the same shadow as the private vehicles travel on the same street. Ignoring time spent on stopping and starting at stops, what is the time-area factor and per person time-area factor in this case?

Solution.

The solution to the average shadow per vehicle can be found using the speed *u* and rearranging the given equation to solve for vehicle density *k* as a function of *u*:

$$u = 55 - 0.381k$$

$$k = \frac{55}{0.381} - \frac{1}{0.381}u = 144.4 - 2.62u$$

Substituting $u = 17$ mi/h gives $k = 144.4 – 2.62(17) = \sim 100$ veh/mi. On this basis, $L_{\text{total}} = 52.8$ ft including the shadow, and subtracting the vehicle length of 16 ft leaves 36.8 ft for the shadow. The time-area factor can then be calculated from the flow value *q*:

$$q = uk = (17)(100) = 1700 veh/hr$$

$$\overline{TA} = \frac{WL}{q} = \frac{(15)(52.8)}{1700} = 0.466\ ft^2 hr/veh$$

Next, we use the average occupancy value to calculate $\overline{TA}_p$:

$$\overline{TA}_p = \frac{WL}{\alpha q} = \frac{792}{(1.2)(1700)} = 0.388\ ft^2 \cdot hr/pers$$

For the bus, use the length of the bus of 40 ft and the shadow of 36.8 ft to calculate *k* and *q*.

Hint to instructor: for simplicity, the bus density can be treated as a continuous lane of buses traveling at the same speed as car traffic. Thus the total length occupied per bus including physical length and shadow is 76.8 ft. Divide feet per mile by length per bus to calculate density:

$$k = \frac{ft/mi}{L} = \frac{5280}{76.8} = 68.75 \text{ veh/mi}$$

Since the buses are moving at 17 mi/h, the values of q, $\overline{TA}$, and $\overline{TA}_p$ are:

$$q = uk = (17)(68.75) = \sim 1169 veh/hr$$
$$\overline{TA} = \frac{WL}{q} = \frac{(15)(68.75)}{1169} = 0.986\, ft^2 hr/veh$$
$$\overline{TA}_p = \frac{WL}{\alpha q} = \frac{(15)(68.75)}{(22)1169} = 0.0448\, ft^2 hr/\,person$$

Note the substantial reduction in per person time-area factor, which is to be expected from a well-utilized urban bus.

Problem 8-2.

An origin-destination (OD) matrix for a bus route with 8 stops where the distance between each stop is 0.5 mi is given below. (a) What is the boarding and alighting count for each stop 1 through 8? (b) What is the space-averaged load factor for this run? (c) What is the segment of the run with the highest number of passengers on board, and what is the number on board in that segment? If the maximum occupancy of the bus is 22 seated and 44 standing, is the maximum capacity exceeded?

Stops	S2	S3	S4	S5	S6	S7	S8
S1	1	2	2	4	3	1	9
S2		0	1	0	1	3	6
S3			1	1	0	2	4
S4				3	2	0	8
S5					1	3	6
S6						2	5
S7							6

Solution.

Part (a): Boarding and alighting count, based on adding up all passengers going to other stops boarding at each stop, and all passengers arriving from other stops: (22, 0), (11, 1), (8, 2), (13, 4), (10, 8), (7, 7), (6, 11), (0, 44), where each parentheses contains (number boarding, number alighting).

Part (b): The load values for the seven segments are 22, 32, 38, 47, 49, 49, and 44. Since each segment has 0.5 mi of length, the actual load distance value is the sum of each count multiplied by 0.5 mi, or 140.5. The capacity of the bus is 60 and there is only one transit unit, so m = 1 and

$$LF_{avge} = \frac{\sum_j P_j l_j}{mC_v L} = \frac{140.5}{(1)(60)(3.5)} = \frac{140.5}{210} = 66.9\%$$

Part (c): The maximum occupancy on the bus is 49 passengers on segments 5 and 6 (i.e., between stops 5 and 6, and again between stops 6 and 7). Since the maximum is 60, the maximum capacity is not exceeded.

Problem 8-3.

A transit bus costs $300,000 and has an expected life of 12 years with $0 salvage value; it drives 25,000 mi/year. It gets 3.5 mpg diesel, and diesel costs the transit agency $3/gal. The bus completes a round-trip route of 10 mi/p. The bus must recover $50/h for wages to pay not only for the driver wages but office and support staff as well. The discount rate is 7%. (a.) If the cost of the bus is amortized on an annual (as opposed to monthly) basis, what is the annualized cost of the bus? (b) Calculate the total cost per round trip for the bus, taking into account capital cost, fuel cost, and wage cost values given above; do not include any other costs. (c) Calculate the number of riders that must board at an average fare of $1.50 for the bus to breakeven. (d) Name one cost component that the calculation does not include (more than one possible right answer).

Solution.

Part (a): The $300,000 capital cost of the bus is discounted using the 12-year lifetime and 7% discount rate incorporated in the standard discounting equation (see appendix):

$$A/P = \frac{i(1+i)^N}{(1+i)^N - 1}$$

$$A/P = \frac{(0.07)(1+0.07)^{12}}{(1+0.07)^{12} - 1} = 0.1259$$

$$(A/P)(P) = (0.1259)(\$300{,}000) = \$37{,}771$$

Part (b): Dividing annualized capital cost by distance per year gives ($37,800)/(25,000 mi) = $1.51/mi. Based on fuel cost of $3/gal, distance of 25,000 mi/year, and 3.5 mpg, the fuel cost is $0.86/mi or $8.60 per 10 mi round trip. Total cost per 1 hour round trip is wages + capital + fuel = $50 + $15.10 + $8.60 = $73.70 per round trip.

Part (c): At $1.50 per fare, the required number of boarding is:

$$\frac{\$73.70}{\$1.50} =\sim 49$$

Part (d): An example of a cost not included is repairs and maintenance.

Problem 8-4.

A light rail transit (LRT) system runs once every 15 minutes from 6 AM to 11 PM and carries a total of 15,000 riders in to the city per day (the same 15,000 riders then return via LRT to their point of origin). The line maintains a uniform schedule with departures at :00, :15, :30, and :45 past the hour, with the first departure leaving at 6 AM from each end point and the last departure leaving at 10 PM from each end point. The line runs for 20 mi in each direction. The LRT averages 17 mi/h when in operation from one end point to the other, and then waits for the next departure time before leaving to return in the other direction. Among the riders of the new service, 90% are previous car commuters who traveled in single occupant vehicles (SOVs) an average of 15 mi one way to work, but now leave their cars at home. (a) To the nearest minute, how long does the LRT vehicle wait at the end of its run before starting its return run in the opposite direction? (b) How many LRT vehicles are required to maintain the schedule with 15-minute headways and each run ending and starting as described in part (a)? You can ignore the need for spare vehicles. (c) If both car and LRT vmts are taken into account, how many net vmts per day are reduced by shifting the SOV commuters to LRT?

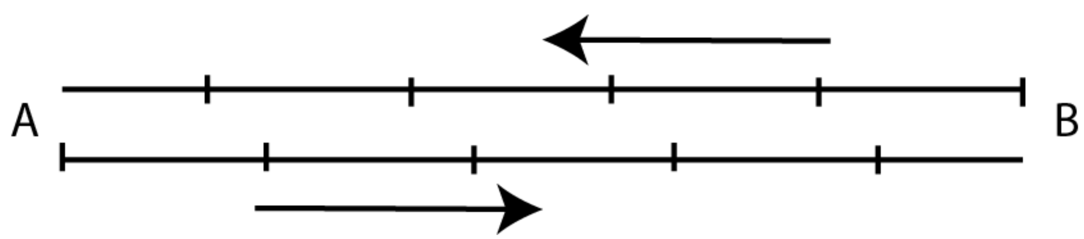
Solution.

Part (a): Call the two end points of the system A and B. First note that every 15 minutes an LRT leaves from each end point. Take an LRT departing from A: the time required to reach B is:

$$\frac{20mi}{17mph}\cdot\frac{60\min}{h} =\sim 71\min$$

The LRT will then depart from B when 75 minutes have passed since it left A. Thus the wait time is 4 minutes.

Part (b): Since the LRTs travel at 17 mi/h, in 15 minutes they travel 4.25 mi. Therefore, when the fifth LRT is departing from either terminal, LRTs number 1 to 4 are located 4.25 mi, 8.5 mi, 12.75 mi, and 17 mi from the terminal. Thus there are five LRTs traveling in each direction or 10 total. The diagram below shows the position of the LRTs at the instant that a service departs from either end point, where each short-line segment crossing the route represents an LRT:



Part (c): Note to instructors: You may wish to clarify with students that, as simplifying assumptions, the service starts from both directions at the beginning of the day, and that LRTs can go out of service at the end of the day whenever they reach either end point. For LRT, the number of vmt traveled requires an analysis of the total number of runs of the LRTs required to complete the schedule. LRTs leaving either end point at 6 AM will make their next departure at 7:15 AM from the opposite end point, the following at 8:30 AM, etc. Continuing this process leads to the same vehicle making its last departure at 9 PM from the terminal from which it started (either A or B). There are five LRTs operating in each direction, so after the 9 PM departure, LRTs 2 to 5 will depart at 9:15, 9:30, 9:45, and 10 PM. Thus each LRT makes exactly 13 departures per day, and ends its day at the other terminal from the one where it started. Total vmt are therefore the product of the number of LRTs, the number of departures per LRT, and the miles traveled per departure (i.e., the one-way distance on the route):

$$VMT = (10Veh_{LRT})\left(13\frac{trips}{Veh_{LRT}}\right)\left(20\frac{mi}{trip}\right) = 2600mi/day$$

On the SOV side, the total vmt per day is the product of fraction of riders who previously drove SOVs and the round-trip distance per day:

$$VMT = (15{,}000riders)(0.9)\left(30\frac{RTmiles}{day}\right) = 405{,}000mi/day$$

Thus the reduction is the difference or 405,000 – 2,600 = 402,400 mi/day. Note that this calculation does not take into account the larger spatial footprint of the LRT or the larger resource requirement per vmt.

Problem 8-5.

A city builds a light rail transit (LRT) system to reduce traffic on the roadways. The system runs once every 15 minutes during the peak from 6 AM to 10 AM and again from 4 PM to 8 PM. During the off-peak (10 AM to 4 PM and 8 PM to 11 PM) it runs every 30 minutes, i.e., the last LRT run departs at 10:30 PM. The average occupancy of the vehicle is 50%. Each multiunit LRT train has a capacity of 360 riders. Among the riders of the new service, 70% are previous car commuters who traveled in single occupant vehicles (SOVs) an average of 28 mi per round trip from home into the city. The rest used previously existing public transportation, walked, or bicycled. Assuming 250 working weekdays per year and ignoring reduced driving from LRT operations on weekends, what is the total reduction in SOV travel in vehicle-miles per year thanks to the new system?

Solution.

The approach to this problem is to calculate total daily savings and then multiply by the number days per year. There are 32 departures during the peak hours and 18 departures in the off-peak, for a total of 50 departures per day. The occupancy is on average 50% or 180. Assuming the figure of 180 is equivalent to the total number of boardings in one direction on one round trip of the LRT (it could reasonably be assumed that on average the same passenger will return by LRT, but that the return boarding should not be counted for calculating avoided vmt), and that 70% of those boardings represent avoided 28-mi round-trip car trips, the savings are the following:

$$VMT_{daily} = (50)(180)(0.7)(28) = 176{,}400mi$$

Annual savings are then (176,400)(250) = 44.1 million vmt avoided.

Problem 8-6.

An LRT vehicle, or rail car, costs \$4.2 million and has an expected life of 20 years; it travels about 112,000 mi/year. It consumes 2.3 kWh of electricity per mile of travel and electricity costs \$0.08/kWh to the transit agency. The rail car travels on a route that is 32 mi long and requires 1 hour and 40 minutes to travel one way. The rail car must recover \$65/h for wages to pay not only for the driver but office and support staff as well. (a) If the interest rate for loans is 7% and the transit company borrows the cost of the rail car to repay it over 20 years with one annual payment per year, what is the annualized cost of amortizing the rail car? (b) Calculate the total cost per round trip for the rail car, taking into account capital cost (use the value with interest from part (a), not simple payback), energy cost, and wage cost values given above; do not include any other costs. (c) Calculate the number of riders per year that must board at an average fare of $2.25 for the rail car to breakeven. (d) Name one cost component that the calculation does not include (more than one possible right answer).

Solution.

Part (a): The calculation of the discounting factor needed is:

$$(A/P, i\%, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$$

$$(A/P, 7\%, 20) = \frac{0.07(1+.07)^{20}}{(1+.07)^{20} - 1} = \$0.0944$$

The annualized cost is therefore (0.0944)($4.2M) = $396,450. Alternatively, one could use the spreadsheet function '=-PMT(7%,20,$4.2M) = $396,450.

Part (b): The approach here is to calculate annual cost and then divide by number of round trips per year. Since the LRT goes 32 mi in 1.67 hours, it is averaging 19.2 mi/h. Therefore, 112,000 mi will require 5845 hours per year. The wage cost @ $65/h is then $379,925.

The cost of electricity per year is:

$$C_{elec} = (112{,}000 mi)\left(2.3 \frac{kWh}{mi}\right)(\$0.08 / kWh) = \$20{,}608$$

Total cost per year is then:

$$C_{tot} = \$396{,}450 + \$379{,}925 + \$20{,}608 = \$796{,}983$$

Since each round trip is 64 mi, the total number per year is 1750. The cost per round trip is then:

$$C_{roundtrip} = \frac{\$796{,}983}{1750} = \$455$$

Part (c): The number of riders for breakeven is:

$$N_{breakeven} = \frac{\$796{,}983}{\$2.25} = 354{,}215$$

Note that since there are 1750 round trips or 3500 one-way trips, the number of boardings per trip is ~101.

Part (d): Different answers are possible, but one example would be maintenance.

Problem 8-7.

A bus system runs once every 15 minutes from 6 AM to 11 PM, with the last departure leaving each end point at 10:30 PM. The bus line maintains a uniform schedule with departures at :00, :15, :30, and :45 past the hour, with the first departure leaving at 6 AM from each end point and the last departure leaving at 10:30 PM from each end point. The line runs for 5.2 mi in each direction, and buses average 12.48 mi/h when in operation from one end point to the other. When each bus reaches its end point, it waits for the next departure time before leaving to return in the other direction. How many minutes does each bus wait between finishing one run and starting the next, to the nearest minute? How many buses are required to maintain this schedule, not including spare buses in case one or more are out of service for repairs, etc.?

Solution.

For waiting time: The distance is 5.2 mi, and the speed is 12.48 mi/h, so the time required is 0.4167 hour, or 25 minutes. A bus that leaves at :00 after the hour would therefore arrive at :25 after at the terminal, and leave again at :30. Therefore, the total wait time is 30 – 25 = 5 minutes.

For number of buses required: The bus described previously will leave the far-end terminal at :30 past, and arrive at the originating terminal at :55 past, so that 60 minutes after it starts it can leave again. There are three other such start times at :15, :30, and :45 past, and they all are able to start again after 60 minutes from the original terminal. Therefore, the number of buses is four.

Problem 8-8.

A transit agency is considering adopting a route deviation (RD) system for a route that currently carries 300,000 passengers per year. In the "catchment area" of the route, there are also 100,000 boardings per year for the agency's paratransit system. The paratransit ridership includes passengers who for various reasons (lack of mobility, illness, etc.) are not able to travel to and from the regular bus stops.

If the agency adopts RD, they estimate that half of the current paratransit ridership will begin to use the buses rather than continuing to use paratransit. In addition, RD is expected to generate 50,000 new boardings per year that did not previously travel by the route.

The fare for riders on the transit route, whether it uses RD or not, is $1.50 per ride. The agency charges twice the regular fare for use of the paratransit system, which is the maximum paratransit fee allowed by law, reflecting the higher cost of operating paratransit. The cost for each type of rider for the regular bus rider, rider using the RD equipped bus, and paratransit rider is $1.10, $1.10 + cost of fuel for deviation, and $16.00 per passenger, respectively. Note that this is a simplistic interpretation of cost: in reality, adding a single passenger to a bus that is already operating incurs some varying "marginal cost" per passenger, but this interpretation is outside the scope of the problem.

In assessing the effect on cost of adopting RD, the agency must take into account the extra cost of fuel to cover route deviations. The average deviation from the route to pick up an off-route rider is 0.375 mi (for either paratransit or general population riders); ignore any deviation that might occur to drop off riders at their destination. The vehicles have a fuel economy of 6-mpg diesel, and diesel costs the agency $2/gal (agencies do not pay sales or highway taxes).

The agency can potentially pay for the investment in RD by reducing paratransit cost and increasing ridership on the route, but they will also incur an annual capital cost of $460,000 to pay for the needed equipment. Note that this is a discounted annual cost that takes into account the initial cost and investment lifetime; you do not need to calculate any discounted cash flow in this problem.

Question: use appropriate calculations to evaluate whether the investment in RD benefits the agency or not. Hint: It may simplify the "book keeping" in this problem to calculate the net revenue or cost from each type of passenger, taking into account fare received and cost incurred, and then multiplying by the number of passengers in each scenario to add up total revenues and cost.

Solution.

The approach is to calculate the net revenue or cost per passenger for each type, and then multiply by the total number of passengers to add up annual cost.

Note that for the route deviation the cost of fuel must be added on:

$$(0.375mi)\left(\frac{1gal}{6mi}\right)(\$2/gal) = \$0.13$$

$$C_{RD} = \$1.10 + \$0.13 = \$1.23$$

Net revenue for regular passengers is $1.50 − $1.10 = $0.40. For RD passengers it is $1.50 − $1.23 = $0.27. For paratransit the net cost is $16 − $3 = $13.

We can then present tables for total net cost per year. For the base case, since there are zero RD passengers:

Psgr type	Number	Net revenue	Total
Reg psgr	3.00E+05	$ 0.40	$120,000
Deviation psgr	0	$0.27	$ -
Paratransit	1.00E+05	$13.00)	$1,300,000
Total			$1,180,000

In the case of the RD system, there are 300,000 regular passengers, plus a reduced number of 50,000 paratransit passengers, plus 100,000 RD passengers (50,000 shifted from paratransit plus 50,000 new passengers):

Psgr type	Number	Net revenue	Total
Reg psgr	3.00E+05	$0.40	$120,000
Deviation psgr	1.00E+05	$0.27	$27,000
Paratransit	5.00E+04	$13.00	$650,000

Total			$503,000

In addition, the total cost per year for RD includes the annual cost for the technology, giving $503,000 + $460,000 = $963,000.

Therefore, $1,180,000 − $963,000 = $217,000 in savings per year accrue, so the investment is beneficial.

Problem 8-9.

Cars and buses travel at rush hour with average occupancy of 1.2 and 30 persons, respectively, with a lane width of 15 ft. For the car, use travel speed of 27 mi/h, speed to density relationship of u = 54.4 – 0.48 u, and an average car length of 16 ft. For the bus, use 40-ft length, the same shadow length as for the car, and an average speed of 15 mi/h as the bus needing to stop for boarding/alighting and time to merge in and out of traffic. What is the average time-area consumption in ft^2-h/person for each mode?

Solution.

The solution is first obtained for the cars. Since the given speed is 27 mi/h, it can be shown that the corresponding density is 57.08 veh/mi; this can be confirmed as shown:

$$u = 54.4 - 0.48(57.08) = 54.4 - 27.4 = 27 mph$$

The total length occupied by each vehicle including shadow is therefore:

$$L_{tot} = \frac{5280\, ft/mi}{57.08 veh/mi} = 92.5\, ft/veh$$

The per-person time-area factor is then:

$$TA_{pers} = \frac{(92.5)(15)}{(1.2)(27)(57.08)} = 0.750$$

The answer is therefore 0.75 ft^2-h/person. For the bus, the shadow must first be calculated from the car values:

$$L_{shadow} = L_{tot} - L_{veh} = 92.5 - 16 = 76.5\, ft$$

The total length for the bus is then 76.5 + 40 = 116.5 ft/veh. Density is based on length per mile divided by length per bus, and flow in turn based on density and speed of 15 mi/h:

$$k = \frac{5280}{116.5} = 45.32 veh/mi$$
$$q = uk = (15)(45.32) = 679.85 veh/h$$

The factors needed for the time-area factor are now known:

$$TA_{pers} = \frac{(116.5)(15)}{(30)(679.85)} = 0.0857$$

The result of 0.0857 ft^2-h/person is an indication of the density improvements that are possible with well-used bus service.

Problem 8-10.

An urban traveler is considering a trip from their home to a destination in a city that is laid out in a perfect grid of square blocks, each block 0.25 km on a side. They live on a corner on one block, and are contemplating a trip to a destination on another corner 20 blocks due east of their home and also on the corner. Either car or bicycle is the option for this trip (ignore extra distance incurred to find parking).

Alternatively, the traveler could walk one block south from their house, catch a bus on the corner there, travel 20 blocks east, alight, and walk one block north to the destination. The bus operates at 12-minute headway.

Solution.

Since the distance is 20 blocks or 5 km, the travel distance by car and bike are 10 minutes and 19 minutes, respectively.

For travel by bus, the traveler must travel a total of 0.5 km to and from bus stops to make the trip, for a total of 6 minutes of walking. In addition, they could expect to spend half of the average headway or 6-minutes waiting and travel the distance of 5 km in 15 minutes. Substituting into Eq.8-9:

$$t_{OD} = t_a + t_w + t_{line} + t_e = 3 + 6 + 15 + 3 = 27m$$

Therefore the total time spent via bus is 27 minutes. Lastly, the ratios to driving are 19/10 = 1.9 and 27/20 = 2.7 for bicycle and bus, respectively.

Problem 8-11.

A municipal transit agency is considering an investment in a small shuttle bus service with the following parameters. The initial cost is $360,000 for two vehicles, and the ridership averages 200,000 riders per year. The total operating cost per year for the bus for wages, fuel, repairs, and overhead is $325,000. The average fare paid by each rider is $1.50. The investment is to be evaluated over a 12-year time horizon, and the expected salvage value at the end of the project is $20,000. The MARR is 6%.

a. Calculate the NPV of this investment. Is it financially attractive?
b. Calculate the operating credit per passenger boarding which the government would need to give to the investment in to make it breakeven financially.

Solution.

The annualized value of the buses is =-PMT (6%, 12year, $360K) = $42,940. The value of fares paid per year is 200,000 @ $1.50 per rider or $300,000. Thus the total cost per year and net surplus or shortfall per year are:

$$Cost = \$325{,}000 + \$42{,}940 = \$367{,}940$$
$$Net = \$300{,}000 - \$367{,}940 = -\$67{,}940$$

Therefore, there is a net shortfall of $67,940 per year, and the investment is not financially attractive by itself.

Part (b): The shortfall can be covered by giving a credit per passenger based on dividing the total shortfall by the total number of riders:

$$\frac{\$67{,}940}{200{,}000} =\sim \$0.34 / rider$$

Problem 8-12.

Calculate the conventional and modified B/C ratio for Problem 8-11, without the government operating credit per passenger. Refer to Appendix C for background on B/C ratio, and use annual worth values.

<u>Solution.</u>

Based on annual worth values, the initial cost, operating cost, and total benefit are $42,940, $325,000, and $300,000, respectively. Substituting into B/C equations gives:

$$Conv B/C = \frac{\$300{,}000}{\$42{,}940 + \$325{,}000} = 0.82$$
$$Mod B/C = \frac{\$300{,}000 - \$325{,}000}{\$42{,}940} = -0.58$$

Problem 8-13.

Note to instructor: You may wish to provide additional background for this problem by discussing the supporting material on transportation economics and decision making in Chap. 4.

This problem extends the discussion of urban public transportation modes by considering demand for travel as a function of transit fare, consumer surplus accrued to passengers by selling fares at a price lower than the maximum that they would be willing to pay, and fare elasticity of demand in response to a change in price. Fare elasticity is defined qualitatively as the percent change in demand in response to a 1% change in price, or stated mathematically, elasticity e_p is defined at location *i* on the demand curve as:

$$e_p = \frac{\partial Q_i}{\partial P_i} \cdot \frac{P_i}{Q_i}$$

Here P_i is the price at point *i* on the demand curve, Q_i is the quantity demanded, and the partial derivative is taken of *Q* as a function of *P* at point *i*.

The public transit operator in a major city offers an express bus service from a downtown location to the airport at a fare of $5.00 for a one-way trip. Suppose the demand function for this service is:

$$Q = 1000 - 160\ P$$

where Q is in daily one-way trips and P is the fare in dollars.

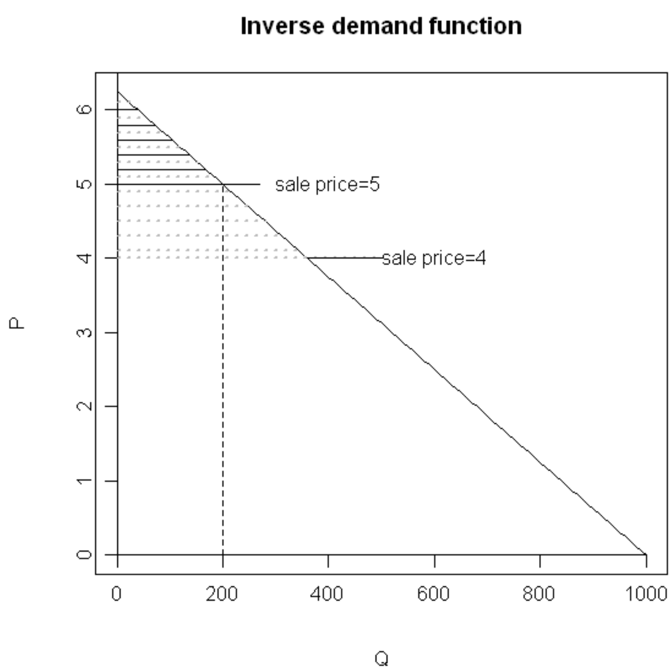
a. What is the current daily revenue associated with this service?
b. What is the current value of daily consumer surplus?
c. What is the fare elasticity of demand at the current price?
d. The operator is considering a fare reduction to $4.00 per trip. What would be the resulting daily revenue at that fare? What would be the consumer surplus?
e. Considering both the revenue impact on the operator and the change in consumer surplus as a measure of social welfare, would you recommend the change? Why or why not?

Solution.

Part (a): At a fare of $5.00, the demand function indicates that the number of daily one-way trips are $Q = 1000 - 160\,(5) = 200$. The daily revenue is thus 200 × 5 = $1000.

Part (b): The consumer surplus is defined as the area under the inverse demand function, but above the price, as shown in the figure below. The inverse demand function is $P = 6.25 - Q/160$, as shown in the following figure. Consumer surplus is the area of triangle shaded with solid black lines:

$0.5 \times (6.25 - 5) \times 200 = \$125$.



Part (c): The fare elasticity of demand is defined by:

$$e_p = \frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i} = -160 * \frac{P_i}{1000-160P_i} = -160 * \frac{5}{1000-160*5} = -4$$

Part (d): At the proposed fare of $4.00, the demand function indicates that the number of daily one-way trips would be $Q = 1000 - 160\,(4) = 360$. The daily revenue would be 360 × 4 = $1440.

The consumer surplus would be the area of triangle shaded with dotted gray lines in the previous figure: 0.5 × (6.25 − 4) × 360 = $405.

Part (e): Yes, we should certainly recommend this change. Both revenue and consumer surplus will increase if the fare is reduced from $5 to $4. This benefits both the supplier and users, assuming the capacity is available to handle the increased demand.

Problem 8-14.

Total public transportation boardings in the U.S. are provided in the table below from the American Public Transportation Association. Choose a transit agency such as the local agency serving your college or university and obtain time series data for that agency. (a) Construct a figure for relative year-on-year boardings for a start year of your choosing, showing both the national trend and the specific agency. (b) How have your chosen agency either grown or declined compared to the national value? What factors, if any, can you give to explain the relative values for your agency compared to the U.S. national average? Note: If you do not have time series data available for the entire period 1970 to 2012, you can solve the problem with a shorter time series based on whatever data are available.

Year	Trips, bil.	Year	Trips, bil.
1970	6.9	1992	8.4
1972	6.6	1994	8
1974	6.8	1996	7.9
1976	7.1	1998	8.7
1978	7.5	2000	9.4
1980	8.3	2002	9.6
1982	8.1	2004	9.6
1984	8.5	2006	10
1986	8.7	2008	10.3
1988	8.7	2010	10.2
1990	8.8	2012	10.6

Solution.

For this problem, we will use as an example the boarding data for the Tompkins Consolidated Area Transit (TCAT) in and around Ithaca, NY, the home of Cornell University. The TCAT data are given for the years 2002 to 2012 inclusive, so the focus will be on this time period only, with the figure indexed to 2002 = 1.00. The relevant data are the following:

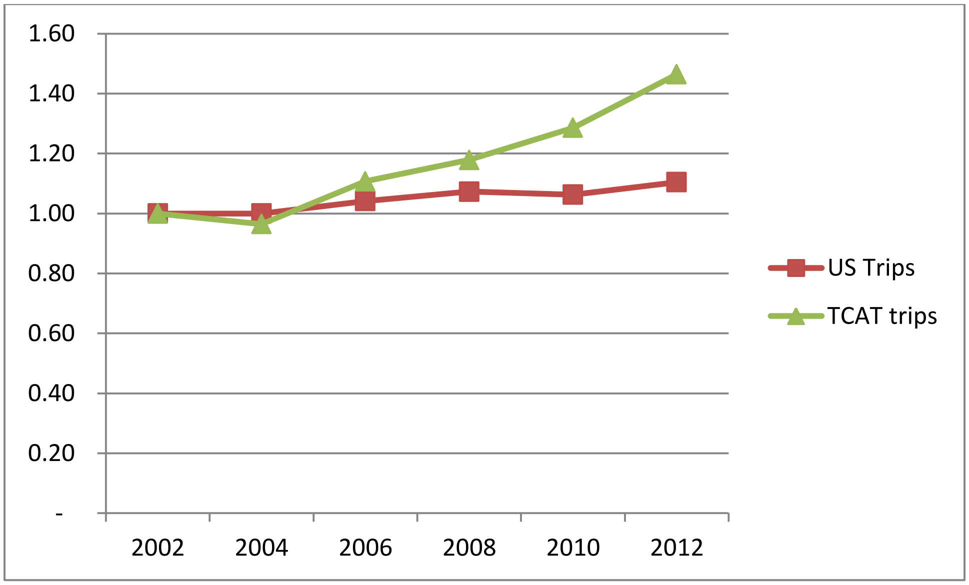
Year	U.S. Trips	TCAT Trips
	(bil.)	(mil.)
2002	9.6	2.8
2004	9.6	2.7
2006	10	3.1
2008	10.3	3.3
2010	10.2	3.6
2012	10.6	4.1

Recalculating indexed values with 2002 = 1.00 gives the following:

Year	U.S. Trips	TCAT Trips
2002	1.00	1.00
2004	1.00	0.96
2006	1.04	1.11
2008	1.07	1.18
2010	1.06	1.29
2012		1.46

	1.10	

The resulting figure is then:



Discussion.

The local agency growth outpaces the national average, 46% to 10%. This difference could be due to the strong local and New York state support for public transportation during this period, and also the demographic served, which is largely college students (the system serves three colleges, Ithaca College, Cornell, and Tompkins Cortland Community College); it has been noted nationally that young people riding transit has been growing faster than the overall population, as they are less likely to own cars, are more likely to use their smart phones or do other work while riding transit, and other reasons. On the other hand, the trend is arguably more volatile in the smaller system, with a drop of 4% 02-04, whereas the largest drop in the national trend is 1% 08-10.

Problem 8-15.

For urban transportation systems, a metric related to the throughput per hour presented in Table 8-1 is the "productive capacity," which is defined as the product of spaces per hour and operating speed. Thus productive capacity combines throughput, which is of interest to the operator, and operating speed, which is of interest to the passenger as a measure of quality of service. Suppose that a proposed rapid transit line for a city has an operating speed of 50 km/h and a line capacity of 40,000 spc/h. The investment required is $90 million per km for one track in each direction, in other words, $180 million per km for a pair of tracks. The investment is being compared to the alternative of building a freeway with productive capacity of 100,000 space-km/h^2 per pair of lanes and cost of $30 million per km per pair of lanes.

a. What is the productive capacity of the rapid transit line in space-km/h^2?
b. What is the total cost for a 20-km length rapid transit line?
c. Suppose the city instead builds a freeway with equal productive capacity. What is the number of pairs of lanes required and total cost for the system for the same distance?

d. Infrastructure decisions are not made on the basis of cost alone. Give an example of one non-cost factor that the city might consider that favors the rapid transit line and one factor that favors the freeway.

Solution.

a. Productive capacity

$$= (OpSpeed)(LineCap)$$

$$= (50km/h)\left(40{,}000\frac{spc}{hr}\right)$$

$$= 2M\ \frac{\text{space-km}}{\text{hr}^2}$$

b. Total cost = \$90 M/km × 2 × 20 km = \$3.6 B.

c. Lane pairs needed for freeway:

$$\frac{2M\_spc-km/h^2}{100K\_spc-km/\ln/h^2} = 20\ \text{lane pairs}$$

Total cost:

$$(20\_LanePair)\left(\frac{\$30M}{km}\right)(20km) = \$12B$$

d. Comment about this problem: The calculation is not complicated once you understand the question, but the problem makes a point about comparisons of RRT to freeways taken to their logical conclusion. A well-used, cost-effectively built RRT provides a level of productive capacity that is much more expensive for freeways to provide because of the vast number of lanes required (20 lanes in each direction!). Other non-cost components: for RRT, there may be ecological advantages from lower energy requirement and fewer emissions per unit of energy thanks to using electricity; for freeway, travelers have more flexibility in getting to/from the freeway compared to the RRT line where travelers must probably change conveyance when entering/leaving the RRT system.

Problem 8-16.

Note to instructor: See discussion at the beginning of Problem 8-13 above.

The public transit operator in a major city offers an express bus service from a downtown location to the airport at a fare of \$6.00 for a one-way trip. Suppose the demand function for this service is:

$$Q = 800 - 100\ P$$

where Q is in daily one-way trips and P is the fare in dollars.

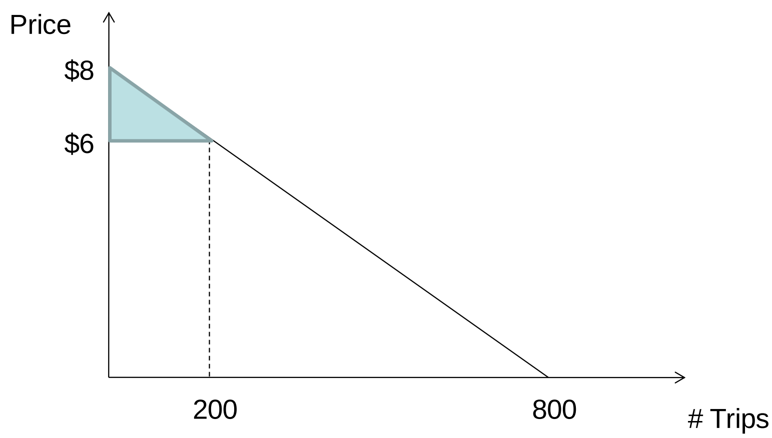
a. What is the current daily revenue associated with this service? What is the current value of daily consumer surplus?

Solution:

At a fare of \$6.00, the demand function indicates that the number of daily one-way trips are $Q = 800 - 100\,(6) = 200$. The daily revenue is thus 200 × 6 = \$1200.

The consumer surplus is defined as the area under the inverse demand function, but above the price. The inverse demand function is $P = 8.0 - Q/100$, as shown in the following figure. Consumer surplus is the area of the shaded triangle:

0.5 × (8 − 6) × 200 = \$200.



b. What is the fare elasticity of demand at the current price?

Solution:

The fare elasticity of demand is defined by:

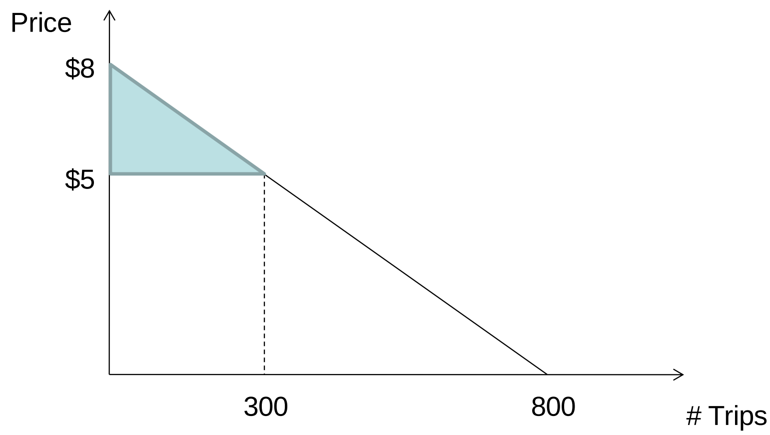
$$e_p = \frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i} = -100 * \frac{P_i}{800 - 100P_i} = -100 * \frac{6}{800 - 100 * 6} = -3$$

c. The operator is considering a fare reduction to \$5.00 per trip. What would be the resulting daily revenue at that fare? What would be the consumer surplus?

Solution:

At the proposed fare of \$5.00, the demand function indicates that the number of daily one-way trips would be $Q = 800 - 100\,(5) = 300$. The daily revenue would be 300 × 5 = \$1500.

The consumer surplus would be the area of the shaded triangle in the figure below: 0.5 × (8 − 5) × 300 = \$450.



d. The operator's long-run total cost function for providing the service is:

$$TC = 5Q - 0.015Q^2 + 0.00004Q^3$$

where cost is measured in dollars per day and Q is in one-way trips per day. At the current operating point (price and quantity), what are the values of total cost, average cost, and marginal cost for the operator? How will those values change if the pricing change is made?

<u>Solution:</u>

At the current operating point (p = 6; Q = 200), the total cost for providing the service is $720 per day.

The average cost is then: $AC = \frac{TC}{Q} = \frac{720}{200} = \$3.60$ per trip; and the marginal cost is:

$$MC = \frac{d\,TC}{dQ} = 5 - 2(0.015)Q + 3(0.00004)Q^2 = \$3.80 \text{ per trip.}$$

If the price is reduced, the new operating point will be (p = 5; Q = 300). The total cost for providing the service would be $1230 per day. The average cost at that volume would be: $AC = \frac{TC}{Q} = \frac{1230}{300} = \$4.10$ per trip; and the marginal cost would be:

$$MC = \frac{d\,TC}{dQ} = 5 - 2(0.015)(300) + 3(0.00004)(300)^2 = \$6.80 \text{ per trip.}$$

e. Considering both the revenue and cost impacts on the operator, and the change in consumer surplus as a measure of social welfare, would you recommend the pricing change for this service? Why or why not?

<u>Solution:</u>

If the pricing change is made, the revenue for the operator increases by $300 and the cost increases by $510. Thus, from a profit standpoint, this does not look like a good change. However, the transit operator is a public agency, so profit maximization is not necessarily their objective. At the new fare of $5.00, the agency is still more than covering their average cost, so they are not losing money on the service. The consumer surplus will increase by $250 per day if the fare is reduced from $6.00 to $5.00, indicating that the public as a whole is better off with the lower fare. On the whole, the change is beneficial, since the public is made better off, and the transit operator still covers its costs.

[Note: There is another aspect to the pricing issue, dealing with the idea that prices should be equal to marginal costs for greatest efficiency in the economy. In this sense, the $6.00 price is too high, because at the resulting volume MC is only $3.80; the $5.00 price is too low, because at the volume of Q = 300, MC is $6.80. Thus, neither is an "optimal" price. However, for now, it is sufficient to recognize that overall, the $5.00 price is better than the $6.00 price.]

Problem 8-17.

Note to instructor: See discussion of elasticity at the beginning of Problem 8-13 above.

As we saw in class, average peak-period bus fare elasticity in the U.S. is about −0.23. For a transit operator under financial pressure to increase revenue, does a fare increase make sense as a way to do that? Why or why not?

Solution.

A fare elasticity of −0.23 implies that the demand is not very sensitive to a price change. When the fare elasticity is between 0 and −1, raising the fare will indeed increase revenue. If the elasticity is −0.23, a 1% increase in price will reduce demand by 0.23%, but since the demand reduction is less than the fare increase percentage, total revenue rises.

Politically, a fare increase is often a difficult thing to accomplish, but that is a different issue. From the standpoint of increasing revenue, it does make sense.

Problem 8-18.

Time-area factor: Cars travel at 20 mi/h on an arterial street during rush hour where the relationship between vehicle density and speed is $k = 130 - 2.82\,u$, for speeds of 46 mi/h and under, where u is given in mi/h. The average occupancy is 1.6 passengers per vehicle. Buses also travel on the same street with density determined by the length of the vehicle and the shadow, rather than by a function $k(u)$. Each bus is 40-ft long and has a shadow of 45 ft; buses travel at 15 mi/h due to the need to stop and pick up or discharge passengers. The bus occupancy is 33 passengers on average, and the lane width is 15 ft.

a. What is the per person time-area factor for each mode?
b. Suppose under the described conditions 1000 commuters switch from driving in cars to traveling by bus. What is the reduction in time-area occupancy, measured in ft^2-h?

Solution:

For car mode: given a speed of 20 mi/h, the density per mile is:

$$k = 130 - 2.82u = 130 - (2.82)(20) = 73.6 veh/mi$$

The value of L = total length per vehicle occupied is therefore the number of feet in a mile divided by the number of vehicles per mile, or

$$L = \frac{5280}{73.6} = 71.7\, ft/veh$$

Solving for the per person time-area factor then gives:

$$\overline{TA}_p = \frac{WL}{\alpha ku} = \frac{(15)(71.7)}{(1.6)(73.6)(20)} = 0.457\, ft^2 \cdot hr / pers$$

For the bus, the total length occupied per vehicle is the sum of the physical length plus shadow:

$$L_{tot} = L_{veh} + L_{shadow} = 40 + 45 = 85\, ft$$

The density per mile is thus:

$$k = \frac{5280}{85} = 62.1 veh/mi$$

Solving for the per person time-area factor for the bus therefore gives:

$$\overline{TA}_p = \frac{WL}{\alpha ku} = \frac{(15)(85)}{(33)(62.1)(15)} = 0.0415\, ft^2 \cdot hr / pers$$

Part (b): The figures from part (a) are used to calculate the savings per 1000 commuters. First calculating the difference in time-area factor and then applying to the number of commuters gives:

$$Savings = 0.457 - 0.0415 = 0.415\, ft^2 \cdot hr / pers$$
$$TotalSavings = (0.415\, ft^2 \cdot hr / pers)(1{,}000\, pers) = 415\, ft^2 \cdot hr$$

Problem 8-19.

Consider two lanes of traffic on the expressway eastbound into the Lincoln Tunnel under the Hudson River in the NYC area: one lane is rush hour traffic of cars with average occupancy 1.5 persons per vehicle, and the other is the dedicated, counterflow bus lane with motor coaches carrying on average 30 commuters per vehicle.

The length of the average car and bus is 16 ft and 40 ft, respectively. Both have a 69-ft shadow, given that it is rush hour. The cars travel 35 mi/h average compared to 30 mi/h for the buses, reflecting the cars' superior ability to accelerate and decelerate. The density of vehicles per mile for cars and buses can be determined from their length and shadow (Hint: 1 mi = 5280 ft), and both lanes are 15-ft wide.

a. What is the value of the per person time-area factor $TA_{p\text{-}avge}$, for the cars and for the buses, in units of ft^2/1000 persons/h?

b. What advantage does the result in part (a) suggest for commuting by bus as opposed to car? Short answer, one or two sentences maximum.

Solution:

For the private cars, use the total length to calculate *k*, *q*, and the time-area factor:

$$L_{tot} = L_{veh} + L_{shadow} = 16 + 69 = 85\,ft$$

$$k = \frac{5280\,ft/mi}{85\,ft/veh} = 62.1 veh/mi$$

$$q = uk = (35mph)(62.1veh/mi) = 2{,}173 veh/h$$

$$\overline{TA}_p = \frac{WL}{\alpha_p q} = \frac{(1000)(15)(85)}{(1.5)(2173)} = 391\,ft^2/1000\,pers-h$$

Repeating the same calculation for the buses gives the following:

$$L_{tot} = L_{veh} + L_{shadow} = 40 + 69 = 109\,ft$$

$$k = \frac{5280\,ft/mi}{109\,ft/veh} = 48.4 veh/mi$$

$$q = uk = (30mph)(48.4veh/mi) = 1452 veh/h$$

$$\overline{TA}_p = \frac{WL}{\alpha_p q} = \frac{(1000)(15)(109)}{(30)(1452)} = 37.5\,ft^2/1000\,pers-h$$

Part (b): The calculation shows the motivation for why New York City and the state of New Jersey have established counterflow bus lanes for buses at rush hour to and from the crossings into Manhattan. The per person time-area factor for 1000 travelers is less than a 10th of that of cars, even though the buses travel more slowly.

Chapter 9 Personal Mobility and Personal Accessibility

Problem 9-1.

A carsharing organization offers two rate tiers, $50 per year and $8/h, or $200 per year and $5/h. For either tier, the mileage charge is $0.20/mi. The anticipated hours required for 1 mi of driving is 0.125 hours. A prospective carshare member anticipates reserving the car for 3 hours on each occasion. What is the minimum frequency at which they could reserve the vehicle and still prefer the cheaper hourly rate?

Solution.

Since each rental lasts 3 hours and each mile of driving on average generates 0.125 hours of rental time, the total miles driven per rental is 24 mi, at a cost of $4.80 in either case. Call the high annual cost option case 1 and the low cost option case 2. In case 1, each rental costs $15 in hourly charges so the total cost is $19.80 per rental. For case 2, the respective values are $24 and $28.80. Accordingly, the break-even number of rentals per year *N* can be solved for using the following calculation:

$$FC_1 + VC_1 N = FC_2 + VC_2 N$$
$$N = \frac{FC_1 - FC_2}{VC_2 - VC_1} = \frac{200 - 50}{28.80 - 19.80} = 16.7$$

Therefore, the breakeven occurs between 16 and 17 rentals per year. It follows that if the user rents 17 or more times per year, they will be better off with the higher annual fee and lower charge per hour.

Note that since the mileage charge of $4.80 is constant for both options, it could be ignored and the calculation would give the same answer, that is:

$$FC_1 + VC_1 N = FC_2 + VC_2 N$$
$$N = \frac{FC_1 - FC_2}{VC_2 - VC_1} = \frac{200 - 50}{24 - 15} = 16.7$$

Inclusion of the mileage charge shows that at the break-even value of $N = 17$, the total cost incurred by the user per year for annual fee, hours, and mileage, is approximately $540.

Problem 9-2.

An urban resident is debating whether or not to either join a carsharing organization or purchase a new car. Joining carsharing costs $300 per year, and after that, for simplicity of this problem the carsharing rate structure is based entirely on miles driven. The chosen vehicle is a compact hatchback, purchase cost $16,000, salvage value $8,000, life 5 years, insurance $2500/year, fuel economy 28 mi/gal, and maintenance cost $400 per year. Use a discount rate of 10% throughout. The resident is concerned on the vehicle purchasing side that she will not have enough space for the occasional long trip, large group of friends, or large load, so has chosen a midsize Sedan as the vehicle she will purchase, to meet these occasional needs. (With carsharing, if these needs arise, she will address them some other way since in any case she is not tied to renting a carshare vehicle for every trip.) The Sedan lists for $28,000, has a fuel economy of 24 mpg overall, costs $1500 per year for insurance, and incurs $0.04 per mile for repairs and maintenance. It should be discounted over 10 years with zero salvage value. Assume $4/gal for gas for either option. (a) Calculate the cost per mile for carsharing. (b) Calculate the fixed and variable cost components for buying the Sedan. (c) If the carsharing agency charges twice the cost of driving per mile to cover all of their overhead costs, what is the break-even distance in miles? (d) If the resident expects to drive 4000 mi or 10,000 mi/year, what is the difference in cost between the less and more expensive option for meeting their driving needs?

Solution:

Part (a): Net annual capital cost is:

$$A = (P)(A/P, i\%, N) - (F)(A/F, i\%, N) = (16000)(0.2638) - (8000)(0.1638)$$
$$\$4221 - \$1310 = \$2910$$

Annual fuel cost is:

$$10000 mi/yr\left(\frac{1}{28mpg}\right)(\$4/gal) = \$1429/yr$$

Total cost: (insurance + capital + fuel + maintenance)

$2500 + $2910 + $1429 + $400 = $7239/year

Cost per mile: $7239/10K = $0.724/mi

Part (b): Annual capital cost and overall fixed cost including insurance of the Sedan is:

$$A = (P)(A/P, i\%, N) = (28000)(A/P, 10\%, 10) = \$4557/yr$$
$$FC = \$1500/yr + \$4557/yr = \$6057/yr$$

Gas cost per mile and overall variable cost including maintenance of the Sedan is:

$$\left(\frac{1gal}{24mi}\right)\left(\frac{\$4}{gal}\right) = \$0.167/mi$$
$$VC = \$0.167/mi + \$0.04/mi = \$0.207/mi$$

Therefore, the fixed cost is $6097/year, and the variable cost $0.207/mi.

Part (c): Carshare costs $0.724/mi. Twice this value is $1.45/mi. Breakeven is:

$$D_{breakeven} = \frac{FC_1 - FC_2}{VC_2 - VC_1} = \frac{6057 - 300}{1.45 - 0.207} = 4631mi$$

Part (d): If driving 4K miles per year, carshare is preferred, and the own car would cost $785 more. For 10K miles per year, buying the car is preferred, and carshare costs $6673 more. Costs are shown in the table below:

Miles	Carshare	own car	Difference
4000	$6100	$6885	$785
10000	$14,800	$8127	$6673

Problem 9-3.

A carsharing organization uses a subcompact hatchback car as a fleet vehicle, with the following characteristics: list price new, $18,000; resale value after 5 years, $8000; fuel economy, 27 mpg; and average maintenance cost, $500 per year. The price of gas is $3.80/gal, the insurance cost is $2,700 per year, and the discount rate is 6%. Recent records show that each mile of driving corresponds to 0.125 hours of rented time. The organization desires to earn 25% of revenue from per mile charges and the rest from per hour charges. The total miles driven by all customers using the vehicle over the course of 1 year is projected to be 11,000 mi. (a) Calculate the average cost per mile to the nearest whole cent. (b) Calculate the percent of per mile cost allocated to each of capital cost, insurance, maintenance, and fuel. (c) Suppose the carshare company needs to earn 1.5 times the annual cost of the vehicle to cover their overhead cost. What should they charge per mile and per hour for this vehicle? (d) What percent of hours in a 24-hour day must the vehicle be rented to achieve the hourly goals?

Solution:

Part (a): The net annual capital cost of the vehicle after taking into account resale after 5 years is:

$$ACC = P \cdot (A/P,5,6\%) = \$18,000(0.2374) = \$4273$$
$$Salvage = F \cdot (A/F,5,6\%) = \$8000(0.1774) = \$1419$$
$$Net = \$4273 - \$1419 = \$2854$$

Fuel cost per year is:

$$(11000mi)\left(\frac{1gal}{27mi}\right)(\$3.80/gal) = \$1548$$

Since insurance is $2700 and maintenance is $500, the total annual cost is $2854 + $1548 + $2700 + $500 = $7602/year. Dividing by 11,000 mi driven gives $0.69/mi.

Part (b): The following table shows percentages for each of the categories:

Capital	$2853.96	37.5%
Fuel	$ 1548.15	20.4%

Insurance	$2700	35.5%
Maintenance	$500	6.6%
Total	$7602.11	100.0%

Part (c): Since the company wishes to bring in revenue from the car that is 1.5 times the cost, the desired revenue is $11,403 per year. Because each mile driven generates 0.125 hour of reserved time, the number of hours of reserved time expected per year is:

$$(11{,}000mi)(0.125hr/mi) = 1375hr/yr$$

Accordingly, the total collected per mile and per hour are respectively the following:

$$PerMile = (0.25)(\$11{,}403) = \$2851$$
$$\frac{\$2851}{11000mi} = \$0.26/mi$$
$$PerHour = \$11403 - \$2851 = \$8552$$
$$\frac{\$8552}{1375hr} = \$6.22/h$$

Part (d): Since the number of hours per year is 1375, the number of hours per day and the fraction of the day are:

$$\frac{1375h/y}{365day} = 3.8h/day$$
$$\frac{3.8h}{24h} = 15.7\%$$

Problem 9-4.

In this problem, you are to build a Monte Carlo simulation model with 1000 iterations for the B/C ratio for telecommuting from the employee's perspective, taking into account the following components in the ratio: (1) driving cost for vehicle, (2) time cost of time spent on driving, (3) home utility cost, and (4) annual IT cost to be able to telecommute. A table of five stochastic inputs is given below; these should be "sampled" to calculate the B/C ratio for each run of the simulation. In addition, the following inputs are fixed ("deterministic") in each run of the simulation: fuel economy of 28 mpg, average travel speed of 30 mi/h, 46 work weeks per year, 1.5 days per week of telecommuting, driving cost other than fuel of $0.10/mi, and gas cost of $4/gal. (a) Before conducting the simulation, calculate the fixed-value output from the model. What is the expected value of B/C? (b) Suppose that RAND() returns for the four stochastic inputs driving distance, value of time, utility cost, and IT cost the following four values: 0.724, 0.812, 0.728, and 0.932, respectively. What is the resulting value of B/C in this case? (c) Run a simulation in the softward package of your choosing and compute 1000 runs for your simulation model. Deliver (1) a page of run results, with the input values and resulting B/C value for each run (one line per run is expected); (2) the overall average B/C ratio across the 1000 runs; (3) the percent deviation of the simulation average B/C value from the fixed-value average from part (a). (d) Produce a histogram with bin sizes of your choosing for the 1000 results from the simulation. €(e) Short answer: what information do the simulation and histogram give you that is not already available from the fixed-value average B/C in part (a)? Two to three sentences maximum.

Variables	Distn	Units	(μ, σ)*	(a, b)
Average one-way distance	Normal	Miles	(11.6,4)	n/a
Value of avoided time	Uniform	$/h	n/a	(6,10.20)
Home utility costs	Normal	$/event	(2.75,0.5)	n/a
Annual IT costs	Normal	$/year	(250,40)	n/a

Solution.

Part (a): First calculate the expected value for value per hour of lost time, which is the average of the low and high end of the range because the random variable is uniformly distributed:

$$\mu = \frac{(\$6 + \$10.20)}{2} = \$8.10/hr$$

The expected fuel cost is calculated based on the expected distance of 11.6 mi, fuel economy of 28 mi/gal, and gas cost of $4/gal:

$$11.6mi\left(\frac{1gal}{28mi}\right)(\$4/gal) = \text{\$1.66 per one-way trip}$$

Since the non-fuel cost per mile is $0.10, the cost per one way is (11.6 mi)($0.10) = $1.16, the total cost per one way is $1.66 + $1.16 = $2.82, and the avoided cost per round trip is ($2.82) (2) = $5.63.

Next, the value per trip of avoided time is:

$$(11.6mi)\left(\frac{1}{30mph}\right)(\$8.10/h) = \$3.13$$

Therefore, the round-trip value is $6.26.

The expected value for the remaining inputs is the mean for each and can be read directly from the table. Therefore, the expected value for B/C is:

$$B/C = \frac{(TripCost + SavedTime)N}{HomeUtilCost \cdot N + ITCost} = \frac{(\$5.63 + \$6.26)(46 \cdot 1.5)}{\$2.75 \cdot 46 \cdot 1.5 + \$250} = 1.87$$

Part (b): Since you are given four randomly generated uniform (0, 1) values, you can use the NORMINV function in Excel to calculate the parameter returned for the Monte Carlo simulation. As an illustration, the way that NORMINV works is presented here by using instead the standard normal table, and showing that NORMINV arrives at the same value.

The first value introduced is 0.724 for infrastructure cost. The value sought from the standard normal table is the value of *z* such that there is a 72.4% chance that a randomly chosen standard normal variate would fall below *z*. From the standard normal table, for this to be true, $z = 0.595$. Then because distance is distributed ($\mu = 11.6$, $\sigma = 4$), the value for infrastructure is:

$$Dist = \mu + z \cdot \sigma = 11.6 + (0.595)(4) = 13.98 \text{ mi}$$

Thus, the round-trip distance is 27.96 mi. Repeating these steps for office utility cost, home utility cost, and number of days per year using input values of 0.812, 0.728, and 0.932 gives values of \$9.41/h, \$3.05 per telecommuting event, and \$309.61 per year for IT cost, respectively. For value of time saved and driving cost, we need to make additional calculations. For value of time:

$$Val = 27.96mi\left(\frac{1}{30mph}\right)(\$9.41/h) = \$8.77/event$$

For travel cost including gas and non-fuel cost:

$$Gas = 27.96mi\left(\frac{1}{28mpg}\right)(\$4.00/gal) = \$3.99/event$$
$$Nonfuel = 27.96mi(\$0.10/mile) = \$2.80/event$$
$$Total = \$3.99 + \$2.80 = \$6.79/event$$

The number of events per year is 46 weeks times 1.5 events per week, or 69 per year. Based on these inputs, the value for B/C is now:

$$B/C = \frac{(InfraCost + OfficeUtil\,Cost)N}{HomeUtilCost \cdot N + ITCost} = \frac{(\$8.77 + \$6.79)65}{\$3.05 \cdot 65 + \$309.61} = 2.06$$

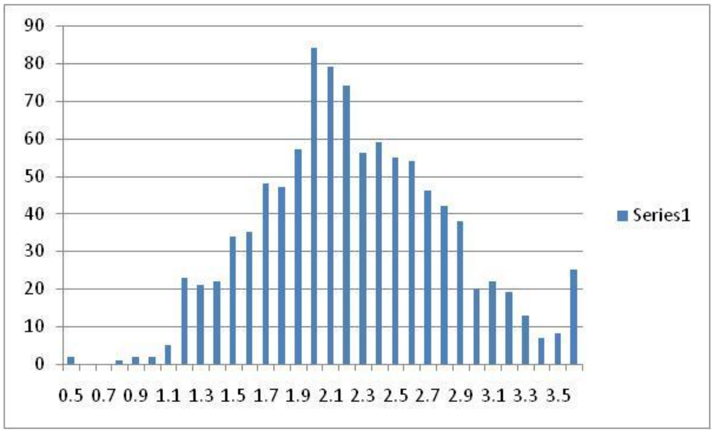
Part (c): Table with 10 iterates: this part of the answer will be unique for each run of the Monte Carlo simulation, but here is a sample:

Trial	Events	Drive cost	Avd'd time	Util cost	Setup cost	b/c	
	[days/week]	[\$]	[\$]	[\$]	[\$]	net	ratio
1	1.50	315.05	63.43	168.22	190.15	\$20.10	1.06

2	1.50	622.60	669.40	166.16	196.86	$928.99	3.56
3	1.50	798.20	610.26	160.87	255.90	$991.68	3.38
4	1.50	$ 235.82	295.38	180.16	321.10	$29.95	1.06
5	1.50	368.31	143.29	169.00	239.13	$103.47	1.25
6	1.50	378.82	407.45	152.49	208.33	$425.45	2.18
7	1.50	290.27	332.69	99.09	253.89	$269.98	1.76
8	1.50	375.46	605.45	242.36	279.05	$459.51	1.88
9	1.50	461.22	425.29	182.58	218.77	$485.16	2.21
10	1.50	263.33	305.84	139.63	298.27	$131.28	1.30

In each cell, the value has been calculated using RAND() plus the appropriate formula for either a normal or uniformly distributed parameter. The important point is that each cell should call RAND() separately, i.e., not call RAND() once and then use the same value for each parameter in the iterate. The rightmost column is the B/C value, calculated using the same formula as in parts (a) and (b). In this particular run, the simulation returned an average value of B/C = 1.89. The difference in B/C value is 0.02, or approximately 2% different from the expected value from part (a). Values from other simulation runs may be slightly different, but not much: a correctly created simulation should come very close to the expected value.

Part (d): Histogram from a representative run of the simulation, again noting that each histogram will be unique but should look something like this one. The B/C ratio values on the horizontal axis are the maximum value for the bin in question, e.g., the bin labeled “1.1” counts all iterates with a B/C value between 1.0 and 1.1, etc. The rightmost bin is unusually large because it captures all iterates with a value larger than 3.5, thus any runs on the right-hand side tail of the distribution.



Part (e): Open-ended answer, but you could in general talk about how the histogram provides not only the expected value but the percent of trials that resulted in a B/C value of 1 or less. In this case, there are very few, so you could be confident that telecommuting will be cost-effective.

Problem 9-5.

A carsharing organization is considering adding a new line of luxury cars to their offering. Joining their carsharing organization costs $300 per year. After that, the carsharing rate structure is based on cost per hour and miles driven. Assume $4/gal for gas. The chosen vehicle is a Mercedes Benz C-350 Sedan, which costs $52,400 new. Fuel economy according to the USEPA is 20 mpg city and 29 mpg highway, for a combined fuel economy of 24 mpg; you can use the 24-mpg value for purposes of calculating fuel costs in the problem.

Since the vehicle is valuable, collision insurance is expensive, so the insurance cost is $4800 per year. The organization buys the vehicle from a dealer with an auto loan at 3.9% interest, with one annual payment per year. After 5 years of useful life, they sell the vehicle for $25,000, which can also be annualized at a 3.9% discount rate. Maintenance is expected to cost $800 per year. The car is expected to be driven by members 15,000 mi/year, and each hour of reservation is expected to generate 6 mi of driving.

The carsharing agency will set prices per hour and per mile for the vehicle based on the total revenue they anticipate being necessary. They wish to achieve revenue per year that is exactly twice the total cost per year, and this cost should be contributed 75% from hourly charges and 25% from mileage charges.

a. Calculate the total cost per mile, to the nearest $0.01.
b. What values of cost per hour and cost per mile should be adopted to exactly meet the organization's revenue goals?
c. What percent of hours in a 24-hour day must the vehicle be rented to achieve the hourly goals? Do you think this is realistic, given the value of the vehicle? Why or why not?

<u>Solution:</u>

Part (a): Total cost per mile starts with annual capital cost. We can use the Excel function –PMT as follows to calculate annualized purchase cost, salvage value, and net value per year:

$$A(P) = -PMT(i\%, N, P) = -PMT(3.9\%, 5, \$52{,}400) = \$11{,}737/yr$$
$$A(F) = -PMT(i\%, N, F) = -PMT(3.9\%, 5, , \$25{,}000) = \$4{,}625/yr$$
$$Net = \$11{,}737 - \$4625 = \$7113/yr$$

Since the mileage driven is 15,000 mi/year, the cost of driving for gas is:

$$(15{,}000 mi/yr)\left(\frac{1}{24mpg}\right)(\$4/gal) = \$2500/yr$$

Total cost and cost per mile are therefore:

$$TotCost = Cap + Fuel + Ins + Mtc$$
$$= \$7113 + \$2500 + \$4800 + \$800 = \$15{,}213$$
$$Cost/mi = \frac{\$15{,}213}{15{,}000 mi/yr} = \$1.01/mi$$

Part (b): The desired revenue is twice the cost, or \$30,426. Given the 75/25 breakdown, of the total, \$22,819 should come from hourly charges, and the remaining \$7606 from mileage charges. Based on 6 mi driven per hour rented, the total number of hours is:

$$\frac{15{,}000 mi/yr}{6 mi/hr} = 2500 hr/yr$$

The cost per hour and per mile desired is therefore:

$$Cost/hr = \frac{\$22{,}819}{2500} = \$9.13/hr$$
$$Cost/mi = \frac{\$7606}{15{,}000} = \$0.51/mi$$

Part (c): Since there are 8760 hours in a year (ignoring leap years), the percent of hours required is 2500 /8760 = 29%. Under most circumstances, this level of utilization would be unrealistic for an expensive luxury car with high cost per hour and per mile. Typical members would only want the luxury amenities at certain times, and at other times it would not be competitive with ordinary carshare vehicles. Students could also mention in their answer, however, that the carsharing agency might use the luxury vehicle as a "loss leader": although they lose money on luxury rentals, the presence of the vehicles in the fleet makes it possible for them to attract enough new members to make up for the losses.

Problem 9-6.

Evaluation of carsharing based on your own personal experience: you are to pick a 7-day period within which you should evaluate 5 to 10 local trips that you make.

If you own a car, track the number of miles in each trip and the amount of time taken. Use the 2012 American Automobile Association figure of \$0.59/mi (midsize car, midrange miles per year). Then estimate your cost for the same trip using either public transportation or carsharing: if it is realistic to choose public transportation then you can consider the cost to be one round-trip fare. If public transportation is not realistic (schedule or nearest stop not sufficient), then calculate the cost using carsharing at $5.00 per hour (or $1.25 per quarter hour) and $0.20 per mile, or whatever is the local rate.

If you do not own a car, calculate the cost of using carsharing in place of using public transportation or bicycle, again taking into account both time and distance, and the same carsharing costs as above. Count up the total public transportation cost based on $3.00 per round trip, and the total carsharing cost that would replace it.

If you do not make as many as five qualifying trips in 7 days, you may consider trips that your friends make as your own in order to reach five trips. If you make more than 10, you can stop reporting at 10.

a. Report the total cost for the trial, for your current mode and for carsharing. *Open-ended, no model response.*

b. In our in-class discussion, the assumption was that your choices are "rational," i.e., whether you own a car or not, your current arrangement is cheaper than joining carsharing. Considering only hourly and mileage costs (ignore yearly membership) and based on the costs in part (a), is this assumption correct? *Open-ended, no model response.*

Problem 9-7.

Compare the cost of transportation for an individual living in an urban area who can either own a car, or be a member of carsharing, but not both. For simplicity, the individual's urban travel pattern is reduced to just three types: work commuting, leisure trips (linear forth-and-back to a single location), and "running errands" (a circuitous route involving several stops and ending up back at home). Parameters for each type of trip are in the following table. Some caveats: for professional reasons, the work trip cannot be done by bicycle, and the errands trip must use car or carshare because of the items that must be dropped off or picked up. (r/t = "round trip")

	Distance	Time	Times/week
	r/t mi	hours	
Work	24	10	5
Leisure	8	2	3
Errands	12	3	1

The individual has the following modes at their disposal, with costs shown:

	Sunk cost	Out-of-pocket	
	[$/mi]	cost [$/mi]	[$/h]

Pers-car	$ 0.35	$0.15	-
Carshare	-	$0.20	$6.50
Transit	-	$2.00/trip	-
Bicycle	-	Negl.	-

Explanation for the cost table.

- Personal car: the traveler makes trip-by-trip decisions based on the out-of-pocket cost, however accounting for weekly transportation expenditures must include both sunk and out-of-pocket cost.
- Carshare: the individual pays the hourly cost for the entire duration of the trip, not just when the vehicle is moving. Also, carsharing is the only mode that has a specific time component in the cost. For the work trip, if the traveler uses carshare, they must rent the carshare car for the entire time, including while they are at work, so that they can return it to their place of origin.
- Transit: this option serves only the work trip. Also, it is a flat fare, $2.00 each way, rather than by the mile.
- Bicycle: although realistically a bicycle would have some life cycle costs, these are ignored in this problem to make it simpler.

Questions.

a. Given the cost structure shown, why might the individual commute by car to work if they own a car, but definitely not use carshare for the work commute if they did not own a car but were a carsharing member? Provide calculations to support your answer.
b. For the leisure trip, since it is the only trip for which bicycle is allowed, you can individualize your answer: you decide how often the individual uses bicycle (always, sometimes, never). State the expected number of trips per week by bicycle that you choose, and explain your reasoning.
c. Based on insights from (a) and (b), calculate the weekly transportation cost for both the “own-car” and “carshare” case.
d. State whether the user will choose own-car or carshare and defend your answer in one or two sentences based on your cost values from part (c) and any other assumptions. Remember, your answer may depend on your assumptions and explanations in parts (a) to (c).

Solution.

Part (a): For the work trip, keeping the carshare car sitting at the workplace all day is very expensive, but if the owner owns their own car, there is no additional cost other than the per mile cost. So, for example, a 24-mi round trip to work that took 10 hours would cost $69.80, of which $65.00 is the hourly charge.

Part (b): This is a personal answer, but for the purpose of providing and example we will assume that the bicycle is used for half the trips, or 1.5 trips every week. The reasoning is that the example traveler might wish to cycle some of the time to get exercise, but also not cycle at night or during inclement weather.

Part (c): For carshare, commuting by transit costs $20 per week. The errands trip by carshare costs $21.40 for 3 hours and 12 mi of driving. Depending on your assumptions about bicycling for leisure trips, total cost per week might be $41.40 (all bicycling), to $63.30 (50% bicycling), to $85.20 (no bicycling). In each instance some explanation is needed to make clear your assumptions around the mix of bicycling and carshare for the leisure trips. For own car, and assuming the full cost per mile of $0.50 and not the $0.15 out of pocket is used, the comparable values are $66 to $78, depending on how much cycling is used. However, if you use transit for the work trip, the cost falls to $26 to $38.

Part (d): The main implication is that by trading in their own car for carsharing, the user ends up spending about the same for transportation, but they do less driving, so it is better for the environment. But, looking from the opposite perspective, if the user cannot save much money by trading in their car for carsharing, they may be inclined to keep the car for reasons of convenience.

Problem 9-8.

Monte Carlo simulation of benefit-cost (B/C) ratio for telecommuting. In this problem, you are to build a Monte Carlo simulation model for the B/C ratio for telecommuting, this time from the employer's rather than the employee's perspective, taking into account the following components in the ratio: the benefits to the employer will include (1) avoided infrastructure cost since they will need less office and parking space, (2) avoided office utility cost from the worker not being present. Additionally, suppose that to encourage telecommuting, the employer offers to pay (3) home utility cost, in proportion to the amount of telecommuting cost and (4) annual IT cost to be able to telecommute, which is a lump sum payment for the year, but is variable depending on how much the employee actually spends, within the parameters below. The last parameter, (5) number of days of telecommuting per year, is also variable.

Table of data: (*Important:* for "uniform" parameters, the two values are "low" and "high" for the range of possible values, not mean and SD)

Parameter	Type	Mean or low	SD or high
Infrastructure	Normal	7.42	1.08
Office utility	Normal	2.57	0.55
Home utility	Normal	5.42	0.88
IT cost	Uniform	117	205
Num TC days	Normal	70	6

1. What is the expected value for the simulation?
2. Compute a probabilistic, rather than deterministic, value for B/C, based on randomly generated numbers. Suppose that for the five parameters above, the corresponding randomly generated values are 0.528, 0.733, 0.410, 0.477, and 0.205, respectively. What value of B/C is returned?
3. Now conduct a Monte Carlo simulation with 1000 runs and based on the results provide a table with 10 iterates (including the five independent input values and the B/C value that results) and a

histogram with all findings. Note: you do *not* need to print out all 1000 iterates—save some trees! ☺ What is the mean and standard deviation of the simulation? What is the percent difference between the expected value for B/C from part (a) and your average B/C value?

4. Suppose the employer wishes a B/C value of B/C = 1.1 to justify the investment in telecommuting. Based on the histogram from part (c), in approximately what percent of iterates did your simulation fall short of this value?

Solution:

Part (a) First calculate the expected value for IT cost, which is the average of the low and high end of the range:

$$\mu = \frac{(117+205)}{2} = 161$$

The expected value for the remaining inputs is the mean for each and can be read directly from the table. Therefore, the expected value for B/C is:

$$B/C = \frac{(InfraCost + OfficeUtil\,Cost)N}{HomeUtilCost \cdot N + ITCost} = \frac{(\$7.42+\$2.57)70}{\$5.42 \cdot 70 + \$161} = 1.29$$

Part (b): Since you are given five randomly generated uniform (0, 1) values, you can use the NORMINV function in Excel to calculate the parameter returned for the Monte Carlo simulation. As an illustration, the way that NORMINV works is presented here by using instead the standard normal table, and showing that NORMINV arrives at the same value.

The first value introduced is 0.528 for infrastructure cost. The value sought from the standard normal table is the value of z such that there is a 52.8% chance that a randomly chosen standard normal variate would fall below z. From the standard normal table, for this to be true, $z = 0.0702$. Then because infrastructure cost is distributed (μ = \$7.42, σ = \$1.08), the value for infrastructure is:

$$InfraCost = \mu + z \cdot \sigma = \$7.42 + (0.0702)(\$1.08) = \$7.50$$

Repeating these steps for office utility cost, home utility cost, and number of days per year using input values of 0.733, 0.410, and 0.205 gives values of \$2.91, \$5.22, and 65 days, respectively. For IT cost, we need to calculate the value using the uniform distribution. Using U as the randomly generated value, IT cost is:

$$ITCost = a + U(b-a) = 117 + 0.477(205-117) = \$159$$

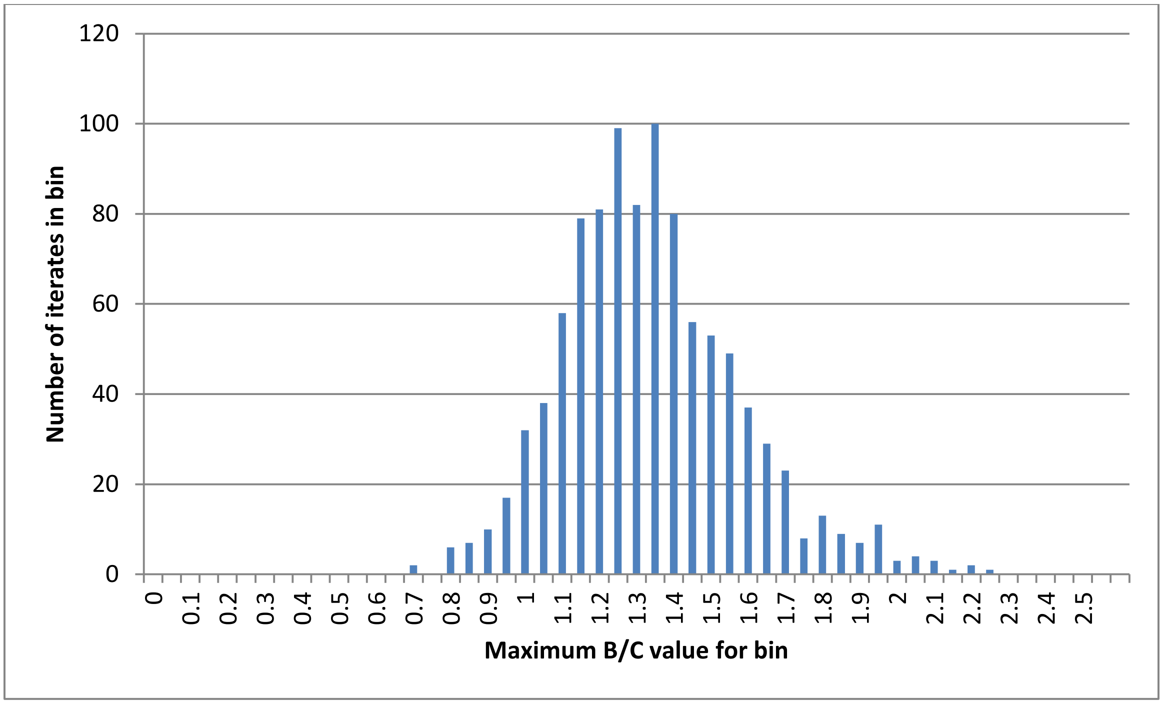
Based on these inputs, the value for B/C is now:

$$B/C = \frac{(InfraCost + OfficeUtil\,Cost)N}{HomeUtilCost \cdot N + ITCost} = \frac{(\$7.50+\$2.91)65}{\$5.22 \cdot 65 + \$159} = 1.36$$

Part (c): Table with 10 iterates: this part of the answer will be unique to each student, but here is a sample:

Iterate	Infra	Office	Home	IT Cost	Num Days	B/C
1	7.61	2.98	5.15	139.47	60.52	1.42
2	7.86	2.84	3.95	199.32	70.18	1.57
3	6.71	2.71	5.68	165.74	67.19	1.16
4	7.18	2.33	6.01	146.96	77.45	1.20
5	7.01	2.90	4.81	149.50	55.60	1.32
6	8.94	2.68	4.87	198.33	68.62	1.50
7	7.49	3.41	4.76	164.60	71.38	1.54
8	6.58	2.42	5.57	131.85	67.17	1.20
9	6.51	1.63	6.81	152.46	70.62	0.91
10	7.28	2.82	5.26	149.26	62.12	1.32

In each cell, the value has been calculated using RAND() plus the appropriate formula for either a normal or uniformly distributed parameter. The important point is that each cell should call RAND() separately, i.e., not call RAND() once and then use the same value for each parameter in the iterate. The rightmost column is the B/C value, calculated using the same formula as in parts (a) and (b). In this particular run, the simulation returned an average value of B/C = 1.31, SD = 0.226. The difference in B/C value is 0.02, or approximately 2%. Your value may be slightly different, but not much: a correctly created simulation should come very close to the expected value. Sample histogram is below.



Part (d): In the sample histogram in part (c), 171 out of 1000, or 17.1% of iterates were at or below B/C = 1.1. Values in this range accepted for full credit.

Chapter 10 Intercity Passenger Transportation

Problem 10-1.

A high-speed rail trainset with 10 passenger cars and a power car at either end has a capacity of 460 passengers. It operates on a line between a large city and a rural "hinterland," which is broken up into six segments, each approximately 100-mi long (in other words, there are seven stations on the line. You could picture a situation like the HSR that operates in Great Britain between London and the north of England and Scotland, where clearly the major population center is at the southern end, and the market is more sparsely populated at the northern end). On a given service, the train leaves the major city on segment 1, and carries the numbers of passengers shown in the table below on each of the six segments. It then turns around and comes back carrying the numbers of passengers for the return journey (note that you would read up to know the order of carrying passengers, since the train travels segment 6, then 5, 4, etc.) (a) calculate seat-miles, passenger-miles, and utilization. (b) Why might it be difficult for this route to achieve utilization at or near 100%? Explain in one or two sentences.

Segment	Outward journey	Return journey
1	404	422
2	399	344
3	289	279
4	224	160
5	154	171
6	65	60

<u>Solution.</u>

Note to instructor: solutions are calculated on a whole-route and not per segment basis in this solution; it may be helpful to advise students in advance.

The total seat-miles provided is the product of the distance of 600 mi and capacity of 460 seats, doubled to give the round-trip value:

$$SeatMiles = (600mi)(2)(460seats) = 552{,}000 \text{ seat - miles}$$

Total passenger-miles delivered is the product of passengers and distance of 100 mi on each segment. These values can then be summed up to calculate total seat-miles in each direction, as shown in the table below:

Segment	Out	Return

1	40400	42200
2	39900	34400
3	28900	27900
4	22400	16000
5	15400	17100
6	6500	6000
Total	153500	143600

Adding the total passenger-miles in each direction gives 153,500 + 143,600 = 297,100 passenger-miles.

Utilization is the quotient of passenger-miles delivered and seat-miles available:

$$Utilization = \frac{PassMi}{SeatMi} = \frac{297100}{552000} = 53.8\%$$

Part (b): The limitation that prevents utilization close to 100% is that the line cannot provide adequate capacity on the high population density end of the line without leaving empty space at the other end. If the line were to hypothetically reduce seat-miles so that the utilization would be high at the low-demand end, there would no longer be room to meet demand at the high-demand end. (This statement assumes that the train operator is not able to simply drop off cars along the way as they are no longer needed, which might prove a complex and time-consuming operation.)

Problem 10-2.

You are to evaluate the total operating cost and profit or loss of a high-speed rail (HSR system, based on four cost components: (1) energy cost (electricity), (2) capital repayment cost, (3) wages of the workforce, and (4) overhead (which includes the usual business overhead plus all maintenance cost and any other cost associated with the business). Energy costs are based on an average operating speed of 150 mi/h (240 km/h) and use the formuli presented in the chapter. You can assume the HSR trains are 75% efficient and the electricity delivery is 90% efficient. The operation owns 160 HSR trainsets, each travel 648,000 mi/year. For calculating electricity consumption, assume a cost of 9 cents per kWh and treat the trains as if they travel constantly at the operating speed—for simplicity, ignore start-up and stopping, or slowing to go around turns, etc. Capital repayment cost is $10 million per trainset per year. For wages, assume that each trainset requires 50 employees in the company workforce for all functions (train crew, repair shop, ticketing, etc.) and that the average cost to the company of an employee is $100,000. Lastly, overhead is 25% of the sum of the first three cost components. For revenues, the system carries 45 million passengers per year, and the average fare is $85.00. (a) Calculate total revenue and total operating cost for the operation. What is its annual profit or loss? (b) What percent of the total operating cost does the energy cost constitute? (c) Suppose the HSR trainsets in this example operate at 250 mi/h instead of 150 mi/h. Compare the power consumption of the two: for a 67% increase in speed, by what percent does power increase?

Solution.

Part (a): Solving for total operating cost requires that the electricity consumption be calculated. The operating speed of 240 km/h is equivalent to 66.7 m/s. Therefore, using appropriate coefficients in the drag equation the drag *D* is:

$$\begin{aligned} D &= a + bV + cV^2 \\ &= 3.82 + (0.1404)(66.7) + (0.00653)(66.7)^2 \\ &= 42.2kN \end{aligned}$$

The power requirement based on drag and velocity is therefore:

$$P = FV = (42.2kN)(66.7m/s) = 2{,}814kW = 2.81MW$$

Total electricity consumption comes from multiplying power by the number of hours of operation per year. The 160 trainsets travel 1,040,000 km each, for a total of 166 million km per year. At a speed of 240 km/h, the time required is (166M)/(240) = ~693,000 hours per year. Energy required is therefore:

$$E = (2.81MW)(693{,}000h) = 1{,}951{,}000MWh$$

Electricity purchased takes into account losses between purchase and delivering energy to propulsion of the trainset:

$$E_{purchased} = \frac{1{,}951{,}000MWh}{(0.75)(0.9)} = 2{,}890{,}000MWh$$

The price of electricity of \$0.09/kWh is equivalent to \$90/MWh. Therefore, total expenditure on electricity is (2.89 million MWh)(\$90) = ~\$260 million per year

The 160 trainsets require 50 employees at a cost of \$100,000 each, for a total of \$800 million per year. The capital cost of each trainset is \$10 million per year, so the total capital cost per year is \$1.6 billion. Overhead is calculated by adding 25% to the sum of cost components, so the total cost per year is:

$$C_{tot} = (\$260M + \$800M + \$1.6B)(1.25) = \$3{,}320M = \sim \$3.32B$$

Thus net cost = \$3.32 billion/year.

Gross revenue is (\$85/ticket)(45 million passengers) = ~\$3.825 B. Therefore, net profit is \$3.825B − \$3.32B = ~\$500M.

Part (b): Energy cost constitutes \$260 million out of \$3.32 billion, or approximately 7.8%.

Part (c) The speed of 250 mi/h (400 km/h) is equivalent to 111.1 m/s. Repeating the calculations above gives a new power requirement of 11.1 MW. Thus the increase in power requirement is:

$$\frac{11.1 - 2.81}{2.81} = \frac{8.3}{2.81} = 295\%$$

Thus, a 67% increase in speed increases power required by 295%.

Problem 10-3.

An Airbus A319 has the specification values given in Table 10-3 and an aerodynamic drag coefficient of 0.03, a frontal area of 57 m^2, and jet engines with an efficiency of 33%. At cruising altitude with air density of 0.4 kg/m^3 it travels at a speed of 230 m/s (828 km/h). Assume it is operating at maximum weight. Use an energy content and CO_2 emission rate of 35.9 MJ/L and 2.528 kgCO_2/L, respectively. (a) What is the instantaneous power requirement for the A319 to maintain the given speed while maintaining constant altitude? (b) What is its jet fuel consumption per kilometer? (c) If the A319 is filled to capacity, what are the CO_2 emissions per passenger-km under these conditions, ignoring the effects of takeoff and landing?

Solution.

Part (a): Supported area A_s can be calculated from length and width of the A319 from the table:

$$A_s = LW = (33.84)(34.1) = \sim 1154 m^2$$

P_{total} is next calculated using Eq. 10-4:

$$P_{total} = 0.5\rho C_d A_p V^3 + 0.5\frac{(mg)^2}{\rho V A_s}$$

$$= 0.5(0.4)(0.03)(57)(230)^3 + 0.5\frac{\left(64{,}400kg \cdot 9.8m/s^2\right)^2}{(0.4)(230)(1154)}$$

$$= 4.16\times10^6 W + 1.88\times10^6 W = 6.04\times10^6 W = 6.04MW$$

Part (b): To calculate the jet fuel consumption per kilometer, it is first necessary to calculate the time requirement per kilometer in seconds:

$$\frac{1000m}{230m/s} = 4.35s$$

Next, the total energy requirement per kilometer, the energy input based on 33% efficiency, and the fuel requirement based on 35.9 MJ/L gives a jet fuel requirement of:

$$E_{out} = (6.04MW)(4.35s) = 26.25MJ$$

$$E_{in} = \frac{E_{out}}{\eta_{jet}} = \frac{26.25MJ}{0.33} = 79.54MJ$$

$$\frac{79.54MJ}{35.9MJ/L} = 2.22L$$

Part (c): To calculate emissions per passenger-km, use the emission rate of 2.528 kgCO_2/L and the occupancy of 124 passengers:

$$Fuel_{psgr} = \frac{2.22L}{124} = 17.9mL / pkm$$
$$CO2 = (17.9)(2.528) = 45.17 gCO_2 / pkm$$

Problem 10-4.

Solve Example 10-5 using optimization. First write out the objective function and constraints in mathematical form. Then use the software alternative and coding approach of your choice to show that the resulting answer agrees with that of Example 10-5.

Solution.

In general, the use of optimization allows one to avoid reducing the system of equations to a single unknown variable. Instead, revenue can be written in terms of both P_B and P_L, and the solver can then be allowed to find the optimal set of ticket prices. Total revenue is the sum of revenue from business and leisure travelers, and these in turn are the product of passengers multiplied by fare. The constraints are that the total number of seats is limited to 100, and neither fares nor number of passengers can be negative. Writing the optimization problem gives:

$$MaxZ = Q_B P_B + Q_L P_L = (100 - 0.15P_B)P_B + (140 - 0.5P_L)P_L$$
$$s.t.$$
$$Q_B + Q_L = 100$$
$$P_B, P_L, Q_B, Q_L \geq 0$$

For this solution the problem was coded using MS Excel Solver. The following set of cells from the spreadsheet shows the decision variables in yellow and the objective function in green:

Revenue	$ 25,851.28			
	B	L		
P	$ 364.10	$ 170.77	Sum:	Max
Q	45.38	54.62	100	100

Thus from the spreadsheet we see that if non-integer numbers of passenger sales were possible, the optimal revenue value would be $25,851.28 to the nearest whole cent. If we round to the nearest whole number of passengers, we have 45 and 55, slightly different from the 46 and 54 in the analytical solution. Multiplying these passenger figures by ticket prices of $364.10 and $170.77 gives revenue of $25,776.92, which is close to the analytical solution.

Problem 10-5.

An airline offers service between Ithaca and Philadelphia that attracts business travelers and non-business (leisure) travelers. The service is provided by a 50-seat regional jet, and the demand functions for the two types of travelers are $Q_B = 60 - 0.2\,P_B$ and $Q_L = 70 - 0.55\,P_L$, respectively, where the one-way segment prices (P_B and P_L) are in dollars. (a) If the airline seeks to maximize revenue from its flight each morning, what prices should be charged to the two market segments? (b) What are the resulting numbers of business and leisure travelers on each flight? (c) What is the revenue obtained for the flight?

Solution.

Part (a): The optimal ticket prices can be found by setting up the revenue maximization problem directly as a constrained optimization (which is where the price formulae come from). The revenue function is:

$$R = R_B + R_L = Q_B P_B + Q_L P_L = 60P_B - 0.2P_B^{\,2} + 70P_L - 0.55P_L^{\,2}$$

This involves both prices P_B and P_L, but we also have the constraint on total seats available, which allows us to solve for one price in terms of the other:

$$\begin{aligned} &Q_B + Q_L = 50 \Rightarrow 60 - 0.2P_B + 70 - 0.55P_L = 50 \\ &\Rightarrow \quad P_B = 400 - 2.75P_L \end{aligned}$$

If we substitute this relationship into the revenue expression, it is reduced to one variable:

$$R = 60(400 - 2.75P_L) - 0.2(400 - 2.75P_L)^2 + 70P_L - 0.55P_L^{\,2}$$

This can be differentiated and the derivative set to zero, to find the optimal value of P_L:

$$\begin{aligned} &\frac{dR}{DP_L} = -60*2.75 + 0.2*2*2.75(400 - 2.75P_L) + 70 - 0.55*2P_L = 0 \\ &\Rightarrow \quad P_L = \$\,83.64 \approx \$\,84 \end{aligned}$$

The optimal business fare can then be found from the fare relationship derived above:

$$P_B = 400 - 2.75*83.64 = \$170$$

Part (b): Substituting whole number values for ticket prices into equations for the number of leisure and business travelers gives:

$$\begin{aligned} Q_L &= 70 - 0.55\,(84) = 24 \; seats \\ Q_B &= 60 - 0.2\,(170) = 26 \; seats \end{aligned}$$

Note that the total number of seats sold equals the capacity of the aircraft.

Part (c): The revenue obtained for the flight is the sum of revenue from each type of ticket sold:

$$R = R_B + R_L = Q_B P_B + Q_L P_L = 26*170 + 24*84 = \$\,6436$$

Although the problem does not ask the question, it is interesting to note that the number of each type of ticket sold is roughly equal (26 vs. 24), but the business ticket sales account for approximately 69% of the total revenue of $6436. This observation supports the general finding in the airline industry that business travelers are crucial for their financial success.

Problem 10-6.

Note to instructor: You may wish to provide additional background for this problem by discussing the supporting material on transportation economics and decision making in Chap. 4.

In this problem we extend the discussion of occupancy of aircrafts and revenue from ticket sales by considering the total cost as a function of providing capacity, the average cost (the total cost divided by the number of units of capacity), and the marginal cost (cost of adding one more unit of capacity).

Suppose the long-run total cost function for producing available seat-miles (Q) by an airline is:

$$TC = Q - 0.7Q^2 + 0.14Q^3$$

where costs are measured in millions of dollars per day and available seat-miles are measured in millions of seat-miles per day.

a. At an output level of $Q = 3$ million seat-miles, what are the long-run average and marginal costs per available seat-mile?
b. At what level of output would the airline achieve minimum average total cost? What would that average cost be?
c. What is the long-run marginal cost at the output level found in part (b)?
d. Graph TC in millions of dollars and average cost (AC) and marginal cost (MC) in dollars on the same axes. What observation about the optimal quantity Q is confirmed visually by the graph?

Solution.

At an output level of 3 million available seat-miles (ASM) per day, the total cost is:

$TC = 3 - 0.7 \times 3 \times 3 + 0.14 \times 3 \times 3 \times 3 = \$0.48$ (millions), so the average cost per ASM is:

$AC = TC/Q = 0.48/3 = \$0.16$/seat-mile.

The marginal cost is:

$$MC = \frac{dTC(Q)}{DQ} = 1 - 1.4Q + 0.42Q^2 = 1 - 1.4*3 + 0.42*3*3 = \$0.58 \text{ per ASM.}$$

Notice that this is well above the average cost at that output level.

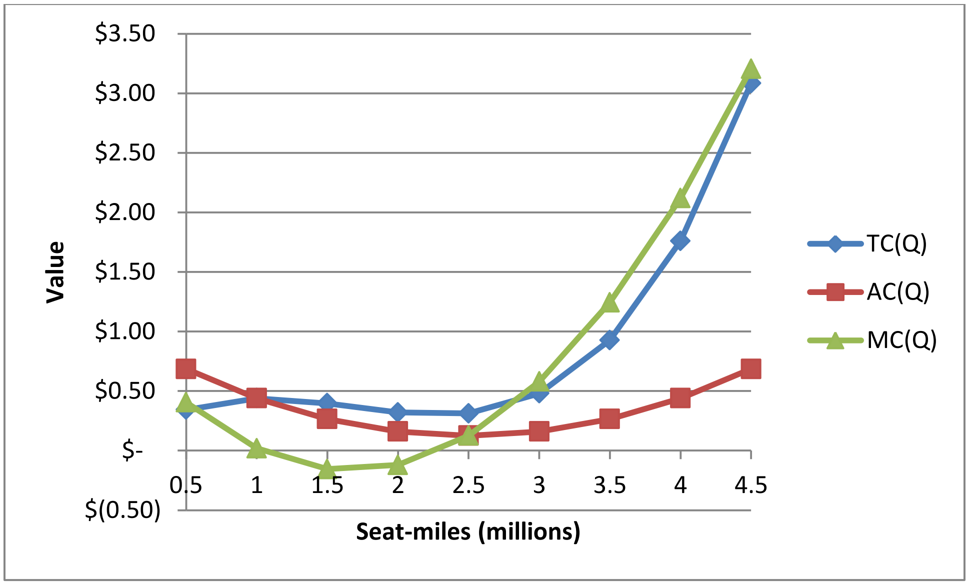
Part (b): To achieve minimum average total cost, average cost (AC) should be equal to marginal cost (MC).

$$AC = \frac{TC(Q)}{Q} = 1 - 0.7Q + 0.14Q^2$$
$$MC = 1 - 1.4Q + 0.42Q^2$$
$$AC = MC$$
$$\Rightarrow 1 - 0.7Q + 0.14Q^2 = 1 - 1.4Q + 0.42Q^2$$
$$Q = 2.5\square million\ ASM\ \ per\ day$$

At that level of output, the average cost per ASM is 1− 0.7(2.5) + 0.14(6.25) = $0.125.

The marginal cost is the same: $MC = 1 - 1.4(2.5) + 0.42(2.5)^2 = \$0.125$ per ASM.

Part (d): The plot from *Q* = 0.5 to *Q* = 4.5 gives the following:



Note that on the *Y* axis, the value scale for *TC* is in millions of dollars, but for *AC* and *MC* the scale is simply dollars. The graph confirms that *Q* = 2.5 million is the point where *AC* reaches a minimum and also where *AC* = *MC*. Below this value, it makes sense to add seat-miles because the marginal cost of an additional seat-mile is less than the average cost, but above this value marginal cost is higher, so it is no longer economically attractive.

Problem 10-7.

In this problem you will solve an airline yield management problem in three different ways. Consider a Boeing 787 Dreamliner with 250 seats all in a common class. (We are ignoring business/first class. Also note that total seating is not a fixed number; airlines are free to order aircraft from companies like Airbus and Boeing with different configurations and total seat numbers.) For a specific origin-destination (OD) pair and day and time, it is observed that at $0 fare, business demand is 270 passengers and leisure demand is 430 passengers. Also, at a fare of $950, leisure demand drops to 0 passengers, and at a fare of $2050, business demand drops to 0 passengers.

Part (a): Write the functions for demand as a function of price on this flight for both business and leisure travelers, and plot both curves on a single set of axes, with demand as a function of price. The graph can be hand-drawn or printed.

Part (b): Solve for the prices that maximize revenue using optimization. Use the solver in MS Excel or some other numerical solver. Report the prices (to the nearest $0.01), the resulting number of each kind of passenger (you do not need to round to whole number of passenger), and the total revenue.

Part (c): Now solve for the optimal values of P_B and P_L using the formuli for these prices that are derived from the closed-form solution to the maximum revenue problem. The formuli for these two prices are presented below. This time round the number of passengers to the nearest whole number, and report prices (to the nearest $0.01), number of passengers, and total revenue. (Note that this is slightly inaccurate because when you round the number of passengers, you actually need a slightly different price to generate that number of passengers, but you can ignore this effect.)

Part (d): In part (c), what is the percent of revenue generated by the business and leisure travelers, respectively?

Part (e): The third and last way to solve for the optimal price structure is to write the equation for the revenue R in terms of P_B and P_L, use the capacity of the aircraft to rewrite R in terms of one variable P_B or P_L, and then solve for the optimal value of that price (Hint: use differentiation). Report prices, number of passengers (no need to round to an integer), and total revenue.

Part (f): Present a table with the two prices found for each solution:

Closed-form solution formula:

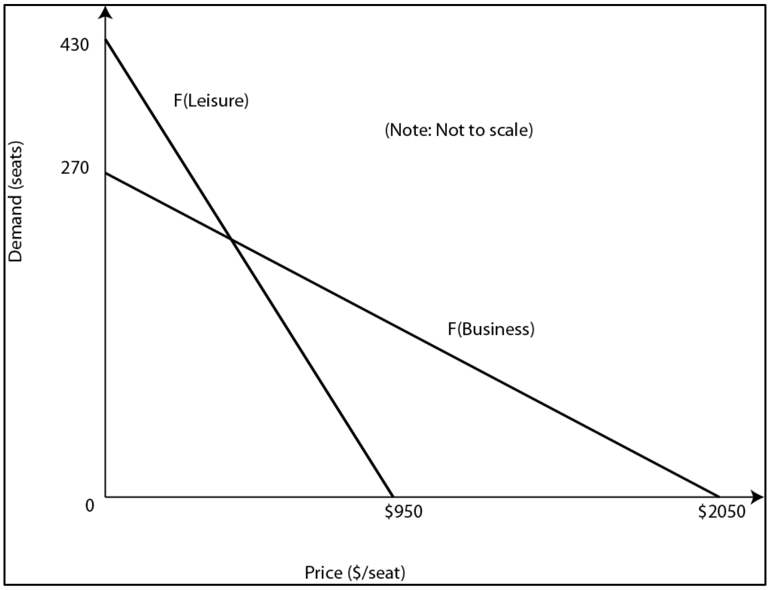
Solution:

$$P_B = 0.5\left[\frac{\alpha_B}{\beta_B} - \frac{2C - \alpha_B - \alpha_L}{\beta_B + \beta_L}\right]$$

$$P_L = \frac{\alpha_B + \alpha_L - C}{\beta_L} - \left(\frac{\beta_B}{\beta_L}\right)P_B$$

Solution.

Part (a): Figure with demand as a function of price:



Note that the question specifically asks for the demand curve, not the inverse demand curve. Points were deducted if you gave the latter.

Solution.

Part (b).

$$max\ R = P_B Q_B + P_L Q_L$$

$$\text{such that } Q_B + Q_L = 250$$

Solution: Maximum *R* = $223,387 when *P* − *B* = $1196.13, *P* − *L* = $646.13. Number of passengers is *Q* − *B* = 112.5, *Q* − *L* = 137.5.

The following table is copied from a spreadsheet with a numerical solver solution:

Rev	223,386.64			
	B	L		
P	$1,196.13	$ 646.13	Sum:	Max
Q	112.46	137.54	250	250
Practical revenue with whole numbers of passengers				
Numbers	112	138		

P-new	$1,199.63	645.12		
Rev	134,358.56	89,026.56	223,385.12	
Revenue	$134,518	$88,869		
Use goal-seek to find price corresponding with Q = 112				
P	$1199.63	$ 645.12		
Q	112.00	138.00		

<u>Solution</u>.

Part (c): Solution using specific formuli for business and leisure pricing:

<u>Solution:</u>

$$P_B = 0.5\left[\frac{\alpha_B}{\beta_B} - \frac{2C - \alpha_B - \alpha_L}{\beta_B + \beta_L}\right]$$

$$P_L = \frac{\alpha_B + \alpha_L - C}{\beta_L} - \left(\frac{\beta_B}{\beta_L}\right) P_B$$

We know that

$$Q_B = 270 - \frac{270}{2050} P_B = 270 - 0.132 P_B \text{ and } Q_L = 430 - \frac{430}{950} P_L = 430 - 0.453 P_L$$

So $\alpha_B = 270, \beta_B = 0.132, \alpha_L = 430, \beta_L = 0.453$

Besides, $C = 250$

$$\text{Hence, } P_B = 0.5\left[\frac{270}{0.132} - \frac{2\times 250 - 270 - 430}{0.132 + 0.453}\right] = 1196.13$$

$$\text{and } P_L = \frac{270 + 430 - 250}{0.453} - \frac{0.132}{0.453} \times 1196.13 = 646.13$$

Results: $P - B$ = $1196.13, P − L = $646.13

The corresponding exact business and leisure demand values are 112.46 and 137.54. Rounding gives 112 and 138 passengers, respectively. Total revenue is then:

$$R = P_B Q_B + P_L Q_L = (\$1196.13)(112) + (\$646.13)(138) = \$223{,}133.39$$

Solution.

Part (d) 60.2% and 39.8%, respectively.

From (c), we can get $Q_B = 270 - \frac{270}{2050} P_B = 270 - \frac{270}{2050} \times 1196.13 = 112.46$

$$\text{and } Q_L = 430 - \frac{430}{950} P_L = 430 - \frac{430}{950} \times 646.13 = 137.54$$

$$\text{So } R_B = Q_B P_B = 112.46 \times 1196.13 = 134516.8$$

$$\text{and } R_L = Q_L P_L = 137.54 \times 646.13 = 88868.7$$

$$\text{Hence } \frac{R_B}{R_B + R_L} = \frac{134516.8}{134516.8 + 88868.7} \times 100\% = 60.2\%, \ \frac{R_L}{R_B + R_L} = 39.8\%$$

Solution.

Part (e): From the description of the aircraft, we know that $C = 250$. The demand functions for business and leisure travelers are written as follows:

$$Q_B = 270 - \frac{270}{2050} P_B = 270 - 0.132 P_B \text{ and } Q_L = 430 - \frac{430}{950} P_L = 430 - 0.453 P_L$$

To solve for optimal prices, solve for P_B in terms of P_L:

$$Q_B + Q_L = 250 \ \Rightarrow (270 - 0.132 P_B) + (430 - 0.453 P_L) = 250$$

$$\Rightarrow 0.132 P_B = 450 - 0.453 P_L \ \Rightarrow P_B = 3409.1 - 3.432 P_L$$

Next, substitute known values into the equations to reduce to an equation with a single unknown P_L:

$$R = Q_B P_B + Q_L P_L$$

$$= (270 - 0.132 P_B) P_B + (430 - 0.453 P_L) P_L$$

$$= [270 - 0.132(3409.1 - 3.432 P_L)](3409.1 - 3.432 P_L) + (430 - 0.453 P_L) P_L$$

$$= -2.008 P_L^2 + 2592 P_L - 613642$$

To maximize R, take the derivative of the equation for R and set equal to zero to solve for P_L and P_B:

$$\frac{dR}{dP_L} = -4.016 P_L + 2592 = 0$$

$$\Rightarrow \ P_L = \$645.4$$

$$\Rightarrow \ P_B = 3409.1 - 3.432 \times 645.4 = \$1194.1$$

So, $Q_B = 270 - 0.132 \times 1194.1 = 112.4$ and $Q_L = 430 - 0.453 \times 645.4 = 137.6$

$$R = 645.4 \times 112.4 + 1194.1 \times 137.6 = \$223024$$

Solution.

Part (f).

Method	Optimization	Part (c) formuli	Differentiation
P-Business	1196.13	1196.13	1194.10
P-Leisure	646.13	646.13	645.40

(Note: The prices need not exactly agree with each other, since solvers etc. can sometimes be slightly erratic. Rather, the purpose is to give you a general sense of how well they agree.)

Problem 10-8.

Budget and breakeven of an HSR operation, loosely based on the Taiwanese HSR system linking Taipei and Kaoshung: You are to evaluate the total operating cost and profit or loss of an HSR system, based on four cost components: (1) energy cost (electricity), (2) capital repayment cost, (3) wages of the workforce, and (4) overhead (which includes the usual business overhead plus all maintenance cost and any other cost associated with the business). Energy costs are based on an average operating speed of 125 mi/h (200 km/h) and use the formuli presented in lecture 1.2. You can assume the HSR trains are 75% efficient and the electricity delivery is 90% efficient. The operation owns 45 HSR trainsets of which each travels 480,000 mi per year. For calculating electricity consumption, assume a cost of 11 cents per kWh and treat the trains as if they travel constantly at the operating speed—for simplicity, ignore start-up and stopping, or slowing to go around turns, etc. Capital repayment cost is $20 million per trainset per year. For wages, assume that each trainset requires 50 employees in the company workforce for all functions (train crew, repair shop, ticketing, etc.) and that the average cost to the company of an employee is $90,000. Lastly, overhead is 25% of the sum of the first three cost components. Questions:

a. What is the total operating cost per year? What percent of the total operating cost does the energy cost constitute?
b. If the average fare paid is $22 per passenger, how many boardings are needed per year for revenues to exactly cover costs?
c. Does the number of boardings from part (b) seem achievable? Why or why not? One- or two-sentence answer.

Solution.

Part (a): Solving for total operating cost requires that the electricity consumption be calculated. The operating speed of 200 km/h is equivalent to 55.6 m/s. Therefore, the drag is:

$$\begin{aligned} D &= a + bV + cV^2 \\ &= 3.82 + (0.1404)(55.6) + (0.00653)(55.6)^2 \\ &= 31.8kN \end{aligned}$$

The power requirement based on drag and velocity is therefore:

$$P = (31.8kN)(55.6m/s) = 1765kW = 1.76MW$$

Total electricity consumption comes from multiplying power by the number of hours of operation per year. The 45 trainsets travel 480,000 mi or 768,000 km each, for a total of 34.56 million kilometer per year. At a speed of 200 km/h, the time required is 172,800 hours per year. Energy required is therefore:

$$E = (1.76MW)(172{,}800h) = 305{,}000MWh$$

Electricity purchased takes into account losses between purchase and delivering energy to propulsion of the trainset:

$$E_{purchased} = \frac{305{,}000MWh}{(0.75)(0.9)} = 452{,}000MWh$$

The price of electricity is equivalent to $110/MWh. Therefore, total expenditure on electricity is $49.7 million per year.

The 45 trainsets require 50 employees at a cost of $90,000 each, for a total of $202.5 million per year. The capital cost of each trainset is $20 million per year, so the total capital cost per year is $900 million. Overhead is calculated by adding 25% to the sum of cost components, so the total cost per year is:

$$C_{tot} = (\$49.7M + \$202.5M + \$900M)(1.25) = \$1440.3M =\sim \$1.44B$$

Energy constitutes $49.7M out of $1.44B, or approximately 3.5%.

Part (b): At $22 per ticket, the number of boardings required is:

$$N = \frac{\$1440.3M}{\$22} = 65.5M / yr$$

Thus, 65.5 million boardings per year are required.

Part (c): The number of boardings seems achievable, but ambitious, or on the high end of the range of possible outcomes. It is equivalent to 179,000 boardings per day; if the 45 trainsets together generate 120 round trips from one end of the line to the other, each round trip would need to generate 1490 boardings, which is within the capacity of the trainset, but quite full.

Problem 10-9.

The Chicago-to-St. Louis Illinois corridor is a prominent example in the U.S. of an expansion of high-speed rail (HSR) as an alternative to flying or driving. Suppose that, at some time in the future, the HSR on this corridor is fully developed and can travel at 280 km/h (175 mi/h). Furthermore, the HSR line has been electrified so that the train runs entirely on electricity. Electricity is derived from the following mix:

Source:	Output	CO_2 rate
	[pct]	[kgCO_2/kWh]

Coal	40%	0.85
Gas	22%	0.325
Nuclear	18%	0
Wind	15%	0
Solar	5%	0

The rate of electricity purchase requirement for the train *P* in kW as a function of speed *V* in km/h is approximated by the following function:

$$P = (1.57)V + (0.0160)V^2 + (2.07x10^{-4})V^3$$

Since the equation for *P* gives the amount of electricity purchased, you can ignore all losses between the point of purchase of the electricity and the train. Between power plants and the point of purchase, the losses in transmission and distribution are on average 11%.

Assume that the travel distance for both the train and the airplane is 454 km (284 mi). You can assume for simplicity that the train travels the entire distance at a cruising speed of *V* = 280 km/h, and that it has room for 460 passengers.

The comparable jet between the two cities is a Boeing 737 with room for 149 passengers. It consumes 540 L of jet fuel in the first 40 km to climb to a cruising altitude of 35,000 ft, consumes 707 L to cruise at this altitude for 320 km, and uses negligible fuel to make the remaining descent into the arrival airport. Each liter of jet fuel releases 2.58 kgCO_2 when combusted, and well-to-tank efficiency to extract crude oil, refine it into jet fuel, and transport to the airport for refueling is 88%.

If the occupancy rate of the train is 65% and of the plane is 75%, how much does each mode of transportation emit per passenger in kgCO_2 per trip, and what is the percent reduction when a passenger switches from the more intensive to less intensive mode?

Solution.

Begin with the average emissions per kWh of electricity. A weighted average gives:

$$\mu_{avge} = (0.4)(0.85) + (0.22)(0.325) = 0.412 kgCO_2 / kWh$$

Adjusting to reflect T&D losses then gives:

$$\mu_{avge} = \frac{0.412}{0.89} = 0.462 kgCO_2 / kWh$$

Power requirement to move the train at the cruising speed of 280 km/h is:

$$P = (1.57)V + (0.0160)V^2 + (2.07x10^{-4})V^3$$
$$P(280) = (1.57)(280) + (0.0160)(280)^2 + (2.07x10^{-4})(280)^3$$
$$= 6238kW$$

To travel the 454-km distance at 280 km/h (ignoring intermediate stops, starting/stopping, etc.) takes 1.62 hours, so multiplying power by time gives 10,123 kWh. Emissions are therefore:

$$(10123)(0.462) = 4681kg$$

Converting to emissions per passenger gives:

$$\frac{4681}{(0.65)(460)} = 15.65kgCO_2 / psgr / trip$$

On the jet side, the total fuel consumption is 540 + 707 = 1247 L. Converting to total emissions and take into account well-to-tank efficiency of 88% gives:

$$(1247L)\frac{2.58kgCO_2 / L}{0.88} = 3{,}656kgCO_2 / trip$$

Converting to emissions per passenger gives:

$$\frac{3656}{(0.75)(149)} = 32.72kgCO_2 / psgr / trip$$

Therefore, HSR is less carbon intensive, and the savings are 32.72 – 15.65 = 17.1 $kgCO_2$, which is equivalent to a 52% reduction.

Chapter 11 Overview of Freight Transportation

Problem 11-1.

Compare two options for shipping a product, either by truck or by intermodal rail (IM). Data for the two modes are the following: for truck: V_{max} = 38,000 lb and shipping time = 2 days; for IM, V_{max} = 43,000 lb and shipping time = 4 days. The product has the following characteristics: the unit cost is $4.03 per pound, and the inventory rate is 18% per year. Demand is given in terms of full truckloads at 12 truckloads per year, regardless of which mode is chosen. The shipping distance in both cases is 734 mi. For trucking, there is a $100 flat charge and then $1.25 per mile additional, and also an order fee of $35 per order. For IM, there is a $400 flat charge and then $0.70 per mile additional, and also an order fee of $35 per order. (a) Calculate the optimal shipment size and total logistics cost per pound for the product, for both truck and IM rail. (b) Indicate which modal option minimizes TLC. (c) If the sales cost is the sum of value of product and total logistics cost (TLC), what is the sales cost per pound and the percent contribution of TLC to sales cost in each of the truck and IM cases? (Hint: Throughout the problem, the differences between truck and rail are small, so carry enough decimal places to show the difference.)

Solution.

Part (a): The cost for 724 mi of distance must be calculated first. Given the flat rates and distance charges involved per shipment, the total cost of the shipment is:

$$C_{truck} = \$100 + (\$1.25/mi)(734mi) + \$35 = \$1052.50$$
$$C_{rail} = \$400 + (\$0.70/mi)(734mi) + \$35 = \$948.00$$

The conversion of 2 and 4 days to years is equivalent to 0.00548 and 0.01096 years, respectively. The flow value per year is Q = (12)(38000) = 456,000 lb/year. The optimal shipment size in pounds per shipment, rounded to the nearest integer value, is:

$$V^*_{truck} = \sqrt{\frac{2F'Q}{PR}} = \sqrt{\frac{2(1052.5)456000}{(4.03)0.18}} = 36376lb$$
$$V^*_{rail} = \sqrt{\frac{2F'Q}{PR}} = \sqrt{\frac{2(948)456000}{(4.03)0.18}} = 34538lb$$

The maximum possible lading per container is 38,000 lb, so in both cases V^* is a feasible value. Solving for total logistics cost for both options gives:

$$C(V^*_{truck}) = \frac{PRV^*}{2Q} + PRT' + \frac{F'}{V^*} = \frac{0.7254(36376)}{2(456000)} + 0.7254(0.00548) + \frac{1052.50}{36376}$$
$$= \$0.06184/lb$$
$$C(V^*_{rail}) = \frac{PRV^*}{2Q} + PRT' + \frac{F'}{V^*} = \frac{0.7254(34538)}{2(456000)} + 0.7254(0.01096) + \frac{948}{34538}$$
$$= \$0.06289/lb$$

Part (b): Since the truck option is slightly cheaper per pound, it is preferred.

Part (c): Each pound of product costs $4.03 before including TLC. Therefore, the final cost is:

$$C_{truck} = \$4.03 + \$0.06184 = \$4.09184 / lb$$
$$C_{rail} = \$4.03 + \$0.06289 = \$4.09289 / lb$$

The percent charge for TLC is the TLC for the respective modes divided by the final cost. The value for truck and rail is 1.51% and 1.54%, respectively.

Problem 11-2.

Ton-mile and energy consumption data for U.S. trucks and railroads for the period 1980 to 2000 are given in the following table. Use Divisia decomposition to create a table and a graph for the period 1980 to 2000, showing four curves: (1) actual fuel consumption, (2) trended fuel consumption, and the contribution of (3) energy intensity, and (4) structural changes to the difference between actual and trended fuel consumption.

Year	Activity		Energy	
	Truck	Rail	Truck	Rail
	Bil.tkm	Bil.tkm	EJ	EJ
1980	836	1342	1.89	0.583
1985	925	1342	1.96	0.485
1990	1045	1565	2.17	0.478
1995	1194	1733	2.4	0.469
2000	1524	2168	3.24	0.555

Solution.

We first create a table of combined truck ("TK" throughout) and rail ("RR" throughout) activity, energy consumption, and energy intensity, for which units of PJ per billion tkm are chosen:

Year	Activity	Energy	Intensity
	bil.tkm	EJ	PJ/bil.tkm
1980	2178	2.473	1.135
1985	2267	2.445	1.079

1990	2610	2.648	1.015
1995	2927	2.869	0.980
2000	3692	3.795	1.028

For the Divisia analysis, a measure of energy intensity for individual modes is also necessary:

Year	TK	RR
	PJ/bil.tkm	PJ/bil.tkm
1980	2.26	0.43
1985	2.12	0.36
1990	2.08	0.31
1995	2.01	0.27
2000	2.13	0.26

Based on the data given, it is possible to calculate the reference overall energy intensity in the base year 1970, which is 1.135 PJ/bil.tkm.

Terms are then generated from the given data according to the presentation in Chap. 3 in the book. The key equation is the following:

$$\Delta e_t = \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2} + (s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$
$$= \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2}\right] + \sum_{i=1}^{n}\left[(s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$

To calculate the structure and intensity terms implied by this equation, it is necessary to calculate using a spreadsheet a table of input values for each of the modes TK and RR.

For TK mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1980	2.26	38.4%				
1985	2.12	40.8%	–0.142	0.024	4.380	0.792
1990	2.08	40.0%	–0.042	–0.008	4.195	0.808
1995	2.01	40.8%	–0.067	0.008	4.087	0.808
2000	2.13	41.3%	0.116	0.005	4.136	0.821

For RR mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1980	0.43	61.6%				
1985	0.36	59.2%	–0.073	–0.024	0.796	1.208
1990	0.31	60.0%	–0.056	0.008	0.667	1.192
1995	0.27	59.2%	–0.035	–0.008	0.576	1.192
2000	0.26	58.7%	–0.015	–0.005	0.527	1.179

Using the above equation, we can now calculate the individual terms needed for the Divisia analysis, including an intensity and structure term for each of TK and RR modes for the years 1975 to 2000.

Year	Intensity		Structure	
	TK	RR	TK	RR
1980	0	0	0	0
1985	–0.056	–0.044	0.053	–0.010
1990	–0.017	–0.033	–0.016	0.003
1995	–0.027	–0.021	0.015	–0.002
2000	0.048	–0.009	0.010	–0.001

To calculate the overall impact of structure and intensity, in the table below we first sum values for TK and RR for the given year (“incremental”) and then add together the value for the given year with the sum of all previous years ("cumulative"). For example, for 1990, the cumulative value for intensity is the sum of the 1985 cumulative value and the 1990 incremental value, that is, –0.100 + (–0.050) = –0.150.

Year	Incremental changes (TK+RR)		Cumulative (Cols.1 and 2)	
	Intensity	Structure	Intensity	Structure
	PJ/bil.tkm	PJ/bil.tkm	PJ/bil.tkm	PJ/bil.tkm
1985	–0.100	0.043	–0.100	0.043
1990	–0.050	–0.013	–0.150	0.030
1995	–0.048	0.013	–0.198	0.043
2000	0.039	0.009	–0.159	0.052

The cumulative values then provide the basis for converting ton-kilometer of activity in a given year into energy use. As shown in the table below, combined TK and RR ton-kilometer are multiplied by the intensity and structure cumulative terms from the preceding table to calculate the contribution to the difference between trended and actual energy consumption.

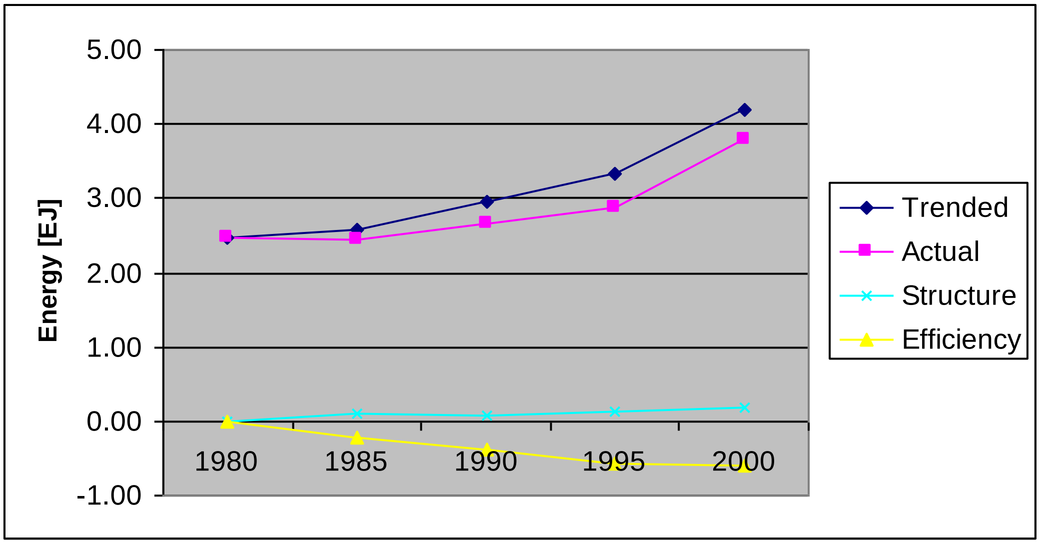
Year	Activity	Intensity		Structure	
	bil.tkm	PJ/bil.tkm	EJ	PJ/bil.tkm	EJ
1980	2178	0	0	0	0
1985	2267	–0.100	–0.23	0.043	0.10
1990	2610	–0.151	–0.39	0.030	0.08
1995	2927	–0.198	–0.58	0.043	0.13
2000	3692	–0.159	–0.59	0.052	0.19

The conclusion of the Divisia analysis is shown by summing trended, intensity contribution, and structure contribution. The trended fuel consumption is obtained by multiplying the vehicle kilometer in each year by the fixed energy intensity from above of 1.135 PJ/bil.tkm. Note that the sum term (second column from the right) exactly equals the actual fuel consumption (both are measured in billions of liters of fuel).

Year	Activity	Trended	Intensity	Structure	Combined	Actual
	bil.tkm	EJ	EJ	EJ	$= T + I + S$	(Check)
1980	2178	2.47	0.00	0.00	2.47	2.47
1985	2267	2.57	–0.23	0.10	2.45	2.45
1990	2610	2.96	–0.39	0.08	2.65	2.65
1995	2927	3.32	–0.58	0.13	2.87	2.87
2000	3692	4.19	–0.59	0.19	3.80	3.80

Interpretation of the Divisia analysis: In general, the actual consumption is lower than the trended consumption because of the significant negative value of the intensity contribution, which dominates the smaller positive contribution of the structure term. The outcome can be visualized using the figure below, and noting that the intensity term is labeled "efficiency" in the figure.

Figure for Problem 11-2:



Problem 11-3.

Below are the data in standard units for U.S. rail and marine for the period 1970 to 2005. Use Divisia decomposition to create a table and a graph for the period 1970 to 2005, showing four curves: (1) actual fuel consumption, (2) trended fuel consumption, and the contribution of (3) energy intensity, and (4) structural changes to the difference between actual and trended fuel consumption.

Solution.

We first create a table of combined rail ("RR" throughout) and marine ("MA" throughout) activity, energy consumption, and energy intensity, for which units of btu/ton-mile are chosen:

Year	Activity	Energy	Intensity
	bil tmi	tril btu	btu/tmi
1970	1361	858	630
1975	1320	840	636
1980	1841	886	481
1985	1770	916	518
1990	1868	778	416
1995	2114	791	374
2000	2112	823	390
2005	2287	894	391

Note the 1970 energy intensity value of 630 Btu/tmi, which is used to calculate the trended data series. For the Divisia analysis, a measure of energy intensity for individual modes is also necessary:

Year	RR	MA
1970	697	545
1975	743	495
1980	605	358
1985	639	399
1990	435	393
1995	370	381
2000	353	474
2005	339	540

Terms are next generated from the given data according to the presentation of Divisia analysis in the book. The key equation is the following:

$$\Delta e_t = \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2} + (s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$
$$= \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2}\right] + \sum_{i=1}^{n}\left[(s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$

To calculate the structure and intensity terms implied by this equation, it is necessary to calculate using a spreadsheet a table of input values for each of the modes RR and MA.

For RR mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1970	696.7	0.562				
1975	742.7	0.571	46.0	0.0091	1439.4	1.1333
1980	605.0	0.499	–137.7	–0.0720	1347.7	1.0704
1985	638.5	0.495	33.5	–0.0037	1243.5	0.9947
1990	435.2	0.554	–203.3	0.0581	1073.7	1.0490
1995	369.8	0.618	–65.4	0.0643	805.0	1.1713
2000	352.7	0.694	–17.2	0.0763	722.5	1.3119
2005	339.0	0.742	–13.6	0.0475	691.7	1.4357

For MA mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1970	545.3					

		0.438				
1975	494.7	0.429	–50.6	–0.0091	1040.0	0.8667
1980	357.9	0.501	–136.8	0.0720	852.6	0.9296
1985	398.7	0.505	40.7	0.0037	756.6	1.0053
1990	393.3	0.446	–5.4	–0.0581	791.9	0.9510
1995	381.2	0.382	–12.1	–0.0643	774.5	0.8287
2000	473.7	0.306	92.5	–0.0763	854.9	0.6881
2005	539.8	0.258	66.1	–0.0475	1013.4	0.5643

Using the above equation, we can now calculate the individual terms needed for the Divisia analysis, including an intensity and structure term for each of RR and MA modes for the years 1970 to 2005.

Year	Intensity		Structure	
	RR	MA	RR	MA
1970	0	0	0	0
1975	26.05	–21.93	6.57	–4.75
1980	–73.70	–63.58	–48.54	30.71
1985	16.68	20.48	–2.30	1.40
1990	–106.65	–2.55	31.17	–22.99
1995	–38.29	–5.01	25.86	–24.88
2000	–11.26	31.82	27.58	–32.63
2005	–9.78	18.64	16.41	–24.05

To calculate the overall impact of structure and intensity, in the table below we first sum values for RR and MA for the given year ("incremental") and then add together the value for the given year with the sum of all previous years ("cumulative"). For example, for 1980, the cumulative value for intensity is the sum of the 1975 cumulative value and the 1980 incremental value, that is, 4.12 + (–137.27) = –133.15.

Year	Incremental		Cumulative	
	Intensity	Structure	Intensity	Structure
1970	0.00	0.00	0.00	0.00
1975	4.12	1.82	4.12	1.82
1980	–137.27	–17.83	–133.15	–16.01
1985	37.16	–0.90	–96.00	–16.91
1990	–109.21	8.18	–205.20	–8.73
1995	–43.30	0.98	–248.50	–7.75
2000	20.56	–5.05	–227.94	–12.80
2005	8.86	–7.63	–219.08	–20.44

The cumulative values then provide the basis for converting ton-miles of activity in a given year into energy use. As shown in the table below, combined RR and MA ton-miles are multiplied by the intensity and structure cumulative terms from the preceding table to calculate the contribution to the difference between trended and actual energy consumption.

Year	Activity	Intensity		Structure	
	bil.tmi	Btu/tmi	tril.Btu	Btu/tmi	tril.Btu
1970	1361	0.00	0.00	0.00	0.00
1975	1320	4.12	5.44	1.82	2.41
1980	1841	–133.15	–245.13	–16.01	–29.47
1985	1770	–96.00	–169.91	–16.91	–29.93
1990	1868	–205.20	–383.31	–8.73	–16.31
1995	2114	–248.50	–525.33	–7.75	–16.38
2000	2112	–227.94	–481.41	–12.80	–27.04

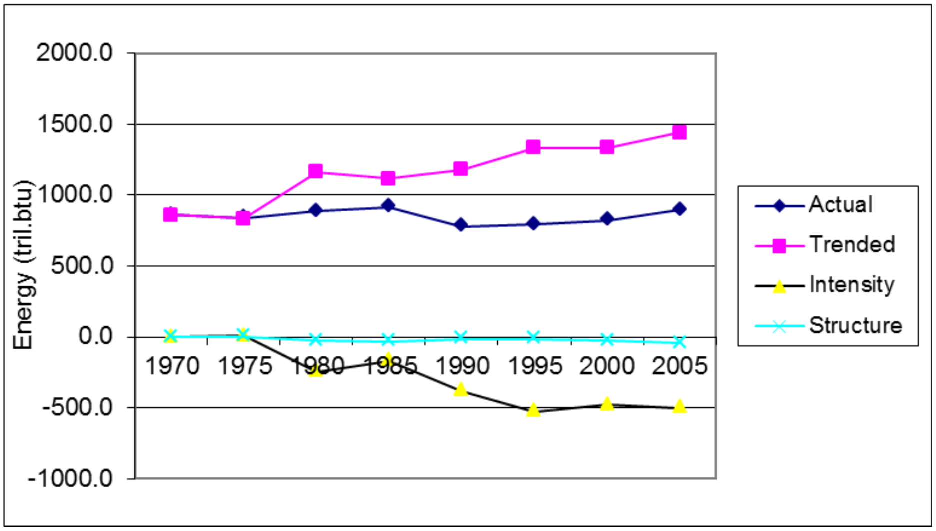
2005	2287	–219.08	–501.03	–20.44	–46.74

The conclusion of the Divisia analysis is shown by summing trended, intensity contribution, and structure contribution, and showing that they exactly equal the actual energy consumption in each year (all measured in trillions of Btu).

	Actual	Trended	Intensity	Structure	*T* + *I* + *S*
1970	858.0	858.0	0	0	858.0
1975	840.0	832.2	5.4	2.4	840.0
1980	886.0	1160.6	–245.1	–29.5	886.0
1985	916.0	1115.8	–169.9	–29.9	916.0
1990	778.0	1177.6	–383.3	–16.3	778.0
1995	791.0	1332.7	–525.3	–16.4	791.0
2000	823.0	1331.4	–481.4	–27.0	823.0
2005	894.0	1441.7	–501.0	–46.7	894.0

Interpretation of the Divisia analysis: In general, the actual consumption is lower than the trended consumption because of the significant negative value of the intensity contribution. Intensity dominates the smaller positive contribution of the structure term, which is also negative but contributes less. The outcome can be visualized using the figure below.

Figure for Problem 11-3:



Problem 11-4.

Note to instructor: This problem considers fuel consumption for ocean-going container ships, which are not directly considered in Unit 4 on Freight, although the power requirement equations are similar to those of other transportation modes considered in Chaps. 5, 10, and 12. Instructors may wish to provide some additional original information not contained in the textbook on container shipping before posting this problem.

Consider the American Presidents Line (or APL) container route linking Shanghai and Savannah, known as the East Coast/North Loop Route. APL provides weekly container ship service on this route. This route has a total round-trip time of 56 days (8 weeks), of which 5 days are in port. Of the remaining 51 days, 2 days are spent traversing the Panama Canal (1 day each direction), so there are 49 days of normal sailing time. The route is served by a vessel with a max of 8000 TEUs like the one shown in Fig. 11-3 moving out of a lock in the Canal.

The total route distance is 24,700 nmi, or 12,350 nmi in each direction, and the normal average cruising speed V is V = 21 knots (1 knot = 1 nmi per hour). Fuel consumption is a function of travel speed. Let P be the power requirement to maintain speed measured in Btu/h, then the functional relationship is $P = KV^3$, where K is a constant parameter with value K = 18,850 Btu-h^2/nmi^3. You may ignore the fuel consumed during the days in port (and in this case, also during the Panama Canal traversal) because that quantity will not be affected by a change in cruising speed.

The vessel costs are $18,000 per day for ownership costs and another $9000 per day for non-fuel operating costs, for a total of $27,000 per day. Bunker fuel costs $700 per ton and has an energy content of 40 million Btu/ton (40 MBtu/ton).

a. Under current operating conditions, what is the total cost per day to operate a ship on the route, including both fuel and non-fuel costs?
b. Consider a shipper who moves toys for retail sale from a manufacturer in Shanghai to a distribution center near Savannah. That shipper moves five 40-ft containers per week (a total of 13,500 ft^3) from Shanghai to Savannah, and the average value of the toys is $40,000 per container. As indicated on the schedule in Fig.11-1, the transit time by sea from Shanghai to Savannah is 26 days. On average, there are three additional days for the containers at each end of the trip (delivery to the port in Shanghai, processing and waiting for loading, unloading at Savannah and waiting for pickup by truck, followed by delivery to the distribution center), so the total time-in-transit is 32 days. The freight cost of a container movement is $3500 for the voyage, plus $300 for port charges, plus $200 for drayage (truck movements at either end of the trip), for a total of $4000 per container. Assume that the inventory costs are evaluated at 15% per year (or 0.29% per week). What is the total logistics cost per week for the five containers for this shipper under the current sailing schedule?
c. Suppose the ship operator decides to reduce the travel speed of the container ship that is full with 4000 containers of toys as described in (b) to 18 knots. If the ship operator gains the financial savings of using less fuel on the voyage but must reduce its charge to the shipper by an amount equal to the increase in in-transit inventory cost, what is the net increase or decrease in revenue for one sailing of the ship?

Solution:

Part (a): The non-fuel cost per day is given as $27,000 per day. For fuel cost, the total tons of fuel per day is calculated as follows:

$$P = KV^3$$
$$= (18850)(21)^3 = 1.745\times 10^8 \, Btu/h$$
$$DailyFuel = (1.745\times 10^8)(24h/d) = 4.189\times 10^9 \, Btu$$
$$\frac{4.189\times 10^9 \, Btu}{4\times 10^7 \, Btu/ton} = 104.7 ton/d$$
$$(104.7 ton/d)(\$700/ton) = \$73{,}306$$

Therefore, the total daily requirement is $27,000 + $73,306 = $100,306/day

Part (b): Using the total logistics cost model, we have the following values: *Q* = 5 containers/week, *P* = $40,000 per container, *r* = 0.0029 per week, *F* = $20,000 for five containers, *V* = 5 containers/week, *T* = 32 days = 4.57 weeks.

Plugging in to the TLC cost equation:

$$C = rPV + \frac{FQ}{V} + rPTQ$$
$$= (0.0029)(40000)(5) + \frac{(20000)5}{5} + (0.0029)(40000)(4.57)(5)$$
$$= \$580 + \$20{,}000 + \$2651 = \$23{,}231$$

Therefore, the total logistics cost is $23,231 per week.

Part (c): The fuel savings require knowledge of both the rate of energy consumption *P* and the longer time required to complete the journey. The total distance is 12,350 nmi. Therefore, in the first instance the total number of hours and fuel requirement for the base case is:

$$\frac{(12{,}350 nmi)}{21 knot} = 588.1h$$
$$(1.745\times 10^8)(588.1h) = 1.03\times 10^{11} \, Btu$$

If speed is reduced to *V* = 18 knots, power, energy, and cost requirement change as follows:

$$P = KV^3$$
$$= (18850)(18)^3 = 1.099 \times 10^8 \, Btu/h$$
$$\frac{12{,}350 nmi}{18 knot} = 686.1h$$
$$TotalFuel = (1.099 \times 10^8)(686.1h) = 7.54 \times 10^{10} \, Btu$$
$$Savings = 1.03 \times 10^{11} \, Btu - 7.54 \times 10^{10} \, Btu = 2.72 \times 10^{10} \, Btu$$
$$\frac{2.72 \times 10^{10} \, Btu}{4 \times 10^7 \, Btu/ton} = 681 ton$$
$$(681 ton)(\$700/ton) = \$476{,}563$$

On the logistics cost side, it can be assumed that the only element that changes is the in-transit inventory cost. The increased trip length is 686.1 – 588.1 = 98 hours, or 0.58 week. Therefore, the increase in logistics cost and net change in cost is:

$$\Delta TLC = rPT = 0.0029(\$160M)(0.58) = \$270{,}711$$
$$\Delta Cost = \$270{,}711 - \$476{,}563 = -\$205{,}853$$

Problem 11-5.

This exercise is based on a rail car load freight model that originally appeared in Morlok, 1978 (Morlok, Edward K, *Introduction to Transportation Engineering and Planning,* McGraw-Hill, New York), which has been updated for modern values. A rail freight train can pull a minimum of 10 and maximum of 120 cars. The fixed cost, FC, is \$50,000. The variable cost, VC, is provided in the table below; you can work in units of 10 cars throughout the problem.

a. Create a table showing total cost TC, average cost AC, and marginal cost MC for adding an increment of 10 cars from the given data.
b. If AC is defined as average cost per 10 cars and MC the cost of adding 10 more cars, what is the optimal number of cars from the table from part (a)?
c. Use the VC data and an electronic solver (Excel, Matlab, handheld calculator, etc.) to fit a quadratic curve of the following form: $VC = aN^3 + bN^2 + cN + d$, where *N* is the number of units of 10 cars added to the train, and *a, b, c, d* are parameters chosen by the solver to minimize the sum of squared error.
d. Use the equation from part (b) to find equations for TC, AC, and MC, and plot AC and MC on the same axes using the equation from part (c). Assume that the function for *N* is continuous, even though the number of cars is discrete.
e. Find the value of *N* that minimizes average cost. *N* can be a decimal number.
f. How close is the value of *N* calculated from part (e) to the value from part (b)?

Variable cost as a function of number of cars:

Cars	VC
10	\$50,000

20	$ 77,000
30	$97,000
40	$116,000
50	$134,000
60	$150,000
70	$168,000
80	$190,000
90	$220,000
100	$260,000
110	$310,000
120	$370,000

Solution.

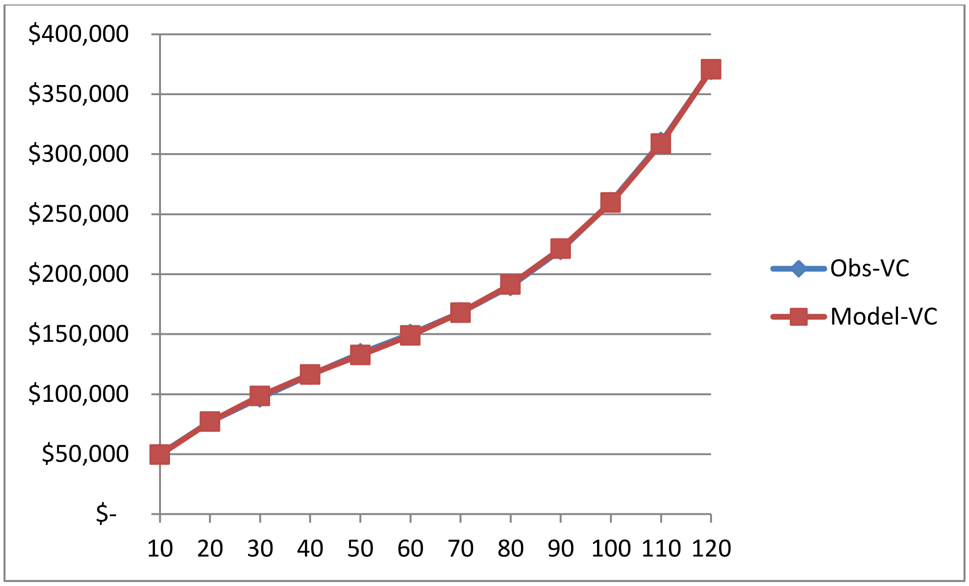
For part (a), TC = FC + VC, AC = TC/N, and MC = $TC_N - TC_{N\text{-}10}$. Compiling all values in a table gives the following:

Cars	FC	VC	TC	AC	MC
10	$50,000	$50,000	$100,000	$100,000	--
20	$50,000	$77,000	$127,000	$63,500	$27,000
30	$50,000	$97,000	$147,000	$49,000	$20,000
40	$50,000	$116,000	$166,000	$41,500	$19,000
50	$50,000	$134,000	$184,000	$36,800	$18,000
60	$50,000	$150,000	$200,000	$33,333	$16,000
70	$50,000	$168,000	$218,000	$31,143	$18,000
80	$50,000	$ 190,000	$240,000	$30,000	$22,000
90	$50,000	$220,000	$270,000	$30,000	$30,000
100	$50,000	$260,000	$310,000	$31,000	$40,000
110	$50,000	$310,000	$360,000	$32,727	$50,000

120	\$50,000	\$370,000	\$420,000	\$35,000	\$60,000

Part (b): From the table, the cost of going from 80 to 90 cars is \$30,000, and the average cost is also \$30,000 at 90 cars. So 90 cars is the optimal value.

Part (c): Take the given VC values in one column and then in an adjacent column calculate a fitted VC of the form $VC = aN^3 + bN^2 + cN + d$. Then in an adjacent column calculate a squared error term $(VC_{observed} - VC_{estimated})^2$. Use the solver to find parameter values that minimize the sum of the terms. A representative solver solution is a = 0.34, b = −49.67, c = 3990.09, and d = 14,271. The goodness of fit can be observed in the figure below, although it was not required as part of the assignment.



Part (d): The relevant values are calculated as follows:

$$FC = 50{,}000$$

$$VC = 0.34N^3 - 49.7N^2 + 3990N + 14271$$

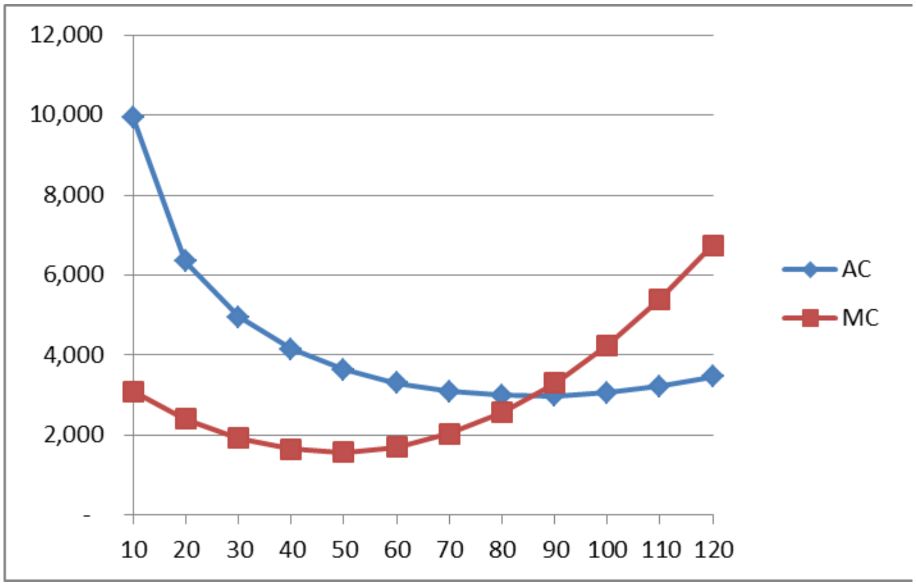
$$TC = FC + VC = 0.34N^3 - 49.7N^2 + 3990N + 64271$$

$$AC = \frac{TC}{N} = \frac{0.34N^3 - 49.7N^2 + 3990N + 64271}{N}$$

$$= 0.34N^2 - 49.7N + 3990 + \frac{64271}{N}$$

$$MC = \frac{dTC}{dN} = 1.02N^2 + 99.4N + 3990$$

The required graph is then:



Part (e): Use AC = MC to solve for the cost-minimizing value of N: At value N = 85.9 cars, AC = MC = ~$2978, using a numerical solver. The solution via this approach is therefore approximately N = 86.

Part (f): The difference is approximately four cars.

Problem 11-5.

A commercial electronic product is to be shipped from Los Angeles to New York, a distance of approximately 2900 mi, by intermodal rail. Each box of product has a value of $2000, and a shipping container moved by IM rail can carry 400 boxes. The IM rail service averages 50 mi/h, and costs $400 flat cost plus $35 processing fee plus $0.80 per mile. The cost of holding inventory is 18% per year, and the total demand for the product is 6100 boxes per year in New York.

a. What is the economic order quantity in boxes per container, and what is the total logistics cost per box at that quantity? Give your answer to the nearest $0.001. (Hint: you can convert shipping time to units of days, and use fractions of a day to solve.)
b. Suppose the shipper decides to ship full containers instead of the EOQ. What is the total logistics cost per box in this case?

<u>Solution.</u>

The transit time for 2900 mi at an average of 50 mi/h is 58 hours, or 2.42 days, or 0.00662 years. Given the flat rates and distance charges involved per shipment, the total cost of the shipment is:

$$Cost = \$400 + \$35 + (2900)(\$0.80) = \$2755$$

The optimal shipment size in boxes per shipment, rounded to the nearest value, is:

$$V^* = \sqrt{\frac{2F'Q}{PR}} = \sqrt{\frac{2(2755)6100}{(2000)0.17}} = 305.6 =\sim 306 units$$

The maximum possible lading per container is 400 boxes, so V_{max} = 400. Solving for total logistics cost for both options gives:

$$C(V^*) = \frac{PRV^*}{2Q} + PRT' + \frac{F'}{V^*} = \frac{340(306)}{2(6100)} + 340(0.00662) + \frac{2755}{306} = 9.016 + 2.384 + 9.016$$
$$= \$20.416 / unit$$
$$C(V\int_{max}) = \frac{PRV_{max}}{2Q} + PRT' + \frac{F'}{V_{max}} = \frac{340(400)}{2(6100)} + 340(0.00662) + \frac{2755}{400} = 11.803 + 2.384 + 6.888$$
$$= \$21.074 / unit$$

Answers to problems:

Part (a), 306 boxes per shipment; TLC = $20.416; Part (b), $21.074/unit.

Chapter 12 Modal and Supply Chain Management Approaches

Problem 12-1.

A tractor-trailer truck weighs 34,500 kg, has a cross-sectional area of 8.8 m^2, a drag coefficient of 0.4, and a coefficient of rolling resistance of 0.0056. Assume an air density of 1.1 kg/m^3. The truck travels a distance of 400 km at a constant speed of 112 km/h without stopping, and at that speed, the tank to wheel (TTW) efficiency is 26%. (a) How many liters of diesel fuel does the truck consume? (b) Suppose that the truck now travels the same distance at a reduced speed of 104 km/h to reduce CO_2 emissions. The TTW efficiency is again 26%. How many kg of CO_2 emissions are saved over the same 400-km distance compared to driving at 112 km/h?

Solution.

Part (a): The tractive power requirement at cruising speed is calculated using the parameters given. The speed of 112 km/h must first be converted to (112)/(3.6) = 31.1 m/s. The instantaneous power, total energy requirement, total energy input, and fuel requirement are then calculated as follows.

$$P_{TR} = 0.5(\rho)(A_F)(C_D)V^3 + mgV\,C_o$$

$$= 0.5(1.1)(8.8)(0.4)(31.1)^3 + (34{,}500)(9.8)(31.1)(0.0056)$$

$$= 117{,}119W = 117.1kW$$

$$t = \frac{400km}{112km/h} = 3.57h$$

$$E_{out} = (117.1kW)(3.57h) = 418.3kWh = 1.51GJ$$

$$E_{in} = \frac{E_{out}}{\eta_{TTW}} = \frac{1.51}{0.26} = 5.79GJ$$

$$Fuel = \frac{E_{in}}{E_{Liter}} = \frac{5.79GJ}{35.87MJ/L}\left(\frac{1000MJ}{GJ}\right) = 161.5L$$

Part (b): Repeating the instantaneous power requirement calculation for 104 km/h (28.9 m/s) gives 101.4 kW. Then the steps for calculating energy and fuel are repeated as follows. Once the new energy requirement is known, the energy savings, energy input savings, fuel savings, and CO_2 reduction can be calculated. The calculation assumes 2.69 kg of CO_2 emitted per liter of diesel combusted.

$$t = \frac{400km}{104km/h} = 3.85h$$

$$E_{out} = (101.4kW)(3.85h) = 390.2kWh = 1.40GJ$$

$$E_{saved} = 1.51GJ - 1.40GJ = 0.11GJ = 110MJ$$

$$E_{in.saved} = \frac{E_{saved}}{\eta_{TTW}} = \frac{110MJ}{0.26} = 423MJ$$

$$Fuel_{saved} = \frac{E_{in.saved}}{E_{Liter}} = \frac{423MJ}{35.87MJ/L} = 11.8L$$

$$CO_2 = (11.8L)(2.69kgCO_2/L) = 31.7kgCO_2$$

Problem 12-2.

Studies show that converting trucks from conventional to wide-based tires can reduce fuel consumption by 2.6% per participating vehicle. It is believed that the maximum potential penetration for this technology into the market is 100%. The technology in the year 2010 has penetrated the market to the level of 5%. For years 2011 to 2013 the penetration values are 5.4%, 5.7%, and 6.6%, respectively. In 2010 there are 1.78 million combination trucks (ignore single-body trucks for this problem), each driving on average 98,000 mi per year. In the base case, trucks that do not have wide-based tires achieve a fuel economy of 5.6 mpg diesel. For the purposes of this problem, you can assume that total number of trucks, miles per truck per year, and fuel economy stay constant for the duration. (a) Using only the penetration achieved in years 2010 to 2013 and the ultimate penetration value, use the technological substitution "*s*-curve" to estimate the percent market penetration in the year 2030 to the nearest 0.1%. Include in your answer the values of the curve-fitting parameters c_1 and c_2 that you deduce. (b) create a graph of year-by-year penetration of the technology from 2010 to 2030, including both observed and modeled curves. (c) Using the data given, how many gallons of fuel are saved in the year 2030 compared to a situation where no wide-based tires have been installed?

Solution.

The technological substitution model is of the form:

$$f = \frac{F \cdot e^{(c_1 + c_2 t')}}{1 + e^{(c_1 + c_2 t')}}$$

Use of the solver with F = 100% returns value of $c_1 = -2.9612$ and $c_2 = 0.0969$. The RMSD for the years 2010 to 2013 is 0.001292. The following table results:

Year	*t'*	Percent		
		Observed	Estimated	(Error)^2
2010	0	5.0%	4.9%	6.28E-07
2011	1	5.4%	5.4%	3.29E-09
2012	2	5.7%	5.9%	4.43E-06
2013	3	6.6%	6.5%	1.62E-06

As an example of the calculation in the table, in the year 2012, the observed value of 5.7% is given. The calculated value is then:

$$f = \frac{\exp(c_1 + c_2 t)}{1 + \exp(c_1 + c_2 t)}$$

$$= \frac{\exp(-2.9612 + 0.0969(2))}{1 + \exp(-2.9612 + 0.0969(2))}$$

$$= 0.059 = 5.9\%$$

The error term is then calculated by squaring the difference:

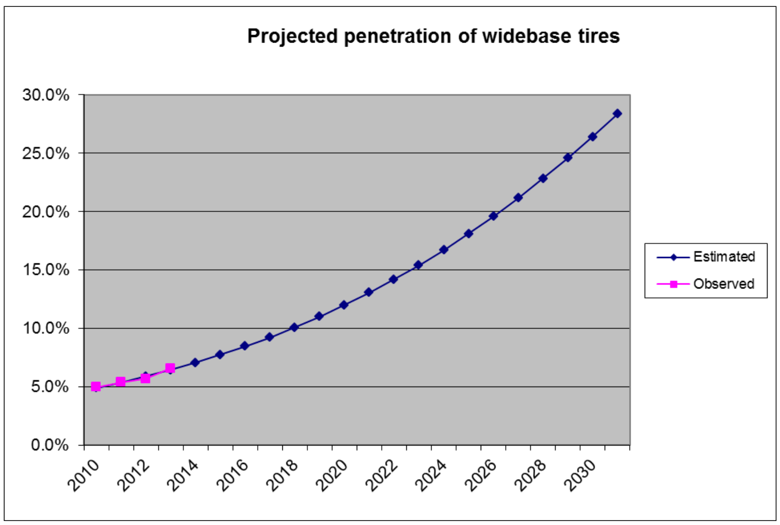
$$Err = (0.057 - 0.059)^2 = 4.0 \times 10^{-6}$$

Note that the value in the table is slightly different because more terms were carried in the calculation. RMSD is calculated by summing the error terms in the right column, dividing by the number of terms n = 4, and then taking the square root.

Having fit the *s*-curve to the observed data we can calculate the expected percentage of penetration in 2030 when t' = 20 years by extrapolating out to that year:

$$f = \frac{\exp(-2.9612 + 0.0969 \cdot 20)}{1 + \exp(-2.9612 + 0.0969 \cdot 20)} = 0.264 = 26.4\%$$

Thus the penetration is 26.4%. This value can also be seen in the graph:



Part (b): The percent penetration is 26.4%, so we need only focus on this fraction of the total fleet of 1.78 million vehicles. Calculating first the number of vehicles affected, the base case fuel consumption for those vehicles, and the number of gallons saved gives:

$$(1.78M)(0.264) = 470{,}000 veh$$

$$(470{,}000)(98{,}000 mi/yr)\left(\frac{1}{5.6 mpg}\right) = 8.23 bil.gal$$

$$(8.23 bil.gal)(0.026) = 214 Mgal$$

Thus the savings in 2030 would amount to 214 million gallons.

Problem 12-3.

A freight train consists of 80 freight cars, each weighing 30 tons empty and carrying a load of 40 tons (typical of consumer goods), at 79 mi/h (the U.S. national rail speed limit) on a level track. The train is to be pulled by a string of locomotives each with maximum power of

3700 hp, weight of 142 tons, and six axles. (a) What is the total resistance of just the pulled load of the train, not including the locomotives, in pounds? (b) What is the minimum number of locomotives required to pull the load at the desired speed, and the resulting total resistance of the train in pounds?

Solution.

Part (a): Use the equation in the chapter to calculate the load per car and then multiply by 80 cars:

$$
\begin{aligned}
R_{CAR} &= aT + b + cTV + dV^2 \\
&= 1.5(70) + 72.5 + 0.015(70)(79) + 0.055(79)^2 \\
&= 603.7 lb
\end{aligned}
$$

The total load for 80 cars is therefore (80)(603.7) = 48,296 lb.

Part (b): The resistance per locomotive is calculated using the formula from the chapter:

$$
\begin{aligned}
R_{LOCOMOTIVE} &= aT + bN + cTV + dV^2 \\
&= (1.3)142 + (29)6 + (0.03)(142)(79) + (0.0312)(79)^2 \\
&= 889.9 lb
\end{aligned}
$$

The available tractive effort at 79 mi/h is then:

$$TE = \frac{aP}{V} = \frac{(308)(3700)}{79} = 14{,}425 lb$$

To find the minimum number of required locomotives, create a table with increasing number of locomotives, increasing tractive effort, and increasing resistance, as shown:

N-locos	TE	*R*
1	14,425	49,186
2	28,851	50,076
3	43,276	50,966
4	57,701	51,856

As shown, the value of TE first exceeds *R* when the number of locomotives is $N = 4$ (57,701-lb effort vs. 51,856-lb resistance). Therefore, four locomotives should be allocated. This result concurs with the observation that some students may have from intercity travel: when they see a long freight train engaged in a fast-line haul on a major railroad, a seemingly large number of locomotives may be allocated so that the train can maintain high speed.

Problem 12-4.

For a given freight market, the cost per shipping container is fixed price $125 and $1.75 per mile for truck, or fixed price $600 plus $0.80 per mile for IM rail. (a) What is the break-even distance below which truck is cheaper and above which IM rail is cheaper? (b) Suppose a truck with a load of one container consumes on average 0.167 gal of diesel per mile, and an intermodal train on average 0.05 gal per container per mile. Each gallon of diesel combusted results in 22 lb of CO_2 produced. Suppose a carbon tax of $100/ton CO_2 is introduced. What is the new break-even distance?

Solution.

(a) The breakeven distance as given is the following:

$$D_{breakeven} = \frac{FC_2 - FC_1}{VC_1 - VC_2} = \frac{600 - 125}{1.75 - 0.8} = \frac{475}{0.95} = 500mi$$

(b) Adjust for carbon tax as follows for the truck and IM rail options, respectively:

$$(0.167 gal/mi)(22lb/gal)(1ton/2000lb)(\$100/ton) = \$0.184/mi$$
$$(0.05 gal/mi)(22lb/gal)(1ton/2000lb)(\$100/ton) = \$0.055/mi$$

These costs can be added to the original cost per mile resulting in $1.93/mi for truck and $0.86/mi for IM rail. Then repeat the distance calculation:

$$D_{breakeven} = \frac{FC_2 - FC_1}{VC_1 - VC_2} = \frac{600 - 125}{1.93 - 0.86} = \frac{475}{1.07} = 440mi$$

Problem 12-5.

A trucking company whose tractor-trailers average 98,000 mi per year is considering an investment in an energy saving technology that costs $20,000 per truck. The device must pay for itself in a maximum of 5 years, simple payback. Fuel costs $4.75 per gallon, the baseline fuel economy is 6 mi per gallon, and the technology increases fuel economy by 5%. Does the technology meet the maximum allowable payback time criterion? Calculate the actual payback time required to support your answer.

Solution.

The new fuel economy from the technology is the following:

$$(6mpg)(1.05) = 6.3mpg$$

The annual fuel savings can be calculated by comparing fuel consumption with and without the device:

$$F_{old} = \frac{98000}{6} = 16{,}333 gal / yr$$

$$F_{old} = \frac{98000}{6.3} = 15{,}556 gal / yr$$

$$F_{saved} = F_{old} - F_{old} = 16{,}333 - 15{,}556 = 778 gal / yr$$

The value of this savings is (778)($4.75) = $3,694 per year. Payback in years is then calculated by dividing initial cost by savings per year:

$$\frac{\$20{,}000}{\$3694} = 5.41 yr$$

Thus, the payback of 5.41 years is slightly too long for the desired limit of 5 years.

Problem 12-6.

Compute energy intensity per pallet-kilometer for a vehicle run around the itinerary presented in the table below, in which the distances are from the previous stop, and the number of pallets is the delivery size at the current location (RDC = regional distribution center). The truck performing the run is part of a fleet of 350 trucks that in the preceding 12 months consumed 28 million liters of diesel and drove 8.48 million kilometer. The capacity of the truck is 30 pallets. Diesel consumption for refrigeration is on average 29-mLdiesel for each loaded kilometer; once the truck is completely unloaded the refrigeration is turned off. (a) What is the energy intensity of the journey, in liters/pallet-km? (Hint: for this problem, calculate N_{pallet} using a weighted average, taking into account the length of each leg of the journey.) (b) What is the weighted average percent utilization of the truck's pallet capacity, including both laden and empty segments?

Location	Kilometer traveled	Pallets delivered
RDC	0	0
1	20	4
2	70	8
3	30	7
4	30	5
5	25	6
RDC	50	0

Solution.

The average number of pallets is calculated from the five loaded segments as follows:

$$N_{pallet} = \frac{\sum_{i=1}^{5} D_i N_i}{\sum_{i=1}^{5} N_i} = \frac{1150\, palletkm}{175km} = 6.57\, pallet$$

The average fuel consumption is based on the previous 12 months, and is calculated as (28 Mkm)/(8.48 ML) = 3.30 km/L. The fuel consumption for refrigeration is (0.029 L/km) (175 km) = 5.08 L. Plugging in to the equation for fuel intensity gives:

$$\eta_{p-km} = \frac{\left(D_{tot} / F + E_{refrig}\right)}{\left(D_{laden} N_{pallets}\right)} \cdot \left(\frac{1000\,\text{mL}}{1\,\text{L}}\right)$$
$$= \frac{\left(225/3.3+5.08\right)}{\left(175\times 6.57\right)} \cdot \left(\frac{1000\,\text{mL}}{1\,\text{L}}\right)$$
$$= 63.67 mLdiesel / palletkm$$

Problem 12-7.

Break-even distance of IM rail and truck: one limitation of IM rail is that, even though it has lower line-haul costs, there is a high fixed cost to get a container into the system. In this problem, compare the cost per container for a given distance, when either shipped by IM rail or mounted on wheels and shipped by truck instead. Data are the following:

	Fixed	Per mile
Truck	125	1.75
Intermodal	600	0.8

a. Calculate the break-even distance above which IM rail is cheaper than truck.
b. Suppose a truck consumes on average 0.167 gal of diesel per mile, and an intermodal train on average 0.05 gal. Each gallon of diesel combusted results in 22 lb of CO_2 produced. Suppose a carbon tax of $100/ton CO_2 is introduced (a large tax by current standards, but for purposes of this problem.). What is the new break-even distance?

Solution.

(a) The break-even distance as given is the following:

$$D_{breakeven} = \frac{FC_2 - FC_1}{VC_1 - VC_2} = \frac{600 - 125}{1.75 - 0.8} = \frac{475}{0.95} = 500mi$$

(b) Adjust for carbon tax as follows for the truck and IM rail options, respectively:

$$(0.167 gal/mi)(22lb/gal)(1ton/2000lb)(\$100/ton) = \$0.184/mi$$
$$(0.05 gal/mi)(22lb/gal)(1ton/2000lb)(\$100/ton) = \$0.055/mi$$

These costs can be added to the original cost per mile resulting in \$1.93/mi for truck and \$0.86/mi for IM rail. Then repeat the distance calculation:

$$D_{breakeven} = \frac{FC_2 - FC_1}{VC_1 - VC_2} = \frac{600 - 125}{1.93 - 0.86} = \frac{475}{1.07} = 440mi$$

Problem 12-8.

Truck and rail freight options are competing with each other from a certain city. Trucking costs \$200 fixed cost, and \$1.35 per mile for each mile driven, for each full truckload. Alternatively, the same truckload can be placed on a freight train with an expected number of loads of 85 loads. For the entire train, the fixed cost is \$72,250, and \$38.25 per mile for the entire train. What is the break-even distance?

Solution.

Break the cost of each mode into fixed cost and variable cost components, or FC,VC, respectively. Costs for truck per load are given. For the train, the cost should be calculated from given values:

$$FC_{RR} = \frac{\$72{,}250}{85} = \$850$$
$$VC_{RR} = \frac{\$38.25}{85} = \$0.45/mi$$

The break-even distance can now be calculated:

$$D_{breakeven} = \frac{FC_{RR} - FC_{TR}}{VC_{TR} - VC_{RR}} = \frac{\$850 - \$200}{\$1.35 - \$0.45} = \frac{\$650}{\$0.90} =\sim 722mi$$

Problem 12-9.

An up-and-coming aerodynamic drag reduction technology for trucks that is starting to appear more frequently on the highways is the trailer fairing that is attached between the van trailer bottom and the road. Studies show that trailer fairings can reduce fuel consumption by 3.8%. It is believed that the maximum potential penetration for this technology into the market is 30%. The technology begins to penetrate the market in the year 2010, being installed on 0.8% of trucks. For years 2011 to 2014 the penetration values are 1.4%, 1.7%, 2.6%, and 3.1%, respectively.

At the beginning of the trailer fairing influx (i.e, when there is negligible penetration), there are 1.78 million combination trucks (ignore single-body trucks for this problem), each driving on average 98,000 mi per year with a fuel economy of 5.6 mpg. For the purposes of this problem, you can assume that all these figures stay constant for the duration.

a. Using only the penetration achieved in years 2010 to 2014 and the ultimate penetration value, estimate the percent market penetration in the year 2020 to the nearest 0.1%. Include in your answer the values of the curve-fitting parameters c_1 and c_2 that you deduce. (Hint: use the cumulative production curve presented in the course pack of Chap. 1 on oil consumption, but this time for technological substitution.)
b. Using the data given, how many gallons of fuel are saved in the year 2020 compared to a situation where no fairings have been installed?

Solution.

To solve part (a), the approach is to use Excel Solver or an equivalent approach in Matlab to fit a curve to the given data and find parameter values c_1 and c_2 that minimize total error. The following table gives the error minimizing values with $c_1 = -3.414$ and $c_2 = 0.323$:

Year	*t'*	Percent		
		Observed	Estimated	(Error)^2
2010	0	0.8%	1.0%	2.42E-06
2011	1	1.4%	1.3%	9.23E-07
2012	2	1.7%	1.8%	5.15E-07
2013	3	2.6%	2.4%	4.26E-06
2014	4	3.1%	3.2%	1.18E-06

The sum of values in the rightmost column is 9.30 E-06. Note that in this instance the year 2010 is treated as year 0 and the observed value of 0.8% is compared to the estimated value of 1.0% for that year for purposes of summing error terms. Other slightly different assumptions are also possible, and would result in different answers, that are also acceptable for full credit.

Market penetration can be calculated for year 2020 using the curve. The value of t' is $t' = 10$. Solving:

$$f(10) = \frac{F \cdot e^{(c_1 + c_2 t')}}{1 + e^{(c_1 + c_2 t')}} = \frac{0.3 \cdot e^{(-3.414+0.32310)}}{1 + e^{(-3.414+0.32310)}} = 0.136 = 13.6\%$$

Part (b): To keep the solution simple, it is possible to focus on only the trucks that are affected by the fairings, or 13.6% of the fleet. Their total fuel consumption in the base case before adding fairings is:

$$(0.136)(1.78M)(98Kmiles)\left(\frac{1}{5.6mpg}\right) = 4.236\times10^{9}\,gal/yr$$

The savings on this amount is 3.8%, or

$$(0.038)(4.236\times10^{10}\,gal/yr) = 1.61\times10^{8}\,gal/yr$$

Thus, the savings is 161 million gallons of fuel.

Problem 12-9.

Train locomotive sizing: An intermodal rail train consists of 80 railcars each weighing 15 tons empty and loaded with 60 tons of containers. It is to be moved at 79 mi per hour (the national rail speed limit) over flat ground by locomotives weighing 210 tons and having six axles per locomotive. What is the minimum number of locomotives required to maintain the desired speed?

Solution.

To solve for resistance per car and per locomotive, we can use the following inputs: V = 79 mi/h; for the cars, T = 15 tons; for the locomotives, T = 210 tons; N = 6 axles. Substituting:

$$R_{CAR} = 1.5T + 72.5 + 0.015TV + 0.055V^2 = 1.5(15) + 72.5 + 0.015(15)(79) + 0.055(79)^2$$
$$= 617.1\,lb/car$$
$$R_{LOCOMOTIVE} = 1.3T + 29N + 0.03TV + 0.312V^2 = 1.3(210) + 29(6) + 0.03(210)(79) + 0.312(79)^2$$
$$= 2891.9\,lb/loco$$

Since there are 80 cars in the train and this quantity is fixed, the resistance trailing the locomotive is also fixed at 80 × 617.1 lb/car = 49,370.4 lb.

The tractive power of the locomotive at 79 mi/h is the following:

$$TE = \frac{308P}{V} = \frac{308(6000)}{79} = 23{,}392\,lb$$

The answer is to find the smallest consist where the tractive effort exceeds the resistance of the train. A train with two locomotives has resistance of 55,154 lb and tractive power of 46,785 lb. A train with three locomotives has resistance of 58,046 lb and tractive power of 70,177 lb.

Therefore, choose three locomotives.

Problem 12-10.

Comparison of truck and rail energy consumption with supply chain energy intensity, based on answers from exercise 12-9. A tractor-trailer has the following characteristics:

Gross weight: 36,000 kg (legal limit)
Speed: 126.4 km/h (79 mi/h)
Cross-sectional area: 9.2 m^2 (approximately 12-ft H × 8-ft W)

Drag coefficient: 0.4
Rolling resistance: 0.006
Air density: 1 kg/m^3

Note that the rolling resistance is reduced, compared to typical light-duty vehicles, due to special low-resistance tires. Also, the truck's speed is over the legal limit, but the purpose is to make a one-to-one comparison to the intermodal rail train in exercise 12-9. Since conditions are close to optimal for the diesel drivetrain, the drivetrain efficiency is 28% in this case. Assume that a liter of diesel contains 35.9 MJ of energy. Use the standard highway vehicle tractive power formula that was applied to passenger cars earlier in the semester.

a. The intermodal rail train from HW 4 has 80 cars and also travels at 79 mi/h. Assume each car can carry four 40-ft shipping containers (the car is long enough for two containers, and the containers can be double-stacked, giving four containers total). If 1-lb force is equivalent to 4.448 N, what is the power requirement per container in units of kW? (Hint: power = force × velocity, or 1 watt = 1 N-m/s.)
b. Suppose the tractor-trailer described above is also carrying one 40-ft container. What is the power requirement in kW to move the truck at the stated speed? What is the percent reduction in power requirement to move one container by intermodal instead of by truck?
c. Now suppose the truck travels for 1 hour at 79 mi/h, and that it is carrying 32 pallets of food products. The food products are dry groceries so they do not require any refrigeration. What is the energy intensity in mL diesel fuel per pallet-km? Hint: 1 kWh = 3.6 MJ.

<u>Solution.</u>

Part (a): From the previous problem, the train has three locomotives and 80 cars, and a total resistance of 58,046 lb. Converted to metric, this amount is equivalent to 258,189 N. Converting 79 mi/h to meters per second gives 35.11 m/s. Thus the equivalent power is:

$$P = FV = (258.2kN)(35.11m/s) = 9065kW$$

Dividing by 320 containers gives per container power requirement of 28.3 kW.

Part (b): The given speed in metric of 126.4 km/h can be converted to 35.1 m/s. In putting all other values into the tractive power requirement equation gives:

$$\begin{aligned} P_{TR} &= 0.5\rho A_F C_D V^3 + mgV\,C_o \\ &= 0.5(1)(9.2)(0.4)(35.1)^3 + (36{,}000)(9.8)(35.1)(0.006) \\ &= 153{,}900W = 153.9kW \end{aligned}$$

The savings is therefore 153.9 – 28.3 = 125.6 kW, which is 82% of the original 153.9-kW value.

Part (c): To calculate the fuel intensity, it is first necessary to calculate fuel consumption. Since the truck travels requiring 153.9 kW for 1 hour, the energy requirement is 153.9 kW. Converting to joules:

$$153.9kWh\left(\frac{3.6MJ}{kWh}\right) = 554.2MJ$$

Since the drivetrain is 28% efficient, the input energy requirement is 1979 MJ. Based on the given energy content per liter of diesel, this energy amount is equivalent to 55.1 L of diesel. The fuel economy F is therefore:

$$F = \frac{Dist}{Fuel} = \frac{126.4}{55.1} = 2.29km/L$$

Note that for distance travel, there is no difference between laden distance and total distance in this problem, both are 126.4 km. The fuel intensity per pallet-kilometer can be presented in a simplified form as follows, since there is no refrigeration energy use:

$$\eta_{p-km} = \frac{D_{tot}/F}{D_{laden}N_{pallets}}(1000) = \frac{126.4/2.29}{126.4(32)}(1000) = 13.63mL/pallet-km$$

Problem 12-11.

An intermodal rail service provides on average eight trains per day over a 365-day year, each carrying 200 containers on average. They have two types of capital costs: $3 billion for infrastructure, which has a 25-year lifetime, and $200 million per train for equipment, which has a 15-year lifetime. The discount rate for both is 9%, and they are to be annualized with the assumption of no residual salvage value at the end of the lifetime. In addition, each train generates $50,000 in non-capital operating cost (wages, fuel, maintenance, etc.) per daily run, and the rail service incurs $50 million per year in fixed overhead costs, regardless of number of trains operated. How much must they charge per container shipped in order to exactly cover all costs?

Solution.

Assume that annual costs for overhead and operating costs remain fixed at given levels in constant dollars, and that the company will replace capital equipment at the end of its life with equipment under the same conditions, so that the same annual capital costs will continue. First solve for the annual capital cost, ACC, for each of the capital components:

$$ACC_{insruc} = -PMT(9\%, 25, \$3B) = \$305.4M$$
$$ACC_{trains} = -PMT(9\%, 15.\$1.6B) = \$198.5M$$

The total cost for train operations at $50,000 per train per day is:

$$(\$50{,}000)(8)(365) = \$146M$$

The total number of containers per year is 1600 per day multiplied by 365 days per year, or 584,000 containers. Therefore, dividing total cost by total containers:

$$CostPerContainer = \frac{TotCost}{TotContainers} = \frac{\$305.4M + \$198.5M + \$146M + \$50M}{584K}$$
$$= \$1198 / container$$

Problem 12-12.

A trucking company whose tractor-trailers average 110,000 mi per year is considering an investment in an energy saving technology that costs $23,000 per truck. The device must pay for itself in a maximum of 6 years, simple payback. Fuel costs $5 per gallon, the baseline fuel economy is 6 mi per gallon, and the technology increases fuel economy by 5%. Calculate the payback time required, and state whether or not the technology meets the maximum payback criterion.

Solution.

A 5% increase in mpg means that it increases by (0.05)(6 mpg) = 0.3 mpg to 6.3 mpg delivered. The payback calculation is the following:

$$\frac{110{,}000}{6} = 18{,}333 gal$$
$$\frac{110{,}000}{6.3} = 17{,}460 gal$$
$$Savings = 18{,}333 gal - 17{,}460 gal = 873 gal / yr$$
$$(873 gal / yr)(\$5 / gal) = \$4{,}365 / yr$$
$$\frac{\$23{,}000}{\$4365 / yr} = 5.27 yr$$

Since the payback is 5.14 years and the limit is 6 years, the investment is marginally feasible.

Problem 12-13.

A food carrying combination truck travels on a 225-km itinerary, of which 175 km are traveled loaded. The average fuel economy is 3.1 km per liter of fuel. In addition, the vehicle consumes 8 L of diesel for refrigeration during the trip. The average lading of the vehicle is 25 pallets for the laden portion of the trip. What is the intensity in mL/pallet-km?

Solution.

Substituting into the equation for intensity gives the following:

$$\eta_{p-km} = \frac{(D_{tot} / F + E_{refrig})}{D_{laden} N_{pallets}} \left(\frac{1000 mL}{L} \right)$$
$$= \frac{(225/3.1 + 8)}{175(25)} \left(\frac{1000 mL}{L} \right) = \frac{(72.58 + 8)}{4{,}375} \left(\frac{1000 mL}{L} \right)$$
$$= 18.42 mL / pallet - km$$

Problem 12-14.

Freight CO_2 emissions: The ton-miles of U.S. freight, excluding pipeline, are given to you in the table below, divided into two columns: "high cost,, which includes truck and air, and "low cost," which includes rail and marine (figures are given in billion ton-miles). In 2008, the CO_2 intensity of these two modes was 316 and 34.2 tons per million ton-miles, respectively. For purposes of this problem, assume that the low-cost modes' intensity will remain fixed going forward (they are already quite efficient), and high-cost modes will decline linearly by a value equal to 0.5% of the emissions per million ton-mile in 2008. As a "reality check," you can verify that the total emissions in 2008 are 497.8 million tons of CO_2.

Year	High cost	Low cost
2000	1214.9	2116
2002	1273.4	2122
2004	1298.5	2281
2006	1309.4	2332
2008	1326.4	2301

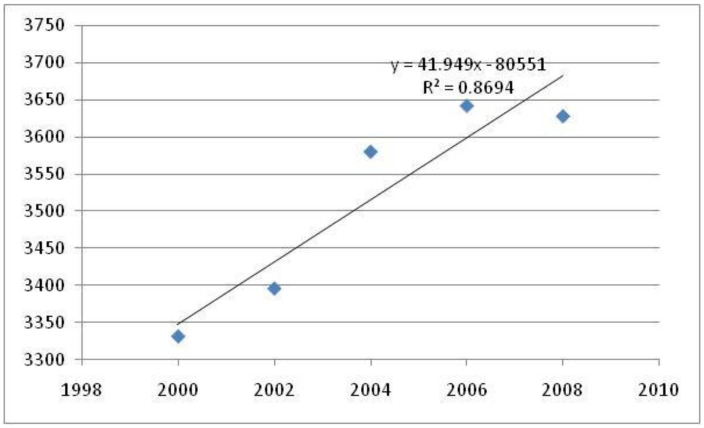
Part (a): Use linear regression to project what the ton-miles and emissions will be in the year 2050 in a "business as usual" scenario, or BAU. Fit a curve to the five data points from 2000 to 2008, and *do not* constrain the curve to pass through (0, 0). You should be able to do this in Excel or Matlab quite easily. You can assume that the percent modal share of hi- and low-cost modes will be the same in 2050 as in 2008.

Part (b): Now consider the following scenario in which aggressive steps are taken to reduce CO_2 emissions compared to the BAU scenario. First, freight is used much more efficiently so that the total ton-miles in 2050 is the same as the value in 2008. Secondly, great efforts are made to move more freight by the low-cost modes, so that the modal share in 2050 is increased by 10 percentage points compared to the value in 2008. 2050 intensity values are the same as in part (a). What is the new value of total CO_2 emissions? By what percent have they been reduced compared to part (a)?

Part (c): Now compare the emissions reduction achieved in 2050 compared to the reductions discussed in the 2008 U.S. presidential campaign, on the order of 60% to 80% compared to 2008. How do the reductions from part (b) compare to this target for 2050? What further steps are missing beyond what was implemented in part (b)? Discuss in maximum one paragraph.

Solution.

The future demand levels are extrapolated from the following graph:



Substituting the year 2050 gives:

$$y = 41.949(2050) - 80551 = 5.44 tril.tmi$$

The emission rate, respective modal emissions, and total emissions in 2050 are then the following:

$$Rduction = (316 ton/mil.tmi)(0.5\%) = 1.58 \text{ ton/mil.tmi}$$

$$E_{2050} = (316 ton/mil.tmi) - (42)(1.58) = (250 ton/mil.tmi)$$
$$Vol_{hi} = 5444\left(\frac{1326}{1326 + 2301}\right) = 1990 bil.tmi$$
$$(1990 bil.tmi)(250 ton/mil.tmi) = 497 mil.tons$$
$$Vol_{lo} = 5444 - 1990 = 3454 bil.tmi$$
$$(3454 bil.tmi)(34.2 ton/mil.tmi) = 118 mil.tons$$
$$TotalEmits_{2050} = 497 + 118 = 615 mil.tons$$

Part (b): Modal shares in 2008 are 36.6% and 63.4%, respectively. Therefore, the modal shares in 2005 are:

$$Vol_{lo} = (5444)(0.734) = 3998 bil.tmi$$
$$Vol_{hi} = (5444)(0.266) = 1446 bil.tmi$$

Emissions are then calculated on the basis of rates of 250 ton/mil.tmi and 34.2 ton/mil.tmi:

$$(1.446 \times 10^6 mil.tmi)(250) = 361 mil.tons$$
$$(3.998 \times 10^6 mil.tmi)(34.2) = 137 mil.tons$$
$$361 + 137 = 498 mil.tons$$
$$Rd = \frac{(615 - 498)}{498} = 19\%$$

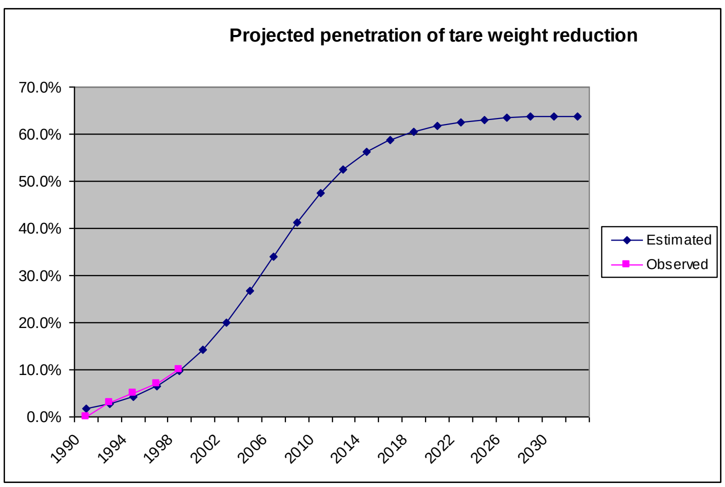
Part (c): The emissions in 2050 from part (b) are about the same as what they were in 2008. Thus, it is a substantial improvement compared to BAU 2050, but not as much as the 60% to 80% target. The missing piece is carbon-free alternative energy sources for freight, without which it is difficult to seriously reduce CO_2 emissions.

Problem 12-15.

Research on energy-efficient trucking suggests that the use of "tare weight reduction" (i.e., taking weight off the combination truck separate from the weight of the cargo carried) represents a method for saving truck fuel consumption. The maximum penetration in the U.S. truck market is expected to be 64%. Suppose the first years of the penetration of the strategy, starting in 1990, are as follows: 0%, 3%, 5%, 7%, 10%, 14%, for years 1990 to 2000 by 2-year increments, respectively. Using the technological substitution model, what is (a) the penetration predicted for 2010, and (b) the first year in which penetration exceeds a value of 63%, i.e., one percentage point less than the maximum?

Solution.

The penetration predicted for the year 2010 is 47%. The year in which penetration first achieves 63% is 2024. The curve below shows the pathway and the answers.



Problem 12-16.

Superior Foods, Inc., is monitoring five vehicles in their fleet over a 24-hour period, to measure certain key performance indicators (KPIs) of interest to management. The specific KPIs are (1) vehicle loading, (2) percent empty running, (3) fuel efficiency, and (4) vehicle time utilization. All five vehicles start from and return to the same regional distribution center (RDC).

Copy the data below for the five vehicles into a spreadsheet, and use the following guidelines for calculating the KPIs:

- Vehicle loading: assume that each vehicle starts with enough pallets of food to meet all demands on the run. The maximum number of pallets is 40 in each trailer. Calculation: average percent lading across the entire run, not including the empty return leg, and not weighted for distance of each leg.
- Percent empty running: only the last leg of the itinerary, where the vehicle is returning to the RDC, is empty. Use the distance of this leg relative to the total distance driven to calculate average percent empty running.
- Fuel efficiency: Superior Foods does not have the capacity to track individual vehicle fuel consumption in real time. Instead, they use average fuel consumption for the entire fleet, multiplied by the number of miles driven, to estimate fuel consumption for a particular itinerary. For the previous 12 months, their entire fleet of 120 trucks averaged 85,000 mi driven per vehicle, and consumed a total of 1.75 million gallons of fuel. Calculate fuel efficiency in gallons of fuel per 1000 pallet-miles. For simplicity, ignore refrigeration fuel consumption throughout this problem.
- Vehicle time utilization: for this exercise, there are just three possible states for each vehicle: (1) running on the road, (2) loading and unloading activities, and (3) idle. Any time during the 24-hour period not spent in states 1 or 2 is spent in state 3. Use percent time in each state as the KPI of interest, i.e., a truck that spends 6 hours driving and 6 hours loading/unloading has the following KPIs: 25%/25%/50%.

Questions:

a. Produce a table with the four KPI values for the five vehicles under study, and the "fleet average", i.e., the average value across all five vehicles for each KPI. For each KPI, calculate the fleet average by taking the arithmetic average of the individual vehicle KPI values.
b. Create a pie chart for utilization based on the average value for all five vehicles. (Note: There are just three possible states for each truck over 24-hours: running on the road, processing, or idle. Any time in 24 hour not spent in the first two is by default spent in the last.
c. (short answer) Answer the following questions in a paragraph: How well is the group of five vehicles performing in your opinion, based on the resulting KPIs? Also, give *one* example of a way that the use of the vehicles could be improved to increase KPI values. Maximum length one paragraph.

Data:

Truck 1:

No. delivs.		Tot pallet	34	Avg:	22.8
Leg	Dist	Avg spd	Proc time	pallets	Load
	[mi]	[mi/h]	[min]	[units]	[units]
RDC	na	na	74	na	na
1	47	40	28	8	34
2	72	35	27	3	26
3	38	33	38	4	23

4	27	42	30	7	19
5	65	42	15	12	12
rtn	62	42	na	na	

Truck 2:

No. delivs.	4	Tot pall	27.0	Avg	17
Leg	Dist	Avg spd	Proc time	pallets	Load
	[mi]	[mi/h]	[min]	[units]	[units]
RDC	na	na	83.717391		na
1	17.9	42.8	36.6	4	27
2	23.0	29.5	37.5	9	23
3	45.9	38.4	33.3	9	14
4	16.8	29.0	21.0	5	5
rtn	43.2	32.9			

Truck 3:

No. delivs.	4	Tot pall	35.0	Avg	22
Leg	Dist	Avg spd	Proc time	pallets	Load
	[mi]	[mi/h]	[min]	[units]	[units]
RDC	na	na	110		na
1	61.9	26.3	30.2	10	35
2	72.4	40.3	17.5	7	25
3	26.3	32.5	33.1	9	18
4	26.5	41.8	37.7	9	9
rtn	19.1	39.4			

Truck 4:

No. delivs.	3	Tot pall	22.0	Avg	16.7
Leg	Dist	Avg spd	Proc time	pallets	Load
	[mi]	[mi/h]	[min]	[units]	[units]
RDC	na	na	129		na
1	38.9	30.3	34.2	5	22.0
2	21.7	40.3	29.2	6	17
3	9.7	40.7	19.9	11	11
rtn	32.2	35.5			

Truck 5:

No. delivs.	3	Tot pall	27.0	Avg	19
Leg	Dist	Avg spd	Proc time	pallets	Load
	[mi]	[mi/h]	[min]	[units]	[units]
RDC	na	na	87		na

1	36.0	38.4	34.1	8	27.0
2	27.1	53.8	34.6	8	19
3	16.6	34.9	26.8	11	11
rtn	129.0	48.0			

Solution.

Part (a): The average truck has the following fuel economy:

$$TotMiles = (85{,}000)(120) = 10.2 mil.miles$$

$$MPG = \frac{10.2M}{1.75 mil.gal} = 5.83 mpg$$

Therefore, taking truck 1 as an example:

$$\eta_{pkm} = \frac{D_{tot}/F + E_{refrig}}{D_{laden} N_{pallets}} = \frac{311/5.83}{(249)(22.8)} = 9.39\, pallet.mile/1000 gal$$

The load percent is the average of all loading levels of 22.8 pallets, divided by 40 pallets maximum, or 57%.

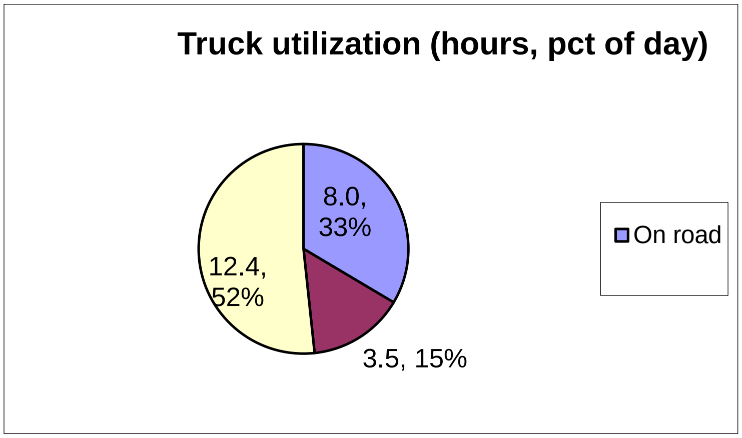
The percent empty is the distance driven empty of 62 mi divided by the total distance of 311 mi, or 20%.

The total of time in processing for truck 1 is the sum of the minutes, or 213 minutes or 3.5 hours. The total time spent running when dividing distance by average speed is 8.05 hours. These two sum to 11.6 hours, so the idle time is 12.4 hours.

The final table for all trucks and all KPIs is the following:

Truck	Load %	% empty	Fuel eff	Run time	Proc tim	Idle time
			gal/1000 pmi	hour	hour	hour
1	0.57	0.2	9.39	8.05	3.55	12.40
2	0.43	0.29	14.1	4.29	3.54	16.17
3	0.54	0.09	8.7	6.08	3.81	14.11
4	0.42	0.31	15.01	2.97	3.54	17.49
5	0.48	0.62	23.65	4.6	3.03	16.37
Total	0.49	0.3	14.17	5.20	3.49	15.31

Part (b): The values for the pie chart are read from the bottom row of the above table, giving the following figure:



Part (c): Based on the KPIs measured, the performance of the fleet could probably be improved. Some ways to do this (although the question only calls for one) are to make sure to deliver more pallets per store in each trip; add more stops to each run to reduce idle time; other answers are also possible.

Chapter 13 Spatial and Geographic Aspects of Freight

Problem 13-1.
Reconstruct the optimization of Example 13-1 to verify the solution.

Solution.
The table of values appears in Table 13-4 as follows:

	From:				
To	OR	WI	GA	ME	Demand
CA	2094	267	134	61	2556
NY	74	283	466	479	1302
FL	0	87	1384	61	1532
IL	44	1353	369	444	2210
NJ	0	94	322	125	541
Supply	2212	2084	2675	1170	

The distances are the following:

[miles]	OR	WI	GA	ME
CA	892	2352	2687	3361
NY	3149	1053	1114	511
FL	3411	1414	343	1689
IL	2409	244	894	1306
NJ	3235	1021	929	488

The optimization model has been set up in a spreadsheet and solved using an electronic solver in this case. In symbolic form, the model is the following. Let the set of origins i include OR, WI, GA, and ME, and let the set of destinations j include CA, NY, FL, IL, and NJ.

$$MinZ = \sum_{i,j} D_{i,j} X_{i,j}$$

$$s.t.$$

$$\sum_j X_{i,j} \leq A_i, \forall i$$

$$\sum_i X_{i,j} = B_j, \forall j$$

$$X_{i,j} \geq 0, \forall i, j$$

With the given pattern of shipments, the total supply in the four origin states and total demand in the destination (market) states are shown in the above table. The supply and demand are exactly balanced at 8.141 million tons. Multiplying the given pattern of shipment by distance for each origin-destination pair (as given in the distance table) gives a total of 6.859 billion ton-miles. Solving the above optimization problem in the spreadsheet gives the following improved pattern:

	From:				
To	OR	WI	GA	ME	Demand
CA	2212	0	344	0	2556
NY	0	0	132	1170	1302
FL	0	0	1532	0	1532
IL	0	2084	126	0	2210
NJ	0	0	541	0	541
Supply	2212	2084	2675	1170	8141

The above allocation of tons multiplied by the distances results in the following amounts of ton-miles, in millions:

	From			
To	OR	WI	GA	ME
CA	1973	0	924	0
NY	0	0	147	598
FL	0	0	525	0
IL	0	508	113	0
NJ	0	0	503	0

Summing the ton-miles in the above table gives 5.292 billion. Thus, the total flow from Example 13-1 has been reconstructed.

Problem 13-2.

Reconstruct the calculation of energy intensity by distance block and the optimization of Example 13-2 to verify the solution. Use the following modal split percentages by distance block that are presented graphically in Fig. 13-4, and energy intensity values of 2844, 533, and 1904 Btu/tmi for truck, rail, and other, respectively.

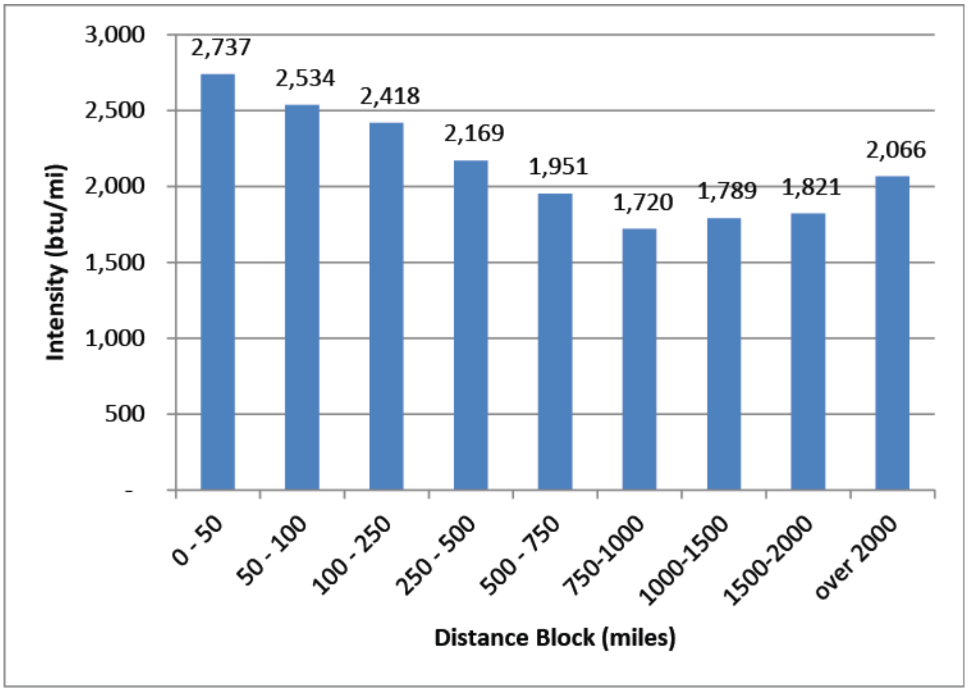
Dist. [mi]	Other	Truck	Rail
0–50	4%	93%	3%
50–100	4%	84%	13%
100–250	3%	80%	16%
250–500	3%	69%	28%
500–750	4%	59%	37%
750–1000	4%	49%	47%
1000–1500	9%	49%	42%
1500–2000	13%	48%	39%
Over 2000	17%	56%	28%

Solution.

Multiply each modal share by the corresponding energy intensity for that mode to create a table of share-intensity products, as shown. Then add up the terms to create the estimate of the average energy intensity for that block. Note that in the table Other + Truck + Rail = Combined.

Mode	Other	Truck	Rail	Combined
Intense	1904	2844	533	
Dist. [mi]				
0—50	76.16	2644.92	15.99	2737
50—100	76.16	2388.96	69.29	2534
100—250	57.12	2275.2	85.28	2418
250—500	57.12	1962.36	149.24	2169
500—750	76.16	1677.96	197.21	1951
750—1000	76.16	1393.56	250.51	1720
1000—1500	171.36	1393.56	223.86	1789
1500—2000	247.52	1365.12	207.87	1821
Over 2000	323.68	1592.64	149.24	2066

From inspection, it can be seen that the combined energy intensity values in the right column concur with the figure from Chap. 13:



Next, consider the table of distances by origin-destination pair presented in the solution to exercise 13-1. Based on the block into which the distance falls, the distance can be replaced by an energy intensity value, as shown:

Energy intensity by O-D pair (Btu/ton-mile):

States:	OR	WI	GA	ME
CA	1720	2066	2066	2066
NY	2066	1789	1789	1951
FL	2066	1789	2169	1821
IL	2066	2418	1720	1789
NJ	2066	1789	1720	2169

Multiplying the intensity value above by the distance for the O-D pair gives the energy requirement to ship 1 ton of product on that pair. Note that values are given in 1000 Btu per ton shipped.

Energy requirement by O-D pair (1000 Btu/ton):

States	OR	WI	GA	ME
CA	1534	4858	5550	6942
NY	6504	1884	1993	997
FL	7046	2529	744	3075

IL	4976	590	1538	2336
NJ	6682	1826	1598	1058

The total energy consumption in the given O-D pattern can now be calculated by multiplying given flow values (see table in solution to exercise 13-1) by the above energy values per ton, resulting in the following values in billion Btu:

	From			
To	OR	WI	GA	ME
CA	3212	1297	744	423
NY	481	533	929	478
FL	0	220	1030	188
IL	219	798	568	1037
NJ	0	172	515	132

Summing the values shown gives a total of 12,975 billion Btu or 12.97 trillion Btu.

The linear programming problem is now rewritten with $E_{i,j}$ replacing $D_{i,j}$ in the objective function to calculate total energy instead of total ton-miles. Otherwise, the constraints are the same:

$$MinZ = \sum_{i,j} E_{i,j} X_{i,j}$$

$$s.t.$$

$$\sum_{j} X_{i,j} \le A_i, \forall i$$

$$\sum_{i} X_{i,j} = B_j, \forall j$$

$$X_{i,j} \ge 0, \forall i, j$$

The allocation of flows to the O-D matrix is in fact the same as in exercise 13-1. The new distribution of energy consumption values by O-D pair in billions of Btu is the following:

	From			
To	OR	WI	GA	ME
CA	3393	0	1909	0
NY	0	0	263	1166
FL	0	0	1140	0
IL	0	1230	194	0
NJ	0	0	865	0

Summing the energy values in the above table gives a total of 10.16 trillion Btu, or a reduction of 21% in energy consumption.

Problem 13-3.

A retail firm operates a "decentralized" distribution system (factory to warehouse to shop), in which the system uses six smaller warehouses distributed around a region to receive a product from manufacturer (called "primary distribution") and then send product to retail outlets (called "secondary distribution"). The firm is offered the opportunity to shift to a "centralized" system, in which each unit of product will still undergo primary and secondary distribution, but now there will only be one warehouse in the middle of the region. The transportation of the product incurs financial cost and energy consumption per vehicle kilometer (vkm) of movement. In the case of the warehouse costs, inventory costs are incurred by virtue of needing to keep stock on hand in the warehouse between the time that the stock is purchased and the time it is sold, and energy is consumed to operate the warehouse. Transportation costs $1.50 and consumes 19 MJ of energy per vehicle-km. Assume that all other costs and rates of energy consumption are the same for either option, and therefore are not included in the calculation since they do not affect the outcome. Note also that the cost of energy is included in the warehouse and transportation cost values given. (a) Based on the data given, determine whether the decentralized or centralized system is preferred. (b) What is the environmental dilemma underlying this decision? Discuss in a short answer, up to one-paragraph long.

Warehouse costs & energy use per warehouse:

Alternative	Inventory Cost	Energy
	($1000/year)	(GJ/year)
Decentralized	190	155
Centralized	710	730

Transportation volume generated per year per warehouse:

1000 vkm/year	Primary	Secondary
Decentralized	35	82
Centralized	181	645

Solution.

Note to instructor: The energy costs for warehouses and transportation are included in the annual cost figures given; the students should not look outside for additional energy cost values for either. Doing so leads to erroneous answers.

Part (a): The approach here is to go through the cost and energy consumption calculation for the decentralized option in detail, then to repeat for the centralized in a streamlined form, and finally to make comparisons.

Decentralized option:

On the warehouse side, there are six warehouses total, so cost and values must be multiplied by six, as follows:

$$\text{Cost}: (\$160\text{K})6 = \$1.114\,\text{M/yr}$$
$$\text{Energy}: (155\,\text{GJ})6 = 930\,\text{GJ/yr}$$

On the transportation side, total cost and energy consumption are based on summing total vkm generated across primary and secondary stages, then converting to cost and energy:

$$\text{vkm per warehouse}: 35\text{K} + 82\text{K} = 117\text{K km/yr}$$
$$\text{Total vkm}: (117\text{K})6 = 702\text{K km/yr}$$
$$\text{Total cost}: (702\text{K})(\$1500/1000\,\text{km}) = \$1.053\text{M/yr}$$
$$\text{Total energy}: (702\text{K})(19\,\text{GJ}/1000\,\text{km}) = 13{,}338\,\text{GJ/yr}$$

Summing the warehouse and transportation components gives total cost and energy per year:

$$\text{Cost}: \$1.114\text{M} + \$1.053\text{M} = \$2.193\text{M/yr}$$
$$\text{Energy}: 930\,\text{GJ} + 13{,}338\,\text{GJ} = 14{,}268\,\text{GJ/yr}$$

Centralized Option

Note that because there is only one warehouse, the solution is somewhat simplified. Warehouse cost and energy:

$$\text{Cost}: (\$710\text{K})1 = \$710\text{K/yr}$$
$$\text{Energy}: (730\,\text{GJ})1 = 730\,\text{GJ/yr}$$

Transportation cost and energy:

$$\text{vkm per warehouse}: 181\text{K} + 645\text{K} = 826\text{K km/yr}$$
$$\text{Total vkm}: (826\text{K})1 = 826\text{K km/yr}$$
$$\text{Total cost}: (826\text{K})(\$1500/1000\text{km}) = \$1.239\text{M/yr}$$
$$\text{Total energy}: (826\text{K})(19\,\text{GJ}/1000\,\text{km}) = 15{,}694\,\text{GJ/yr}$$

Combined cost and energy from warehouse and transportation:

$$\text{Cost}: \$710\text{K} + \$1.239\text{M} = \$1.949\text{M/yr}$$
$$\text{Energy}: 730\,\text{GJ} + 15{,}694\,\text{GJ} = 16{,}424\,\text{GJ/yr}$$

Solution.

Choose centralized option because it reduces overall cost ($1.95 M/year instead of $2.19 M/year). The centralized solution increases energy consumption by 2156 GJ, or 2.156 TJ, per year, or 15%. However, it also reduces annual cost by $244,000 per year, or 11%. Absence of any additional information, a business in a competitive market environment must choose the most cost-effective solution, that is, the centralized one, even if it increases annual energy consumption.

Part (b): The dilemma is that even if the retail firm would like to choose the "green" option and save energy, market forces are pushing them in the opposite direction. The centralized solution reduces inventory cost, since the firm can carry less inventory and incur less cost if they centralize all stored product in a single warehouse. However, the energy savings from this change are small. Looking at the warehousing + freight transportation "life cycle," the energy consumed on the warehousing side is small. Most of the energy is consumed in the freight component, so that when the shift is made under the centralized solution to a more transportation-intensive pattern, there is no counterbalancing reduction in energy on the warehousing side that can offset the increased energy consumption by the additional trucking involved.

Problem 13-4.

A food products company ships foods from three production plants to four markets as follows: The capacity of plants at Boise, Dubuque, and Charleston is 2200 tonnes, 3000 tonnes, and 2000 tonnes, respectively. The demand at San Francisco, New York, Miami, and St. Louis is 2002 tonnes, 1784 tonnes, 1355 tonnes, and 1972 tonnes, respectively. The mode of shipment is by truck, and the energy intensity is 2200 kJ/tkm. A table of distances in kilometers between cities is given below. (a) What is the allocation of shipments from plants to markets that meets all demands, does not exceed supplies available at any plant, and minimizes energy consumption? What is the value of energy consumption in this case? (b) Suppose that for all routes with distance of 1500 km or more, an intermodal rail service is made available with energy intensity of 1400 kJ/tkm. Recalculate Problem 16-5 (a). What is the new value of energy consumption? Does the shipment pattern from part (a) change?

City	Boise	Dubuque	Charleston
San Francisco	1427	3763	4299
New York	5038	1685	1782
Miami	5458	2262	548
St. Louis	3855	390	1431

<u>Solution.</u>

Note to instructor. For students who have no previous experience with optimization, it is useful to build an MS Solver model that solves a very simple problem, such as the two-source problem in Chap. 3, to verify that the model is working correctly. Then the same model can be expanded to solve the actual problem.

The pattern with optimized assignment of origins to destinations is derived by solving the following optimization problem. Let *D* be distance between origin and destination, *X* be the decision variable of the number of tonnes to assign to each O-D pair, *A* be the amount availability at sources *i*, and *B* be the demand at markets *j*. Then if *i* = Boise, Dubuque, Charleston and *j* = San Francisco, New York, Miami, St. Louis:

$$
\begin{aligned}
&MinZ = \sum_{i,j} D_{i,j} X_{i,j} \\
&s.t. \\
&\sum_{j} X_{i,j} \le A_i, \forall i \\
&\sum_{i} X_{i,j} = B_j, \forall j \\
&X_{i,j} \ge 0, \forall i, j
\end{aligned}
$$

The solution is coded in the spreadsheet solution (see the accompanying MS Excel workbook). The optimal allocation that minimizes tonne-km is the following table (in tonnes):

	Boise	Dubuque	Charleston, SC	Total	Demand
San Francisco	2002	0	0	2002	2002
New York	111	1028	645	1784	1784
Miami	0	0	1355	1355	1355
St. Louis	0	1972	0	1972	1972
Total	2113	3000	2000		
Available	2200	3000	2000		

By inspection, it can be seen that the demand at each of the four destinations is met, and the supply is not exceeded at the sources (Dubuque and Charleston are used 100%, while Boise has some tonnes of supply extra). Multiplying tonnes by distance gives the following, in thousand tonne-km:

	Boise	Dubuque	Charleston, SC	Total
San Francisco	2856	0	0	2856
New York	605	1420	1463	3488
Miami	0	0	743	743
St. Louis	0	770	0	770
Total	3460	2190	2206	7856

Multiplying 7.86 million tonne-km by the average energy intensity of 2200 kJ/tkm gives total energy consumption of 17.2 TJ.

Part (b): Based on the distance threshold of 1500 km, the following table indicates whether the freight is shifted to intermodal rail or not:

	Boise	Dubuque	Charleston, SC
San Francisco	No	Yes	Yes
New York	Yes	Yes	Yes
Miami	Yes	Yes	No
St. Louis	Yes	No	No

Solving the model using the optimization in the spreadsheet reveals that in the new optimal, the matching of origins and destinations is the same; however, energy consumption is reduced on links where intermodal is used. This change can be observed in a table of energy consumption by origin-destination pair (measured in GJ):

	Boise	Dubuque	Charleston, SC
San Francisco	6283	0	0
New York	783	2425	1609
Miami	0	0	1634
St. Louis	0	1693	0

Summing all the values in the table gives 14,426 GJ, or 14.4 TJ.

Problem 13-5.

Use the origin-destination (OD) flow data and distance matrix below for 1993 annual pulp and paper movements in the U.S. to answer the following questions. (a) What is the total ton-miles of freight generated in the given pattern (hereafter called the "base case"), based on the tons shipped for each OD pair? (b) What is the estimated energy consumption, considering ton-miles and distance-based average energy intensity? (Hint: to estimate energy consumption by OD pair, you may wish to write a short macro or use conditional formatting in a spreadsheet.) (c) Assuming that the tons shipped from each origin represents the total amount available at that origin, and the tons arriving in the market represents the total tons consumed in that destination, create a table of capacity for origin states and market size for destination states, with tons/year for each one. (d) Use the transportation model in linear programming to find the OD pattern that minimizes total ton-miles. Report the OD flows in the optimal pattern in a new OD matrix, and give the new ton-mile total in this case, and the percent reduction in ton-miles compared to the base case. (e) Pick any origin state and show a map of the paper flows from that state for both the base case and optimal pattern. The map can be either hand-drawn or computer-generated. (f) If the freight flows according to the optimal pattern from part (c) what is the total energy consumption in the optimal case, and the amount of energy saved, in trillion Btu, by shifting to the optimal pattern? (g) Now revise the LP so that it finds the pattern that minimizes total energy consumption by directly adding up the energy consumed in each OD pair. What is the new energy consumption in this case? Is it the same optimal pattern and energy value as in the solution from part (d), or is it different? (h) Although the optimal pattern might be advantageous from a transportation intensity point of view, there are other factors that lead to a different, more transportation-intensive spatial pattern of freight represented by the base case pattern. Explain one such factor (short answer, maximum one to two sentences).

Shipment volumes:

1000 tons	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	660	164	278	270	0	252	523	0	88
IL	369	585	1353	497	87	586	444	44	481
OH	465	203	421	0	0	559	117	0	0
TN	398	133	213	543	0	117	127	0	55
NC	761	170	87	0	0	62	34	0	18
FL	1384	259	87	657	0	55	61	0	41
TX	545	1410	258	802	82	58	60	36	95
NY	466	122	283	142	150	152	479	74	58
NJ	322	184	93	200	0	80	125	0	53
CA	134	436	267	270	1744	192	61	2094	161

Distances:

Miles	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	914	1415	878	1059	2703	730	730	2762	1182
IL	894	884	244	799	2218	319	1306	2409	574
OH	763	1025	570	392	2405	356	1017	2421	766
TN	362	481	718	359	2402	709	1504	2355	1055
NC	356	976	1023	570	2810	742	1118	2764	1278
FL	342	749	1414	559	2993	1327	1689	2947	1780
TX	1215	493	1236	864	2305	1328	2167	2583	1432
NY	1114	1525	1053	1317	3640	579	511	3149	1190
NJ	929	1462	1022	1150	2842	825	488	2902	1472
CA	2687	2037	2352	2289	976	2411	3361	892	2230

Solution.

Part (a): The base case total ton-miles are obtained by multiplying the above tons and distances together, as given below in millions of ton-miles:

	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	603	232	244	286	0	184	382	0	104
IL	330	517	330	397	193	187	580	106	276
OH	355	208	240	0	0	199	119	0	0
TN	144	64	153	195	0	83	191	0	58
NC	271	166	89	0	0	46	38	0	23
FL	474	194	123	367	0	73	103	0	73
TX	662	695	319	693	189	77	130	93	136
NY	519	186	298	187	546	88	245	233	69
NJ	299	269	95	230	0	66	61	0	78
CA	360	888	628	618	1702	463	205	1867	359
Total	4017	3419	2519	2973	2630	1466	2054	2299	1176

Summing the above totals gives 22.6 billion ton-miles in the base case.

Part (b): The energy consumption by distance is calculated using the modal split and energy intensity by mode values at each distance. The table below shows the energy intensity of each mode (first row) multiplied by its percent modal share (not shown). Adding the value for the three modes gives the overall energy intensity by distance block (right column).

Mode	Other	Truck	Rail	Total
Btu/ton-mi	1904	2844	533	n/a
0–50	82.8	2640.9	14.9	2739
50–100	68.9	2380.7	67.5	2517
100–250	64.2	2284.8	86.8	2436
250–500	52.2	1971.3	148.9	2172
500–750	69.2	1683.9	198.0	1951
750–1000	72.1	1406.9	249.2	1728
1000–1500	172.7	1388.2	224.5	1785
1500–2000	238.6	1365.9	210.2	1815
Over 2000	314.4	1582.9	148.3	2046

Next, an energy intensity (Btu/ton-mi) is assigned to each OD pair based on distance, as shown:

State	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	1728	1785	1728	1785	2046	1951	1951	2046	1785
IL	1728	1728	2436	1728	2046	2172	1785	2046	1951
OH	1728	1785	1951	2172	2046	2172	1785	2046	1728
TN	2172	2172	1951	2172	2046	1951	1815	2046	1785
NC	2172	1728	1785	1951	2046	1951	1785	2046	1785

FL	2172	1951	1785	1951	2046	1785	1815	2046	1815
TX	1785	2172	1785	1728	2046	1785	2046	2046	1785
NY	1785	1815	1785	1785	2046	1951	1951	2046	1785
NJ	1728	1785	1785	1785	2046	1728	2172	2046	1785
CA	2046	2046	2046	2046	1728	2046	2046	1728	2046

Since both OD distance and intensity are known, it is possible to convert the table into Btu per ton shipped values by multiplying by distance, giving values in 1000 Btu:

State	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	1579	2526	1517	1891	5529	1424	1424	5650	2110
IL	1545	1528	594	1381	4537	693	2332	4928	1120
OH	1319	1830	1112	852	4920	773	1816	4953	1324
TN	786	1045	1401	780	4914	1383	2729	4818	1884
NC	773	1687	1826	1112	5748	1448	1996	5654	2282
FL	743	1461	2525	1091	6123	2369	3065	6029	3230
TX	2169	1071	2207	1493	4715	2371	4433	5284	2557
NY	1989	2767	1880	2351	7446	1130	997	6442	2125
NJ	1605	2610	1825	2053	5814	1426	1060	5936	2628
CA	5497	4167	4811	4682	1687	4932	6875	1541	4562

These values are then multiplied to calculate energy consumption in billion Btu for each OD pair:

State	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	1042	414	422	511	0	359	745	0	186
IL	570	894	804	686	395	406	1035	217	539
OH	613	372	468	0	0	432	212	0	0
TN	313	139	298	423	0	162	347	0	104
NC	589	287	159	0	0	90	68	0	41
FL	1028	379	220	717	0	130	187	0	132
TX	1182	1510	569	1197	387	138	266	190	243
NY	927	338	532	334	1117	172	478	477	123
NJ	517	480	170	411	0	114	133	0	139
CA	737	1817	1285	1264	2941	947	419	3228	734

Summing energy values in all cells gives 42.6 trillion Btu.

Part (c): The supply from the origin states is equal to the sum of all flows to destination states. The demand at destination states is equal to the sum of all arriving flows. The following table can then be constructed:

Supply		Demand	
State	Capacity	State	Demand
	[1000 t]		[1000 t]

GA	5504	PA	2235
LA	3666	IL	4446
WI	3340	OH	1765
AL	3381	TN	1586
WA	2063	NC	1132
MI	2113	FL	2544
ME	2031	TX	3346
OR	2248	NY	1926
MN	1050	NJ	1057
		CA	5359
Total	25396		25396

Note that the total supply exactly equals the total demand.

Part (d): The transportation problem for minimizing ton-miles with limited supply and minimum demand requirements is presented in symbolic form as follows:

$$MinZ = \sum_{i,j} D_{i,j} X_{i,j}$$

$$s.t.$$

$$\sum_{j} X_{i,j} \leq A_i, \forall i$$

$$\sum_{i} X_{i,j} = B_j, \forall j$$

$$X_{i,j} \geq 0, \forall i, j$$

Solving in MS Excel for the pattern that minimizes ton-miles gives the following table of OD assignments:

1000 tons	GA	LA	WI	AL	WA	MI	ME	OR	MN	Tot Dmd:
PA	1828	-	-	-	-	407	-	-	-	2235
IL	-	-	3340	-	-	754	-	-	352	4446
OH	-	-	-	1765	-	-	-	-	-	1765
TN	-	-	-	1586	-	-	-	-	-	1586
NC	1132	-	-	-	-	-	-	-	-	1132
FL	2544	-	-	-	-	-	-	-	-	2544
TX	-	3346	-	-	-	-	-	-	-	3346
NY	-	-	-	-	-	952	974	-	-	1926
NJ	-	-	-	-	-	-	1057	-	-	1057

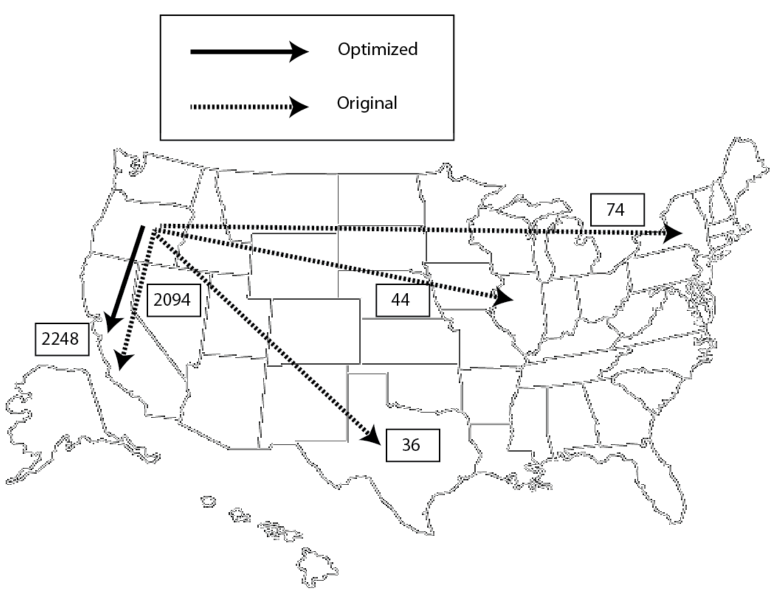
CA	-	320	-	30	2063	-	-	2248	698	5359
Tot Supply	5504	3666	3340	3381	2063	2113	2031	2248	1050	

Consequently, multiplying the above ton assignments by distances from the distance matrix above gives the following ton-miles by OD pair:

State	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	1671	0	0	0	0	297	0	0	0
IL	0	0	815	0	0	241	0	0	202
OH	0	0	0	692	0	0	0	0	0
TN	0	0	0	569	0	0	0	0	0
NC	403	0	0	0	0	0	0	0	0
FL	870	0	0	0	0	0	0	0	0
TX	0	1650	0	0	0	0	0	0	0
NY	0	0	0	0	0	551	498	0	0
NJ	0	0	0	0	0	0	516	0	0
CA	0	652	0	69	2013	0	0	2005	1557

Adding the ton-miles in the above matrix sums to 15.3 billion ton-miles. The original value was 22.6 billion ton-miles, so the reduction is approximately 7.3 billion or 32%.

Part (e): To simplify the task, it may help to choose a source state with relatively few markets in the base case. In this case we have chosen Oregon (OR). In the base case it ships to IL, NY, TX, and mostly to CA. In the optimal case, it ships only to CA. The difference is shown in thousands of tons per year (Map source: www.worldatlas.com):



Part (f): To calculate total energy consumption in the optimized pattern, we can multiply assigned flows by Btu/ton values that were calculated in part (b). The following table of energy consumption values results:

Bil.Btu	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	2887	0	0	0	0	580	0	0	0
IL	0	0	1985	0	0	523	0	0	394
OH	0	0	0	1503	0	0	0	0	0
TN	0	0	0	1237	0	0	0	0	0
NC	875	0	0	0	0	0	0	0	0
FL	1890	0	0	0	0	0	0	0	0
TX	0	3584	0	0	0	0	0	0	0
NY	0	0	0	0	0	1076	971	0	0
NJ	0	0	0	0	0	0	1121	0	0
CA	0	1333	0	140	3480	0	0	3465	3184

Summing all energy values gives 30.2 tril.Btu. The original energy consumption value was 42.6 tril.Btu, so the savings is 12.4 tril.Btu, or 29.1%. Note that this is slightly less than the ton-mile reduction of 32%, due to difference in energy intensity between different OD pairs.

Part (g): To optimize the pattern so as to reduce energy consumption, the constraints remain the same, but the objective function is changed. Let E be total energy consumption and $\mu_{\text{ton}.i,j}$ be the energy consumption per ton for shipment from i to j, the objective is then:

$$MinE = \sum_{i,j} \mu_{ton.i,j} \cdot X_{i,j}$$

Rerunning the objective function gives almost the same OD matrix and the same energy consumption value of 30.2 tril. Btu is out to the first decimal place. (The minimum energy solution is in fact marginally more energy efficient, but the difference is negligible.) The OD matrix is the following:

1000 tons	GA	LA	WI	AL	WA	MI	ME	OR	MN
PA	1828	-	-	-	-	407	-	-	-
IL	-	-	3340	-	-	754	-	-	352
OH	-	-	-	1765	-	-	-	-	-
TN	-	-	-	1586	-	-	-	-	-
NC	1132	-	-	-	-	-	-	-	-
FL	2544	-	-	-	-	-	-	-	-

TX	-	3316	-	30	-	-	-	-	-
NY	-	-	-	-	-	952	974	-	-
NJ	-	-	-	-	-	-	1057	-	-
CA	-	350	-	-	2063	-	-	2248	698

The only change is that the 30K ton flow from AL to TX in the energy-optimal solution does not appear in the ton-mile-optimal solution, and this flow is instead met with product from LA. In turn, there is 30K ton flow from AL to CA in the ton-mile-optimal solution does not appear in the energy-optimal solution, and this flow is instead met with product from LA.

Part (h): Although the optimal pattern might be advantageous from a transportation intensity point of view, there are other factors that lead to a different, there are several possible explanations for the more transportation-intensive spatial pattern of freight that we see in actuality for commodities such as pulp and paper (as well as food products, consumer electronics, and so on). Some possible explanations include the following:

- Some manufacturing facilities may be large and efficient, so that the total cost of delivery including production and transportation is competitive even at a long distance, and the buyer chooses the source at a longer distance to reduce cost.
- The seller-buyer relationship and hence choose of source of the product is based on the specific characteristics of the product, or the strength of the seller-buyer relationship, so that distance is not the deciding factor.
- "System suboptimal subsystem optimal:" Although the choice of source may be suboptimal from a global perspective, if the buyer only sees the network of the seller because they have already chosen that seller, the seller may in fact sourcing the product from the source that to them is the nearest to the buyer and hence optimal.
- Other responses are also possible.

Problem 13-6.

A paper products company ships paper from three production plants to four markets as follows: The capacity per unit of time of plants at Plant 1, Plant 2, and Plant 3 is 2420 tons, 3300 tons, and 2200 tons, respectively. The demand per unit of time at City A, City B, City C, and City D is 2202 tons, 1962 tons, 1490 tons, and 2169 tons, respectively. A table of distances between cities can be found on the course BB site. What is the allocation of shipments from plants to markets that meets all demands, does not exceed supplies available at any plant, and minimizes total ton-miles?

Table of distances in miles between plants and markets:

	Plant 1	Plant 2	Plant 3
City A	892	2352	2687
City B	3149	1053	1114
City C	3411	1414	343

City D	2409	244	894

Solution.

The solution is found using the standard transportation problem linear program:

$$\text{Minimize } Z = \sum_{i,j} D_{i,j} \cdot X_{c,i,j}$$

Subject to

$$\sum_{j} X_{i,j} \le A_i, \forall i$$

$$\sum_{i} X_{i,j} = B_j, \forall j$$

$$X_{i,j} \ge 0, \forall i, j$$

Alternatively, in verbal form:

Minimize Σ (tonnes)(distance)

Subject to

Σ tonnes ≤ tonnes available at each origin

Σ tonnes ≥ tonnes demanded at each destination

Tonnes ≥ 0 (Negative values of tonnes allocated are not allowed)

The optimal allocation that minimizes ton-miles is the following table (in tons):

	Plant 1	Plant 2	Plant 3	Total	Demand
City A	2202	0	0	2202	2202
City B	121	1131	710	1962	1962
City C	0	0	1490	1490	1490
City D	0	2169	0	2169	2169
Total	2323	3300	2200		
Available	2420	3300	2200		

The total shipped from each origin and supplied to each destination is shown in the table as well. By inspection, it can be seen that the demand at each of the four destinations is met, and the supply is not exceeded at the sources (Plants 2 and 3 are used 100%, while Plant 1 has some tonnes of supply extra). Multiplying tons by distance gives the following, in thousand ton-miles:

	Plant 1	Plant 2	Plant 3	Total
City A	1964	0	0	1964
City B	381	1191	791	2363
City C	0	0	510	510
City D	0	529	0	529

Summing the totals column gives a total of 5.37 million ton-miles.

Problem 13-7.

A retail firm operates a *decentralized* distribution system (factory to warehouse to shop) in which the system uses four smaller warehouses distributed around a region to receive a product from manufacturer (called *primary distribution*) and then send product to retail outlets (called *secondary distribution*). The firm is offered the opportunity to shift to a *centralized* system in which each unit of product will still undergo primary and secondary distribution, but now there will only be one warehouse in the middle of the region. The transportation of the product incurs financial cost and energy consumption per vehicle·kilometer (vkm) of movement. In the case of the warehouse costs, inventory costs are incurred by virtue of needing to keep stock on hand in the warehouse between the time that the stock is purchased and the time it is sold, and energy is consumed to operate the warehouse. Assume that all other costs and rates of energy consumption are the same for either option.

a. Based on the data given, determine whether the decentralized or centralized system is preferred.

b. The Chief Financial Officer (CFO) and Chief Environmental Officer (CEnvO) are at the board meeting where the choice is to be made between the two alternative systems, using the results of part (a). What position will each one take? Why? Discuss in up to one paragraph.

Underlying data to be used in problem:

Warehouse cost and energy use:

Option	Inventory Cost	Energy
	($1000/year)	(GJ/year)
Decentralized	190	155
Centralized	475	485

Annual transportation volume per warehouse (in units of 1000 vkm per year)

Option	Primary	Secondary
Decentralized	35	82
Centralized	120	430

Transportation cost and energy use:

Option	Cost ($/1000 vkm)	Energy MJ/1000 vkm
All shipments	1500	19,000

Solution.

The approach here is to go all the way through the cost and energy consumption calculation for the decentralized option in detail, then to repeat for the centralized in a streamlined form, and finally to make comparisons.

Decentralized option:

On the warehouse side, there are four warehouses total so cost and values must be multiplied by four, as follows:

$$\text{Cost}: (\$190\text{K})4 = \$760K/\text{yr}$$
$$\text{Energy}: (155\,\text{GJ})4 = 620\,\text{GJ/yr}$$

On the transportation side, total cost and energy consumption are based on summing total vkm generated across primary and secondary stages, then converting to cost and energy:

$$\text{vkm per warehouse}: 35\text{K} + 82\text{K} = 117\text{K km/yr}$$
$$\text{Total vkm}: (117\text{K})4 = 468\text{K km/yr}$$
$$\text{Total cost}: (468\text{K})(\$1500/1000\,\text{km}) = \$702K/\text{yr}$$
$$\text{Total energy}: (468\text{K})(19\,\text{GJ/1000 km}) = 8892\,\text{GJ/yr}$$

Summing the warehouse and transportation components gives total cost and energy per year:

$$\text{Cost}: \$760K + \$702\text{M} = \$1.462\text{M/yr}$$
$$\text{Energy}: 620\,\text{GJ} + 8892\,\text{GJ} = 9512\,\text{GJ/yr}$$

Centralized option:

Repeat all steps for centralized solution. For compactness, calculations are presented without elaboration. Note that because there is only one warehouse, the solution is somewhat simplified.

$$\text{Cost}: (\$475\text{K})1 = \$475\text{K/yr}$$
$$\text{Energy}: (485\,\text{GJ})1 = 485\text{GJ/yr}$$

$$\text{vkm per warehouse}: 120\text{K} + 430\text{K} = 550\text{K km/yr}$$
$$\text{Total vkm}: (550\text{K})1 = 550\text{K km/yr}$$
$$\text{Total cost}: (550\text{K})(\$1500/1000\text{km}) = \$825K\text{/yr}$$
$$\text{Total energy}: (550\text{K})(19\,\text{GJ}/1000\,\text{km}) = 10{,}450\text{GJ/yr}$$

$$\text{Cost}: \$475\text{K} + \$825\text{K} = \$1.3\text{M/yr}$$
$$\text{Energy}: 485\,\text{GJ} + 10{,}450\,\text{GJ} = 10{,}935\,\text{GJ/yr}$$

Solution.

Part (a): The centralized solution increases energy consumption by 1423 GJ, or 1.423 TJ, per year, or 15%. However, it also reduces annual cost by $162,000 per year, or 11%.

Part (b): The CFO will argue in favor of the centralized solution, since you are not given any additional info about the value to the company of saving energy (other than the direct value, which is included in the cost of inputs). The CEnvO might argue in favor of the decentralized option for green reasons. The CFO will probably win. The dilemma is that even if the retail firm would like to choose the "green" option and save energy, market forces are pushing them in the opposite direction. The centralized solution reduces inventory cost, since the firm can carry less inventory and incur less cost if they centralize all stored product in a single warehouse. However, the energy savings from this change are small. Looking at the warehousing + freight transportation "life cycle," the energy consumed on the warehousing side is small. Most of the energy is consumed in the freight component, so that when the shift is made under the centralized solution to a more transportation-intensive pattern, there is no counterbalancing reduction in energy on the warehousing side that can offset the increased energy consumption by the additional trucking involved.

Chapter 14 Overview of Alternative Fuels and Platforms

Problem 14-1.

A compact passenger car that runs on natural gas emits 0.218 kgCO_2 per mile driven. For simplicity, treat natural gas as being pure methane (CH_4) having an energy content of 50 MJ/kg. (a) What is the mass and energy content of the fuel consumed per mile driven? (b) Suppose the vehicle is fueled with synthetic natural gas that is derived from coal. Of the original energy in the coal, 40% is used to convert the coal to the synthetic gas. Again, for simplicity, treat coal as pure carbon. What is the mass of coal consumed and new total CO_2 emissions per mile? Consider only the effect of using coal instead of gas as the original energy source, and the conversion loss; ignore all other factors.

Solution:

Part (a): Assuming that natural gas is pure methane, the combustion of gas to propel the car uses the following reaction:

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

Since methane has an atomic mass of 16, the ratio of methane to CO_2 mass is 16/44 = 0.3636. The mass of natural gas consumed per mile is then (0.3636)(0.218 kg/mi) = 0.0793 kg methane per mile. Assuming an energy content of 50 MJ per kg of methane gives (0.0793)(50) = 3.95 MJ/mi.

Part (b): Since the vehicle consumes 0.0793 kg of methane per mile, the mass input of carbon from coal is 0.0793(12/16) = 0.0595 kg coal. To take account of the 40% of coal used to provide energy for the conversion process, divide by 60%, giving (0.0595)/(0.6) = 0.0991 kg coal combusted per mile driven. Emissions per mile are then (0.0991 kg coal)(44/12) = 0.363 kgCO_2 per mile.

Problem 14-2.

Use the data in Table 14-3 to complete the Divisia analysis of car and light truck fuel consumption in the period 1970 to 2000 in metric units.

Solution.

First, create a table of combined activity, energy consumption, and fuel economy:

Year	km bil.	Fuel (bil.L)	km/L
1970	1664	304	5.47
1975	1976	355	5.57
1980	2245	355	6.32
1985	2621	375	6.99
1990	3173	398	7.97
1995	3565	431	8.27

2000	4037	477	8.46

For the Divisia analysis, it is necessary to convert fuel economy into a measure of fuel intensity, for which liters per 100 km is chosen:

Year	PC L/100 km	LT L/100 km
1970	17.52	23.86
1975	16.93	23.29
1980	14.90	19.31
1985	13.58	16.61
1990	11.67	14.67
1995	11.21	13.69
2000	10.82	13.54

Based on the data given, it is possible to calculate the reference overall fuel intensity in the base year 1970, which is 18.27 L/100 km.

Terms are then generated from the given data according to the presentation in Chap. 3 in the book. The key equation is the following:

$$\Delta e_t = \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2} + (s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$
$$= \sum_{i=1}^{n}\left[(e_t - e_{t-1})_i \frac{(s_t + s_{t-1})_i}{2}\right] + \sum_{i=1}^{n}\left[(s_t - s_{t-1})_i \frac{(e_t + e_{t-1})_i}{2}\right]$$

To calculate the structure and intensity terms implied by this equation, it is necessary to calculate using a spreadsheet a table of input values for each of the modes of PC and LT.

For PC mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1970	17.52	88.2%				
1975	16.93	83.7%	–0.590	–0.045	34.447	1.719
1980	14.90	79.2%	–2.033	–0.045	31.825	1.629

1985	13.58	76.1%	–1.312	–0.031	28.480	1.554
1990	11.67	71.0%	–1.911	–0.051	25.257	1.471
1995	11.21	64.5%	–0.461	–0.065	22.886	1.355
2000	10.82	63.4%	–0.392	–0.011	22.033	1.280

For LT mode:

Year	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$
1970	23.86	11.8%				
1975	23.29	16.3%	–0.566	0.045	47.150	0.281
1980	19.31	20.8%	–3.979	0.045	42.605	0.371
1985	16.61	23.9%	–2.700	0.031	35.927	0.446
1990	14.67	29.0%	–1.940	0.051	31.287	0.529
1995	13.69	35.5%	–0.987	0.065	28.361	0.645
2000	13.54	36.6%	–0.146	0.011	27.228	0.720

Using the above equation, we can now calculate the individual terms needed for the Divisia analysis, including an intensity and structure term for each of PC and LT modes for the years 1975 to 2000.

Year	**Intensity**		**Structure**	
	PC	LT	PC	LT
1970	n/a	n/a	n/a	n/a
1975	–0.507	–0.080	–0.768	1.051
1980	–1.656	–0.737	–0.710	0.950
1985	–1.019	–0.603	–0.445	0.562
1990	–1.405	–0.513	–0.645	0.799
1995	–0.312	–0.318	–0.739	0.916
2000	–0.251	–0.052	–0.125	0.154

To calculate the overall impact of structure and intensity, in the table below we first sum values for PC and LT for the given year (“incremental”) and then add together the value for the given year with the sum of all previous years (“cumulative”). For example, for 1980, the cumulative value for intensity is the sum of the 1975 cumulative value and the 1980 incremental value, that is, –0.587 + (–2.393) = –2.980.

Year	Incremental changes (PC + LT)		Cumulative (Cols. 1 and 2)	
	Intensity	Structure	Intensity	Structure
	Terms 1	Terms 2	Terms 1	Terms 2
	L/100 km	L/100 km	L/100 km	L/100 km
1975	–0.587	0.283	–0.587	0.283
1980	–2.393	0.240	–2.980	0.524
1985	–1.622	0.116	–4.602	0.640
1990	–1.918	0.154	–6.520	0.794
1995	–0.630	0.177	–7.150	0.971
2000	–0.303	0.029	–7.454	1.000

The cumulative values then provide the basis for converting vehicle·km of activity in a given year into fuel use. As shown in the table below, combined PC and LT vehicle·km are multiplied by the intensity and structure cumulative term from the preceding table to calculate the contribution to the difference between trended and actual fuel consumption.

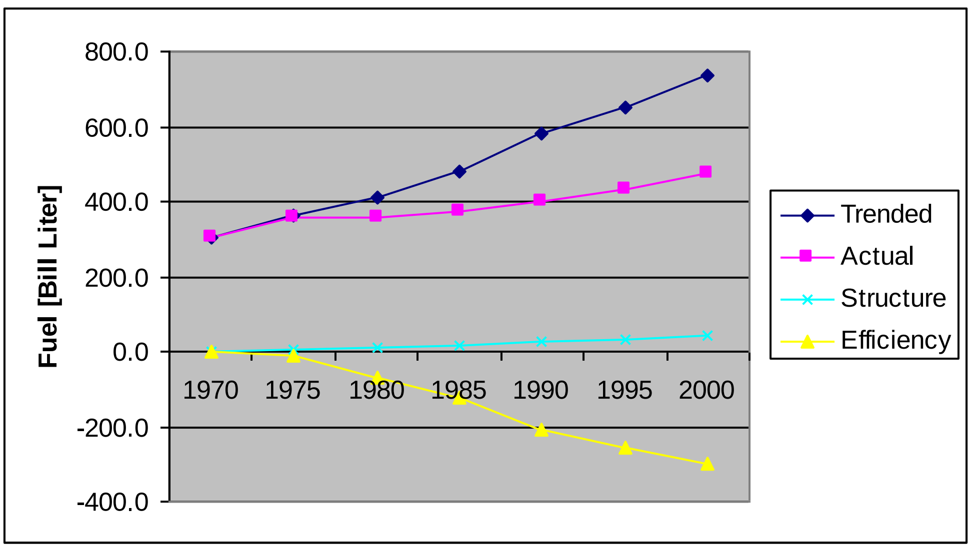
Year	Activity (bil.vkm)	Intensity contribution (L/100 km)	(bil.L)	Structure contribution (L/100 km)	(bil.L)
1970	1664	0	0	0	0
1975	1976	–0.587	(11.59)	0.283	5.59
1980	2245	–2.980	(66.90)	0.524	11.75
1985	2621	–4.602	(120.61)	0.640	16.77
1990	3173	–6.520	(206.88)	0.794	25.20
1995	3565	–7.150	(254.91)	0.971	34.61
2000	4037	–7.454	(300.91)	1.000	40.38

The conclusion of the Divisia analysis is shown by summing trended, intensity contribution, and structure contribution. The trended fuel consumption is obtained by multiplying the vehicle·km in each year by the fixed fuel intensity from above of 18.27 L/100 km. Note that the sum term (second column from the right) exactly equals the actual fuel consumption (both are measured in billions of liters of fuel).

Year	Activity (bil.vkm)	Trended (bil.L)	Intensity (bil.L)	Structure (bil.L)	Sum of $T + I + S$	Check: Actual
1970	1664	304.0	0	0	304.0	304

1975	1976	361.0	–11.6	5.6	355.0	355
1980	2245	410.1	–66.9	11.8	355.0	355
1985	2621	478.8	–120.6	16.8	375.0	375
1990	3173	579.7	–206.9	25.2	398.0	398
1995	3565	651.3	–254.9	34.6	431.0	431
2000	4037	737.5	–300.9	40.4	477.0	477

Interpretation of the Divisia analysis: In general, the actual consumption is lower than the trended consumption because of the significant negative value of the intensity contribution, which dominates the smaller positive contribution of the structure term. The outcome can be visualized using the figure below, and noting that the intensity term is labeled "efficiency" in the figure.



Problem 14-3

Use the data in Table 14-3 to complete the Divisia analysis of car and light truck fuel consumption in the period 1970 to 2000 in U.S. standard units.

Solution.

First, create a table that breaks out intensity and modal share for the PC (passenger car) and LT (light truck) categories. Intensity is measured in gallons per 100 miles.

	PC	PC	LT	LT
	Intensity	**Structure**	**Intensity**	**Structure**
1970	7.42	88.2%	10.0	11.8%

1975	7.16	83.7%	9.9	16.3%
1980	6.29	79.3%	8.2	20.7%
1985	5.73	76.1%	7.0	23.9%
1990	4.94	71.0%	6.2	29.0%
1995	4.74	64.5%	5.8	35.5%
2000	4.57	63.4%	5.7	36.6%

Next, create a table of the elements needed for the decomposition for PC:

	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$	**Intensity Term**	**Structure Term**	**Total Term**
1970	7.42	88.2%							
1975	7.16	83.7%	–0.259	–0.044	14.572	1.719	–0.222	–0.324	–0.547
1980	6.29	79.3%	–0.862	–0.045	13.452	1.630	–0.702	–0.300	–1.003
1985	5.73	76.1%	–0.561	–0.031	12.029	1.554	–0.436	–0.188	–0.624
1990	4.94	71.0%	–0.791	–0.051	10.677	1.471	–0.582	–0.274	–0.855
1995	4.74	64.5%	–0.207	–0.065	9.679	1.355	–0.141	–0.313	–0.453
2000	4.57	63.4%	–0.167	–0.011	9.304	1.280	–0.107	–0.052	–0.159

A similar table for LT:

	e_t	s_t	$e_t - e_{t-1}$	$s_t - s_{t-1}$	$e_t + e_{t-1}$	$s_t + s_{t-1}$	**Intensity Term**	**Structure Term**	**Total Term**
1970	10.00	11.8%							
1975	9.85	16.3%	–0.149	0.044	19.851	0.281	–0.021	0.442	0.421
1980	8.18	20.7%	–1.672	0.045	18.029	0.370	–0.309	0.403	0.093
1985	7.01	23.9%	–1.171	0.031	15.186	0.446	–0.261	0.238	–0.024
1990	6.19	29.0%	–0.816	0.051	13.199	0.529	–0.216	0.338	0.122
1995	5.77	35.5%	–0.419	0.065	11.963	0.645	–0.135	0.386	0.251
2000	5.73	36.6%	–0.041	0.011	11.503	0.720	–0.015	0.065	0.050

Summing the PC and LT terms (rightmost column) gives the total change in energy, as shown:

	PC Total	LT Total	Combined
1970	--	--	--
1975	–0.547	0.421	–0.126
1980	–1.003	0.093	–0.909
1985	–0.624	–0.024	–0.648
1990	–0.855	0.122	–0.733
1995	–0.453	0.251	–0.202
2000	–0.159	0.050	–0.109

In order to graph the effects of “intensity” and “structure,” we need to isolate each term from the PC and LT side and then sum them:

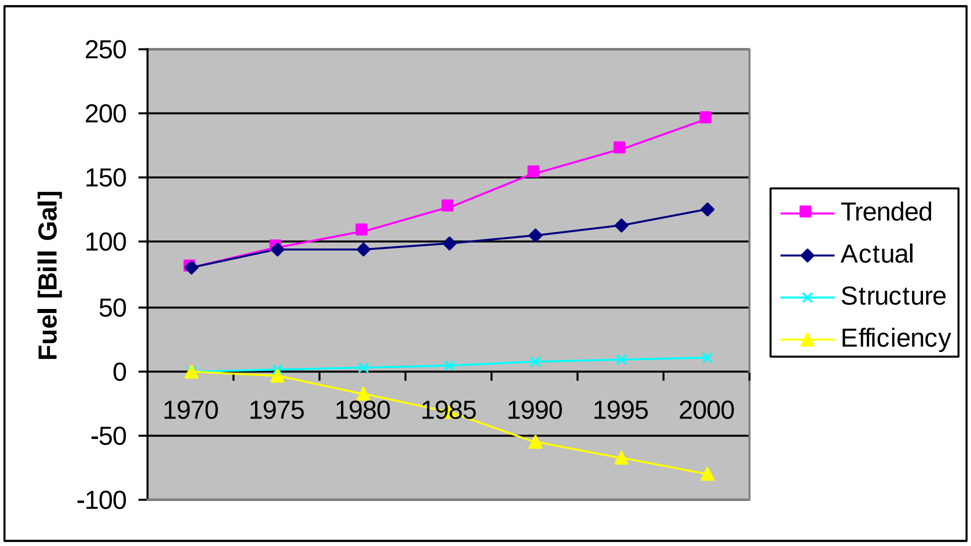
	Sum of chnge (PC+LT)		Cumulative (Cols. 1 and 2)	
	Intensity	**Structure**	**Intensity**	**Structure**
1975	–0.243	0.117	–0.243	0.117
1980	–1.012	0.102	–1.255	0.220
1985	–0.697	0.049	–1.952	0.269
1990	–0.797	0.065	–2.750	0.334
1995	–0.276	0.074	–3.025	0.407
2000	–0.122	0.012	–3.147	0.420

The cumulative change in intensity and structure can be multiplied by the activity (vmt) in a given year to give the impact on fuel consumption, in billions of gallons. This data series then provides the input for the figure that you are asked to draw, below. The “trended” data are the activity level multiplied by 1970 overall intensity (energy use per unit of activity). Next are the intensity and structure terms, followed by the “overall change,” which is the sum of the intensity and structure. As a check, the rightmost column is equal to the difference between the trended and actual fuel consumption, showing that these values are equal to the overall change.

Trend vs actual energy						*Check:*
	Actual (bil. Gal)	**Trended** (bil. Gal)	**Intensity** (bil. Gal)	**Structure** (bil. Gal)	**Overall chg.** (bil. Gal)	***Difference*** ***(bil. Gal)***
1970	80.3	80.3	0	0	0.0	*0.0*

1975	93.8	95.4	–3.0	1.4	–1.6	*–1.6*
1980	93.8	108.3	–17.6	3.1	–14.5	*–14.5*
1985	98.9	126.5	–32.0	4.4	–27.6	*–27.6*
1990	105.2	153.1	–54.5	6.6	–47.9	*–47.9*
1995	113.7	172.0	–67.4	9.1	–58.3	*–58.3*
2000	126	194.8	–79.4	10.6	–68.8	*–68.8*

Figure for Problem 14.3:



Problem 14-4.

Revisit Example 14-3, using the same fuel economy values for ICEVs and HEVs, but this time considering the entire U.S. passenger car fleet. Suppose that HEVs achieve 50% penetration of the national fleet by 2040. The fleet in 2000 consisted of 134 million vehicles; assume this number is fixed for the duration of the transition. Compare this transition to a scenario where there is no influx of HEVs. (a) Calculate the cumulative fuel savings for the period of 2000 to 2040 using a triangle function with peak rate of change in 2020. (b) Calculate the cumulative fuel savings for the period of 2000 to 2040 using a logistics function. Fit the logistics function to the data in years 2000 to 2009 and 2040 only, with the assumed value in 2040 of 50% of the fleet, that is, 67 million vehicles. (c) Compare the results from calculations in parts (a) and (b). Discuss the differences.

Solution.

Part (a): Triangle function: With the numbers given, we determine that parameters a = 2000, b = 2040, so $(a + b)/2$ = 2020. Using the formula for the triangle function:

$$L(t) = 0 \qquad \text{if } t < a$$

$$L(t) = 2\left(\frac{t-a}{b-a}\right)^2 \qquad \text{if } a \le t \le \frac{a+b}{2}$$

$$L(t) = 1 - 2\left(\frac{b-t}{b-a}\right)^2 \qquad \text{if } \frac{a+b}{2} \le t \le b$$

$$L(t) = 1 \qquad \text{if } t > \text{b}$$

we can calculate the number of HEVs in the fleet in any year in the transition, from $N = 0$ in 2000 to $N =$ 67 million in 2040. The number of vehicles and resulting fuel consumption are shown in the table below the logistics function analysis.

The fuel consumption for the fleet in the business-as-usual case is:

$$\text{Fuel} = \left(1.34\times10^{8}\,\text{vehicles}\right)\left(16{,}000\,\text{km/vehicle/yr}\right)\left(\frac{1}{8.5\,\text{km/L}}\right) = 2.52\times10^{11}\,\text{L/yr}$$

The fuel consumption over 40 years is then 40 times this amount, or 1.01×10^{13} L. In the scenario where HEVs penetrate the market to 50% by 2040, the total fuel consumption for both ICEVs and HEVs over the period is 9.22×10^{12} L. Thus, the savings is 1.12×10^{12} L, that is,

$$1.01\times10^{13} - 9.22\times10^{12} = 1.12\times10^{12}\ \text{L}$$

Part (b): Logistics function: The logistics function is fitted by evaluating parameters c_1 and c_2 from the following:

$$f(t) = \frac{F \cdot e^{(c_1 + c_2 t)}}{1 + e^{(c_1 + c_2 t)}}$$

Given the HEV growth data, the assumption used for evaluating c_1 and c_2 is that the value in 2040 is the basis for the maximum penetration value of 67 million HEVs, and that only the values from 2000 to 2009 are used to fit the logistics curve. For the purposes of the calculations, we set $F = 1$ so that $f(t)$ represents the fraction of penetration relative to the maximum of 67 million. The values of c_1 and c_2 are calculated using the solver such that the error between the predicted and actual values for 2000 to 2009 is minimized. The following table gives predicted and actual values using the resulting optimal values for the parameters of $c_1 = -6.809$ and $c_2 = 0.352$:

Year	***f*(*t*) value**		**Error**	**Error**
	Observed	**Modeled**	**= *O* – *M***	**Squared**
2000	1.40E-04	1.10E-03	–9.63E-04	9.27E-07
2001	4.42E-04	1.57E-03	–1.12E-03	1.27E-06
2002	9.65E-04	2.23E-03	–1.26E-03	1.59E-06
2003	1.67E-03	3.16E-03	–1.49E-03	2.22E-06
2004	2.99E-03	4.49E-03	–1.51E-03	2.27E-06
2005	6.08E-03	6.38E-03	–3.02E-04	9.12E-08
2006	9.85E-03	9.05E-03	8.06E-04	6.50E-07

2007	1.52E-02	1.28E-02	2.34E-03	5.45E-06
2008	1.99E-02	1.81E-02	1.80E-03	3.24E-06
2009	2.36E-02	2.56E-02	–2.00E-03	3.99E-06
			Total:	2.17E-05

The numbers of HEVs and ICEVs in each year are shown in the table below alongside the triangle function analysis, and resulting fuel savings in each year are shown in the figure below.

After taking into account the reduced fuel consumption due to the transition to HEVs, the total fuel consumption for both ICEVs and HEVs over the period is 9.19×10^{12} L. Thus, the savings are slightly larger than that of the triangle function approach, at 1.16×10^{12} L, that is,

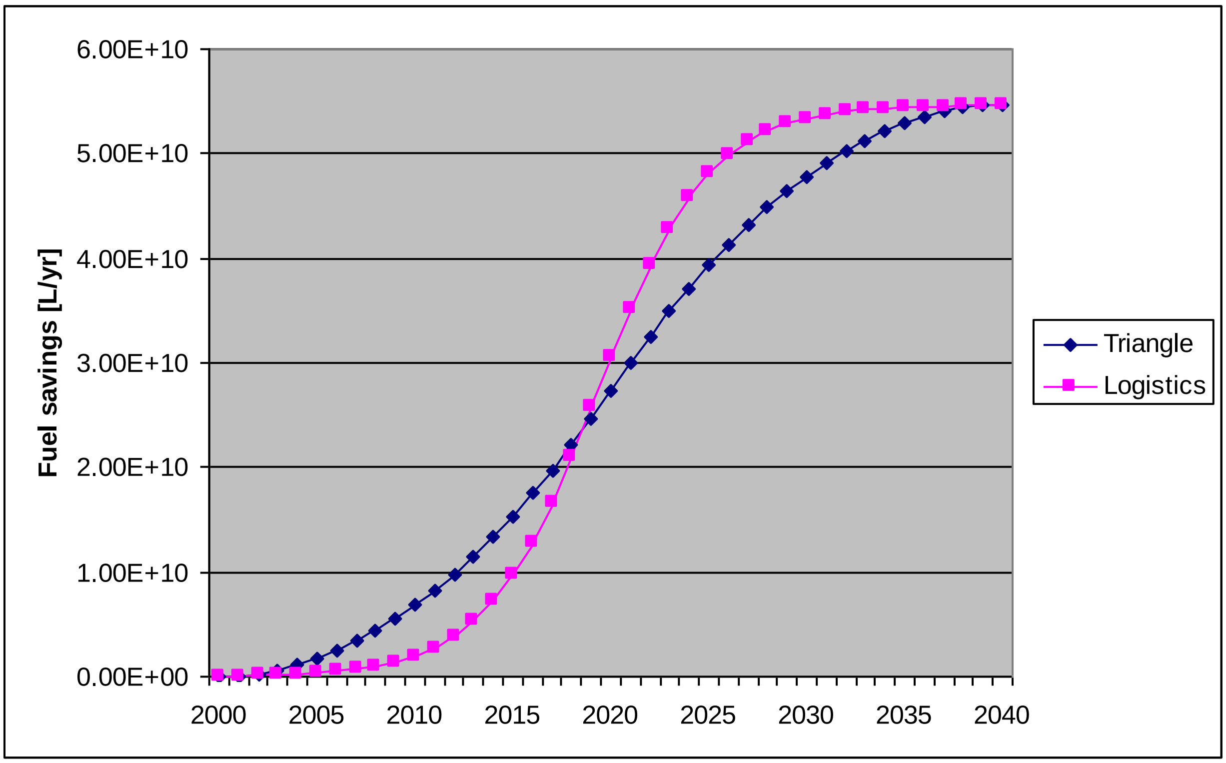
$$1.01\times10^{13} - 9.19\times10^{12} = 1.16\times10^{12} \text{ L}$$

Table comparing triangle and logistics functions:

Year:	Number of HEVs		Total fuel (L)	
	Triangle	Logistics	Triangle	Logistics
2000	0.00E+00	7.39E+04	2.52E+11	2.52E+11
2001	8.38E+04	1.05E+05	2.52E+11	2.52E+11
2002	3.35E+05	1.49E+05	2.52E+11	2.52E+11
2003	7.54E+05	2.12E+05	2.52E+11	2.52E+11
2004	1.34E+06	3.01E+05	2.51E+11	2.52E+11
2005	2.09E+06	4.27E+05	2.51E+11	2.52E+11
2006	3.02E+06	6.06E+05	2.50E+11	2.52E+11
2007	4.10E+06	8.59E+05	2.49E+11	2.52E+11
2008	5.36E+06	1.21E+06	2.48E+11	2.51E+11
2009	6.78E+06	1.71E+06	2.47E+11	2.51E+11
2010	8.38E+06	2.41E+06	2.45E+11	2.50E+11
2011	1.01E+07	3.38E+06	2.44E+11	2.49E+11
2012	1.21E+07	4.70E+06	2.42E+11	2.48E+11
2013	1.42E+07	6.50E+06	2.41E+11	2.47E+11
2014	1.64E+07	8.88E+06	2.39E+11	2.45E+11
2015	1.88E+07	1.20E+07	2.37E+11	2.42E+11
2016	2.14E+07	1.58E+07	2.35E+11	2.39E+11
2017	2.42E+07	2.04E+07	2.32E+11	2.36E+11
2018	2.71E+07	2.58E+07	2.30E+11	2.31E+11
2019	3.02E+07	3.15E+07	2.28E+11	2.27E+11
2020	3.35E+07	3.74E+07	2.25E+11	2.22E+11
2021	3.68E+07	4.30E+07	2.22E+11	2.17E+11
2022	3.99E+07	4.81E+07	2.20E+11	2.13E+11
2023	4.28E+07	5.25E+07	2.17E+11	2.09E+11
2024	4.56E+07	5.61E+07	2.15E+11	2.06E+11
2025	4.82E+07	5.90E+07	2.13E+11	2.04E+11
2026	5.06E+07	6.11E+07	2.11E+11	2.02E+11
2027	5.28E+07	6.28E+07	2.09E+11	2.01E+11
2028	5.49E+07	6.40E+07	2.07E+11	2.00E+11

2029	5.69E+07	6.48E+07	2.06E+11	1.99E+11
2030	5.86E+07	6.55E+07	2.04E+11	1.99E+11
2031	6.02E+07	6.59E+07	2.03E+11	1.98E+11
2032	6.16E+07	6.62E+07	2.02E+11	1.98E+11
2033	6.29E+07	6.65E+07	2.01E+11	1.98E+11
2034	6.40E+07	6.66E+07	2.00E+11	1.98E+11
2035	6.49E+07	6.67E+07	1.99E+11	1.98E+11
2036	6.57E+07	6.68E+07	1.99E+11	1.98E+11
2037	6.62E+07	6.69E+07	1.98E+11	1.98E+11
2038	6.67E+07	6.69E+07	1.98E+11	1.98E+11
2039	6.69E+07	6.69E+07	1.98E+11	1.98E+11
2040	6.70E+07	6.70E+07	1.98E+11	1.98E+11
		Total:	9.22E+12	9.19E+12

The transition can also be compared in a graphical form:



Part (c): The logistics curve predicts a more rapid maximum rate of change in the fleet than the triangle curve. The growth in absolute number of HEVs peaks in 2020 at about 5.9 million vehicles added in that year. Such a transition would be feasible in practice, but would be quite challenging because the total number of HEVs added is so large relative to the total number of vehicles sold in the overall market. In this sense, the triangle curve scenario is more practical, since even in the fastest growth years around 2020, the fleet adds a net of 3.3 million new HEVs, which is well within the rate of turnover of vehicles in general.

Problem 14-5.

Repeat Problem 14-4, but this time with changing overall fleet size and fuel efficiency. Assume that both the fleet size and average fuel economy grow linearly at the average annual rate from 1980 to 2000. Also, assume that HEV fuel economy improves by the same percent each year as the ICEV fleet for the period of 2000 to 2040. Obtain the necessary data to extrapolate rates of change from the internet or other source.

Solution.

We first present calculations of the business-as-usual case (without hybrid transition) needed for the solution of parts (a) and (b).

First, to make projections about future energy use, combine "passenger car" and "light truck" data provided by the U.S. government on the internet into a single data stream for the period 1980 to 2000. Data are provided in gallons, so we will work in gallons and convert to liters. The following table for total registrations, total vehicle miles traveled (VMT), and total fuel results:

Year	Regis, miles	Miles, bil.	Fuel, bil.gal
1980	149.5	1403	93.8
1985	165.1	1638	98.9
1990	182	1983	105.2
1995	194.1	2228	113.7
2000	212.7	2523	126

Regression analysis gives the following values for dependent variables as a function of *x*, the number of years elapsed since 1980:

- For registrations: $y = 15.54(x) + 134.06$ (*x* value is the number of 5-year intervals elapsed since 1975)
- For miles driven per vehicle: $y = 651.17(x) + 8755$ (*x* value is the number of 5-year intervals elapsed since 1975)
- For fuel economy: $y = 0.2633(x) + 17.734$ (*y*-intercept has been adjusted so that the extrapolation passes through the value in 2000)

These data streams as well as total fuel consumption can now be extrapolated outward to the year 2040 as follows, in the business-as-usual case:

-					Base case
Year	Miles driven	Fuel econ	Consump.	Vehicles	Consump.
	(mi)	(mpg)	(gal/veh)	(number)	(1000 gal)
2000	11,852	20.0	592	2.13E+08	1.26E+08
2001	11,928	20.3	588	2.16E+08	1.27E+08

2002	12,003	20.6	584	2.19E+08	1.28E+08

Etc.

2039	14,785	30.3	488	3.34E+08	1.63E+08
2040	14,860	30.6	486	3.37E+08	1.64E+08

Not all rows are shown, for brevity. Summing annual fuel consumption gives 5.95 trillion gallons, or 22.5 trillion liters.

Part (a): Triangle function:

Using the triangle function to model the penetration of hybrids into the fleet, and letting $L(t)$ represent the fraction of vehicles on the road that are hybrids, we have:

$$L(t) = 0 \qquad \text{if } t < a$$

$$L(t) = 2\left(\frac{t-a}{b-a}\right)^2 \qquad \text{if } a \le t \le \frac{a+b}{2}$$

$$L(t) = 1 - 2\left(\frac{b-t}{b-a}\right)^2 \qquad \text{if } \frac{a+b}{2} \le t \le b$$

$$L(t) = 1 \qquad \text{if } t > \mathrm{b}$$

Then, a = 2000, b = 2040, and for example in 2020 $L(t)$ = 0.5, and since there are 275 million vehicles on the road, 50% of the ultimate penetration of 50%, or 25%, are hybrids, equal to 68.7 million vehicles. A spreadsheet table with number of hybrids, miles driven, fuel economy, and so on, by year can be created. Whatever miles are not driven by hybrids are driven by conventional vehicles, and these total annual fuel consumption values can be calculated in the same table. Condensing some intermediate columns to save space, the table looks like as below:

Year	**Hybrid fuel econ**	**Consump.**	**Penetration**	**Vehicles**	**Hybrid fuel**	**Conv.fuel**	**Total fuel**
	(mpg)	**(gal/veh)**	***p*(*t*)**	**(millions)**	**(1000 gal)**	**(1000 gal)**	**(1000 gal)**
2000	38.4	309	0.000	0.00E+00	-	1.26E+08	1.26E+08
2001	38.9	307	0.001	1.35E+05	4.14E+04	1.27E+08	1.27E+08
2002	39.4	305	0.003	5.47E+05	1.67E+05	1.28E+08	1.28E+08

Summing fuel consumption in this case gives 5.20 trillion gallons, or 19.7 trillion liters. (The graph is shown below.)

Savings are 5.95 – 5.20 = 0.752 trillion gallons = 752 billion gallons = 2.85 trillion liters cumulatively, or 12.6% savings compared to the base case.

Part (b): Logistics function:

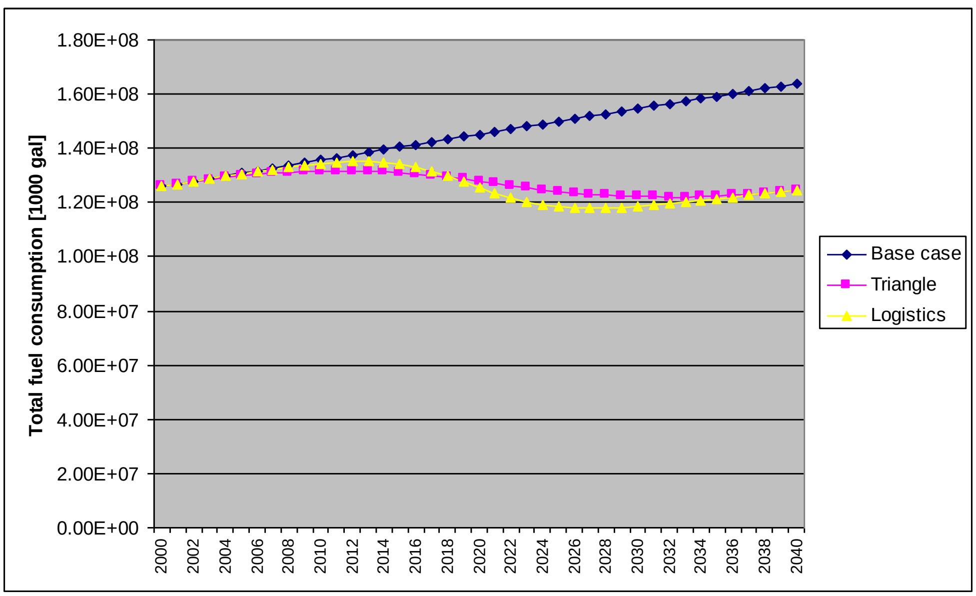
The logistics function is fitted by evaluating parameters c_1 and c_2 from the following:

$$f(t) = \frac{F \cdot e^{(c_1 + c_2 t)}}{1 + e^{(c_1 + c_2 t)}}$$

The annual influx of HEVs from 2000 to 2009 gives best-fit parameters of $c_1 = -6.6197$ and $c_2 = 0.340$. The hybrid transition achieves 50% penetration by the year 2040 in this case (the shape of the curve is reflected in the fuel consumption pattern shown below). Total fuel consumption is 5.17 trillion gallons (19.6 billion liters), and savings are therefore 782 billion gallons, or 13.1%.

Part (c): Comparison: Although the triangle and logistics paths are different, the result in fuel consumption is similar, because the two paths are similar. In either case, much of the original level of fuel consumptions remains, because the eventual influx is a maximum of 50% of the fleet.

Figure: Annual fuel consumption 2000 to 2040 for the case of no influx of hybrids ("base case") and influx of hybrids ("triangle" or "logistics"):



Problem 14-6.

A car buyer is considering whether or not to spend extra for a hybrid in order to save money in the long run on gasoline expenditures. The buyer drives 25,000 km/year. The cost difference between the two vehicles is $4000, and the fuel consumption is 7.9 L/100 km for the ICEV and 5.3 L/100 km for the HEV. The cost of fuel is $1.06/L (about $4/gal). (a) What is the simple payback for buying the HEV, in years? (b) If the buyer expects a 5% return on this investment, and either car will last for 10 years with negligible resale value at the end of that time, what is the NPV? (c) What is the NPV in (b) if the cost of fuel rises by 3% per year for the lifetime of the vehicle?

<u>Solution.</u>

Part (a): The fuel consumption per year for each vehicle is the following:

$$F_{ICEV} = (25000)\left(\frac{1}{100}\right)(7.9L/100km) = 1975L/yr$$
$$F_{HEV} = (25000)\left(\frac{1}{100}\right)(5.3L/100km) = 1325L/yr$$

The annual savings is therefore 1975 – 1325 = 650 L. At $1.06/L, the value is $689/year. Therefore, the breakeven is:

$$\frac{\$4000}{\$689} =\sim 5.8yr$$

Part (b): With an expectation of 5% return, it is necessary to discount 10 years of savings back to the present. This is done using a spreadsheet function for the factor for converting 10 annuities at 5% into a present value:

$$(P/A,5\%,10yr) = -PV(5\%,10yr,\$1) = 7.72$$
$$PV = (7.72)(\$689/yr) = \$5320$$
$$NPV = C_{Fuel} - C_{Vehicle} = \$5320 - \$4000 = \$1320$$

Part (c): This problem can best be solved by using a table in which the price of fuel is inflated by 3% each year. The savings can then be discounted back to the present.

Year	Fuel Cost	Face value	Discounted	NPV
	($/L)		5%	$ (4000)
1	$1.06	$689	$656	$ (3344)
2	$1.09	$710	$644	$ (2700)
3	$1.12	$731	$631	$ (2069)
4	$1.16	$753	$619	$ (1449)
5	$1.19	$775	$608	$ (842)
6	$1.23	$799	$596	$ (246)
7	$1.27	$823	$585	$339
8	$1.30	$847	$574	$913
9	$1.34	$873	$563	$1475

10	$1.38	$899	$552	$2027

Thus at the end of 10 years the NPV is $2027. Note that the increase from NPV = $1320 in part (b) is due to the rising fuel cost.

Problem 14-7.

Note to instructor: Some additional background on price elasticity of demand may be necessary to support this problem.

In this problem we consider the use of price elasticity of demand and observed values from travel behavior to estimate the impact of a change in fuel prices on levels of demand for travel. By extension, this exercise could be applied not only to changes in prices of conventional fuels, but to the impact on demand of the change in price when transitioning to an alternative fuel. The price elasticity is defined as the units of change in demand in response to one unit of change in price, e.g., such as a 1% change. Let ΔP be the change in price, e_p = the price elasticity of demand, and ΔD be the change in demand. Then the following holds:

$$e_p = \frac{\Delta D}{\Delta P}$$

$$\Delta P \cdot e_p = \Delta D$$

Consider the table of fuel price elasticities below (Source: TRACE, 1999). Suppose that in a sample of 1000 commuting trips, there is the breakdown by mode and distance traveled given in the table below:

Mode	**Number of Trips**	**Distance Traveled (km)**
Car driver	780	11,700
Car passenger	100	2000
Public transit	120	1000
Total	1000	14,700

Fuel price demand elasticities: Number of trips

Purpose	Own price elasticity	Public transit cross-elasticity
Commuting	−0.11	+0.20
Business	−0.04	+0.24
Education	−0.18	+0.01
Other	−0.25	+0.15
Total	−0.19	+0.13

Fuel price demand elasticities: kilometers traveled

Purpose	Own price elasticity	Public transit cross-elasticity
Commuting	−0.20	+0.22
Business	−0.22	+0.05

Education	−0.32	+0.00
Other	−0.44	+0.18
Total	−0.29	+0.14

a. Using the elasticities in the table, estimate the number of trips and total distance traveled for car drivers and transit passengers engaged in commuting trips after a fuel price increase from an average of \$3.00 per gallon to \$3.60 per gallon.

<u>Solution.</u>

Note that not all of the data points in the tables are used. The fuel price change of \$0.60 is a 20% increase. Using the elasticities in the table, the elasticity of commuting car trips with respect to fuel price is −0.11, so we would expect a 20(0.11) = 2.2% decrease in the number of car driver trips. This implies a reduction of .022(780) = 17, to 763. The total distance traveled by the car drivers for commuting has an elasticity of −0.2, so it should decrease by 20(0.2) = 4%, to 11,232.

For transit passengers, the respective elasticity values are +0.20 and +0.22. Therefore, we expect a 20(0.20) = 4% increase in trips. This computes to 4.8, and we'll round it to 5. Thus, the resulting total transit trip number is 125. The total distance by transit increases by 20(0.22) = 4.4%, to 1044 km.

b. What is the change in the average trip length for car drivers and transit passengers? What does this imply about which commuters are switching modes?

<u>Solution.</u>

Before the change, the average trip length for car drivers was $\frac{11,700}{780} = 15$ km. After the change, the average trip length is $\frac{11,232}{763} = 14.7$ km. For transit passengers, the average trip length increases, but only slightly, from $\frac{1000}{120} = 8.33$ km. to $\frac{1044}{125} = 8.35$ km.

The answer to the implications part of this question is open-ended and various answers are also possible. One possibility is that the average trip length by transit is considerably shorter than for car drivers. The fuel price change causes some drivers to switch to transit, particularly those making longer-than-average commuting trips (which are more fuel-intensive). This reduces the average trip length by car, and increases it for transit. The overall change for transit is quite small, because there are only five new riders. The difference in the number of people who no longer drive (17) and the number who switch to transit (5) must be explained as either people who no longer commute (e.g., become telecommuters) or who become car passengers by joining carpools. The fact that the average trip length by car drivers decreases more than the trip length by transit increases indicates that some of car drivers with the longest commutes are probably switching to carpools or telecommuting.

Chapter 15 Electricity and Hydrogen as Alternative Fuels

Problem 15-1.

A future midsize EV has a production cost and weight, before adding any batteries, of \$18,000 and 750 kg, respectively. The batteries are available with a charge density of 120 Wh/kg and a cost of \$350/kWh. The energy intensity of the vehicle is 0.11 Wh/kg·km, and the maximum depth of discharge is 90%. (a) Produce two figures for this vehicle, one plotting vehicle range as a function of total mass, and the other plotting total cost as a function of range. (b) Compare this vehicle to an ICEV in the same class that costs \$30,000 and has a typical range per tank of 350 km. What would the EV cost if it were to have the same range? By what percent would the range be reduced compared to the ICEV if it had the same cost?

Solution.

Part (a): From Chap. 15, the formula for range as a function of weight is:

$$R = \frac{CD \cdot DD_{\max} \cdot W_{\text{battery}}}{\mu\left(W_{\text{vehicle}} + W_{\text{battery}}\right)}$$

The values for use in the equation are CD = 85 Wh/kg, μ = 0.11 Wh/kg km, $DD_{\max}$ = 0.9, and W_{vehicle} = 750 kg. The figure for range as a function of total vehicle mass comes from this formula, as shown in the figures:

Example calculation for W_{battery} = 400 kg:

$$R = \frac{120 Wh/kg \cdot 0.9 \cdot 400 kg}{0.11 Wh/kg.km\left(750 + 400 kg\right)} = 341.5\,\text{km}$$

To calculate total vehicle cost, the cost of the batteries can be added to the fixed cost of the vehicle of \$18,000 before adding batteries. In order to calculate the total cost, battery cost must be converted from \$/kWh to \$/kg, as follows:

$$\text{Cost/kg} = \$350/\text{kWh} \cdot \left(\frac{1\text{ kWh}}{1000\text{ Wh}}\right) \cdot 120\text{ Wh/kg} = \$42/\text{kg}$$

The charge density of 120 Wh/kg and cost of \$350/kWh is equivalent to \$42/kg, therefore:

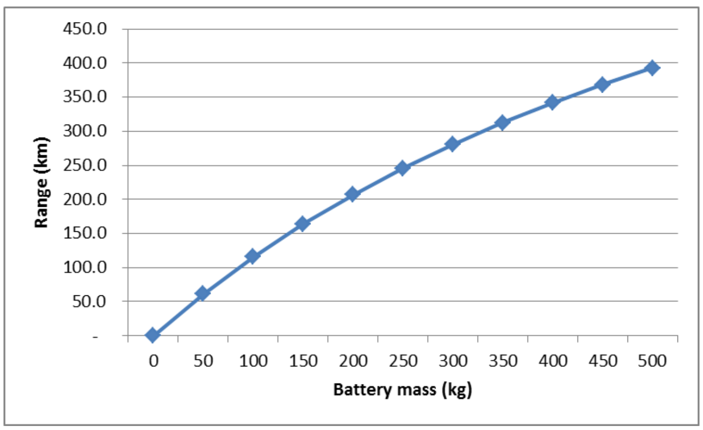
$$\begin{aligned} TC &= C_{\text{fixed}} + W_{\text{battery}} C_{\text{battery}} = \$18{,}000 + \left(400\text{kg}\right)\left(\$42/\text{kg}\right) \\ &= \$18{,}000 + \$16{,}800 \\ &= \$34{,}800 \end{aligned}$$

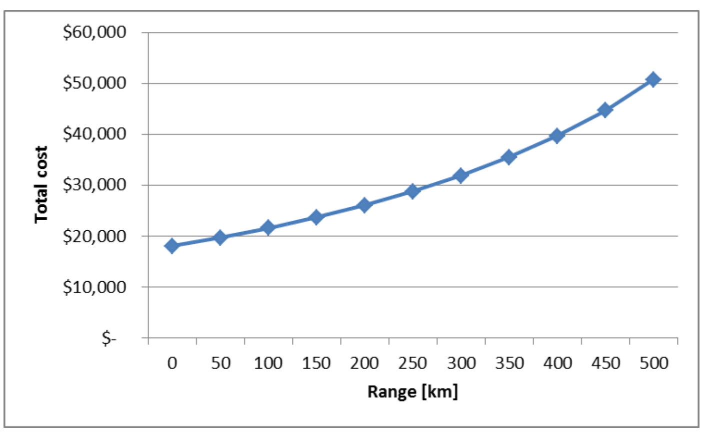
For purposes of graphing, TC can be written in terms of the other variables as follows:

$$TC = C_{\text{fixed}} + \frac{R \cdot \mu \cdot W_{\text{vehicle}}}{\left(CD - R \cdot \mu\right)} C_{\text{battery}}$$

The relationship between mass, range, and cost can now be presented in figures based on the following table:

Battery	Range	Range	TC
[kg]	[km]	[km]	[$]
0	-	0	$18,000
50	61.4	50	$19,690
100	115.5	100	$21,572
150	163.6	150	$23,680
200	206.7	200	$26,058
250	245.5	250	$28,761
300	280.5	300	$31,860
350	312.4	350	$35,450
400	341.5	400	$39,656
450	368.2	450	$44,654
500	392.7	500	$50,689





Part (b): The cost of the EV with a range of 400 km is:

$$TC = \$18{,}000 + \frac{400 \cdot 0.11 \cdot 750}{(120 * 0.9 - 400 \cdot .11)}(\$42/\mathrm{kg})$$
$$= \$18{,}000 + \$21{,}656 = \$39{,}656$$

For the second part of the question, for $30,000, an expenditure of $30,000 – $18,000 = $12,000 can be expended on batteries. This is equivalent to ($12,000/$42) = 286 kg of batteries. Solving for range as a function of battery weight gives:

$$R = \frac{120 \cdot 0.9 \cdot 286}{0.11(750 + 286)} = 270.8\,\mathrm{km}$$

This is equivalent to a 23% reduction in range compared to the ICEV.

Note to instructor: This exercise provides a useful example for comparing an idealized EV of the future that achieves a very desirable combination of total curb weight, range per charge, and overall cost to the EVs of today, as the price is similar to that of the Nissan Leaf, which has 130 to 160 km per charge range, and the range is similar to that of the Tesla Model S, which has a price in the range of $70,000 to$100,000.

Problem 15-2.

A fleet manager is considering replacing a fleet of gasoline ICEVs with equivalent EVs. One factor in the decision is the CO_2 emissions from the vehicles. Each vehicle in the fleet drives an average of 10,000 mi/year, and the ICEVs have an average fuel economy of 20 mpg. The comparable EV consumes 274 Wh of electricity per mile. Use U.S. national average emissions factor from Table 15-2. You can assume that upstream CO_2 emissions (i.e., either from extracting petroleum and converting it to gasoline provided at the pump, or from extracting and transporting coal or gas to the electric plant) are even between the two options and that therefore they are outside the scope of your analysis. (a) How many pounds of CO_2 does each vehicle emit per year? (b) Does the EV reduce CO_2 emissions? (c) Repeat the calculation for your own location, based on local emissions per kWh of electricity. Does the result in (b) change?

<u>Solution.</u>

Parts (a) and (b) only are solved here, since part (c) depends on local emissions values. For the ICEV, 10,000 mi/year @ 20 mpg requires 500 gal of gasoline consumption per year. Therefore, using a figure of 19.5 lb CO_2 per gallon of gasoline consumed gives:

$$(10{,}000\,\mathrm{mi/yr})(19.5\,\mathrm{lbCO_2/mi}) = 9765\,\mathrm{lbCO_2/yr}$$

This amount is equivalent to 4493 kg in metric units. For the EV, from the electricity production mix data, the following average emissions value can be calculated based on total emissions and total electricity production data from Table 15-2:

Source	**Output**	**Total Emissions**		**Emission Rate**	
	Bil.kWh	Mil.tonnes	Mil.std.tons	$kgCO_2$/kWh	$lbCO_2$/kWh
Coal	1734	1723	1895	0.994	2.186

Gas	1017	409	450	0.402	0.885
Oil	28	27	30	0.959	2.109
Nuclear	790	-	-	-	-
Hydro	325	-	-	-	-
Solar	2	-	-	-	-
Wind	120	-	-	-	-
Other RE*	90	-	-	-	-
Total	4106	2159	2375	0.526	1.157

$$\text{Emit rate} = \frac{\text{Total emits}}{\text{Total output}} = \frac{2.159 \times 10^{12}\,\text{kg}}{4.106 \times 10^{12}\,\text{kWh}} = 0.526\frac{\text{kg}}{\text{kWh}}$$

The total emissions per year are then:

$$\left(10{,}000\frac{mi}{yr}\right)\left(0.274\frac{kWh}{mi}\right)\left(0.526\frac{kg}{kWh}\right) = 1441 kg / yr$$

Converting to U.S. customary units gives 3170 lb per year.

Part (b): Yes, the ICEV reduces emissions substantially, by either 9765 – 3710 = 6,595 lb in standard units, or 2998 kg in metric units.

Discussion: Note that the comparison here benefits both from a very efficient EV at 0.274 kWh/mi, and from a large share of zero-carbon electricity in the grid mix. Changing either of these inputs could well reduce the EV's advantage.

Problem 15-3.

You are considering two alternatives, either an ICEV, which includes initial cost of the car and gasoline cost over its lifetime, or an EV, which includes the cost of the car without batteries, the cost of the batteries, and the cost of electricity. For either option, assume a discount rate of 7%, that any capital investment in cars or battery systems will have $0 salvage value at the end of their investment lifetime, that vehicles are owned for 10 years, and that the demand is 8000 mi/year. Ignore maintenance, insurance, and other ownership costs. For the EV, assume a battery life of 40,000 mi, and a required available capacity of 25 kWh. The maximum depth of discharge is 85%. The car without the battery system costs $18,000. Electricity costs $0.16/kWh, and the EV drives 2.7 mi/kWh consumed. The charging efficiency is 90%, meaning that there are 10% losses between the amount of electricity purchased and delivered to the battery system. For the ICEV, the purchase cost is $25,000, the fuel economy is 25 mpg, and the cost of gas is $4/gal. How much does the battery have to cost per kWh of storage capacity to make the EV cost the same as the ICEV?

Solution.

Part (a): The approach to the solution is to calculate the annual cost of the ICEV and use it as a benchmark against which to compare. The annual capital cost *ACC* and fuel cost C_{fuel} are:

$$ACC = -PMT(7\%,10,\$25{,}000) = \$3559$$

$$C_{fuel} = \left(8000 mi/yr\right)\left(\frac{\$4/gal}{25mpg}\right) = \left(320 gal/yr\right)\left(\$4\right) = \$1280/yr$$

Total cost is therefore \$3559 + \$1280 = \$4839/yr.

Next, the total cost per year of the EV not including the battery system is calculated:

$$ACC = -PMT(7\%,10,\$18{,}000) = \$2563$$

$$C_{energy} = \left(8000 mi/yr\right)\left(\frac{\$0.16/kWh}{(0.9)(2.7mi/kWh)}\right) = \left(3292 kWh/yr\right)\left(\$0.16\right) = \$527$$

Total cost is therefore \$2563 + \$527 = \$3089/yr.

Thus the difference between EV and ICEV, which is the amount that can be spent per year on the battery system, is \$4839 − \$3089 = \$1750. This amount can be converted to a present worth *PW* which can be spent initially:

$$PW = -PV(7\%,10,\$1750) = \$12{,}291$$

This amount must cover two capital investments for the battery system, since the lifetime of the vehicle is 80,000 mi over 10 years but the batteries last 40,000 mi or 5 years. The discount factor for the present worth of the investment in year 6 for the battery system for years 6 to 10 at 7% discount rate over 5 years is:

PW = −PV(7%,5,FW=\$1.00) = \$0.7130

Therefore the amount available to spend in year 0 on the battery system is:

$$\frac{\$12{,}291}{1+0.713} = \$7175$$

Because the maximum allowable depth of discharge is 85%, the rated size of the battery system is somewhat larger than the required energy discharge of 25 kWh, as follows:

$$Size = \frac{25kWh}{0.85} = 29.4kWh$$

Finally, the target cost per kWh for the system is the available amount divided by size:

$$Cost/kWh = \frac{\$7175}{29.4kWh} = \$244/kWh$$

Problem 15-4.

A fuel cell stack consists of 75 fuel cells arranged in series, each one 12 cm^2 in size. The stack operates at 80°C and the amount of current generated is 4.257 A/cm^2. The fuel cell operates at a system pressure of 1 MPa using a supply of pure hydrogen and oxygen in ambient air. The partial pressure of the water in the outflow is 0.5 MPa. Assume any losses to be negligible and that the voltage at which it operates is the open circuit voltage; in reality, the voltage would be lower due to the current flow. What is the output of the fuel cell stack when operating at full capacity, in kW?

Solution:

Voltage can be calculated from the Nernst equation:

$$E = E^0 + \frac{RT}{2F}\ln\left(\frac{\alpha\beta^{0.5}}{\delta}\right) + \frac{RT}{4F}\ln(P)$$

From the given data, $\alpha = 1$, β is taken to be the standard value for oxygen in air or $\beta = 0.21$, and δ is the ratio of partial pressure to system pressure or $\delta = 0.5$ MPa/1 MPa = 0.5. Also, P must be converted to bars, that is, $P = 10$ bars.

Plugging in the appropriate values in the equation, that is, $E^0 = 1.18$ V (at 80°C), $R = 8.314$, $T = 353$ K, and $F = 9.64 \times 10^4$, we get

$$E = 1.18 - 0.00133 + 0.0175 = 1.196\,\text{V}$$

Multiplying by current per unit of area gives power $P = (1.196\text{ V})(4.257\text{ A/cm}^2) = 5.09\text{ W/cm}^2$. Multiplying further by the area of each cell gives $P = (5.09\text{ W/cm}^2)(12\text{ cm}^2) = 61.1$ W. Lastly, multiplying by the number of cells gives $P = (61.1\text{ W})(75) = 4.57$ kW.

Problem 15-5.

Problem 5 An electric (EV) vehicle has a battery system with rated capacity of 24 kWh and maximum depth-of-discharge of 80%. For comparison, a hatchback ICEV has a 12-gal tank and averages 31 mpg overall fuel economy. (a) Suppose the system is discharged to its maximum depth and I wish to fully recharge this system in 20 minutes, with a 480-V charger. Assuming the battery is charged at a constant rate from start to finish (i.e., linearly, this is a simplification since batteries actually have a variable charging rate as the battery approaches fully charged), what is the energy flux in kW and the current in amperes? (b) Now suppose instead the battery charges at typical household wall outlet conditions, 20-A current at 120 V. How long does it take to charge in the same situation? (c) Next, suppose the hatchback starts on a full tank, drives 285 mi, and then stops to refuel. If the gas station pump averages 5 gal/min, how long does it take to fill up the tank? Give your answer to the nearest tenth of a minute. D) Short answer: What conclusion could you draw in comparing the refueling of EVs versus ICEVs from the answers in parts (a) to (c)? Comment in one or two sentences.

Solution.

Part (a): The battery system discharges 80% so the total remaining to be charged is 19.2 kWh. Since the amount of time is 20 minutes or 0.33 hour, the rate of charge is 57.6 kW. Therefore, current is the power divided by the voltage, or 57,600480 = 120 A.

Part (b): The current is now 20 A and the voltage 120 V, so the rate is 2400 W = 2.4 kW. Therefore, dividing 19.2 kWh by 2.4 kW rate gives 8.33 hours.

Part (c): Since the vehicle has driven 285 mi with fuel economy of 31 mpg, fuel consumption is 9.2 gal. At 5 gal/min this amount takes 1.8 minutes to refill.

Part (d): The problem is that the user can either wait a long time to recharge or spend a large sum of money on a fast, expensive recharger, but either way the experience does not approach that of the gasoline vehicle. With the most basic charger, the recharge takes 8 hours instead of 2 minutes. With the expensive charger and 20-minute charging time, the current is potentially dangerous at 120 A.

Problem 15-6.

Consider total CO_2 emissions from the PHEV car in Example 15-2, compared to an HEV which is identical except that it uses only gasoline. The CO_2 emissions from electricity generation are the same as in Table 15-2. Assume 88% well-to-tank efficiency for gasoline usage, 11% losses in transmission and distribution of electricity from the generating plants to the vehicle, and 5% average energy losses upstream from the various electric power plants. Furthermore, assume that CO_2 emissions are proportional to energy losses. (a) Does the PHEV reduce energy consumption or CO_2 emissions compared to the HEV? (b) If the electricity is generated one-third from gas and two-thirds from other fossil fuels, does the answer in (a) change?

Solution.

Part (a): From exercise 15-2 above, the emissions from electricity generation is 0.526 $kgCO_2$ per kWh. From Example 15-2, the total electricity consumption for the PHEV is 98.2 kWh of electricity, so the total emissions are

$$(98.2\,\text{kWh})\left(0.526\frac{\text{kgCO}_2}{\text{kWh}}\right) = 51.65\,\text{kgCO}_2$$

Assuming 2.35 $kgCO_2$ generated per liter of fuel consumed (a typical figure), and 27 L of fuel consumed as given in the example, the PHEV emissions from gasoline consumption are:

$$(27.0\,\text{L})\left(2.35\frac{\text{kgCO}_2}{\text{L}}\right) = 63.1\,\text{kgCO}_2$$

$$\text{Emit}_{\text{Tot}} = 25.2 + 63.1 = 88.3\,\text{kgCO}_2$$

For the HEV, total fuel consumption is 66.2 L, so total emissions are:

$$(66.2\,\text{L})\left(2.35\frac{\text{kgCO}_2}{\text{L}}\right) = 155.3\,\text{kgCO}_2$$

Answer: The PHEV reduces CO_2 emissions compared to the HEV.

Part (b): Emissions from the HEV for part (b) do not change. The new emission rate *e* for the electricity used by the PHEV is calculated using data from Chap. 7, as follows. Since two-thirds of the electricity comes from fossil fuels other than gas, according to the exercise, these emissions are assumed to come from coal, and any potential contribution from oil-generated electricity is ignored. Emissions for gas and coal are 0.325 and 0.850 $kgCO_2$ per kWh, respectively. Calculation of *e* gives:

$$e = \left(\frac{1}{3}\right)(0.325) + \left(\frac{2}{3}\right)(0.850) = 0.675$$

Recalculating emissions from electric operation of the PHEV gives 66.3 $kgCO_2$ total, and combined emissions from both electric and gas of

$$\text{Emit}_{\text{Tot}} = 66.3 + 63.1 = 129.4 \text{ kgCO}_2$$

<u>Discussion:</u> This exercise could be adapted in a number of ways. Instructors might include upstream emissions from the petroleum and electricity supply chains, or they might change the efficiency values of HEVs and PHEVs to reflect different vehicles, or they might change the fuel mix for electricity generation, among other possibilities.

Problem 15-7.

Plot efficiency limit as a function of temperature Celsius for a fuel cell and a Carnot heat engine, as follows. For the fuel cell, use the data in the chapter for the efficiency limit at 80°C and 1000°C to plot a line as a function of temperature. For the Carnot engine, assume that the engine exhausts to a low-temperature reservoir at ambient temp (20°C). According to your plot, what is the predicted "cross-over" temperature above which the Carnot engine has a higher efficiency limit than the fuel cell?

<u>Solution.</u>

The figure is shown above. From observation, the cross-over temperature is approximately 600 ºC, or 873 K. The solution can also be found algebraically, as follows. The formula for the line that gives the efficiency η of the fuel cell as a function of temperature T is the following:

$$\eta = (-0.0001837)T + 0.824$$

For the Carnot engine, if we take the outlet temperature to be T_0 = 293 K, efficiency is calculated as follows:

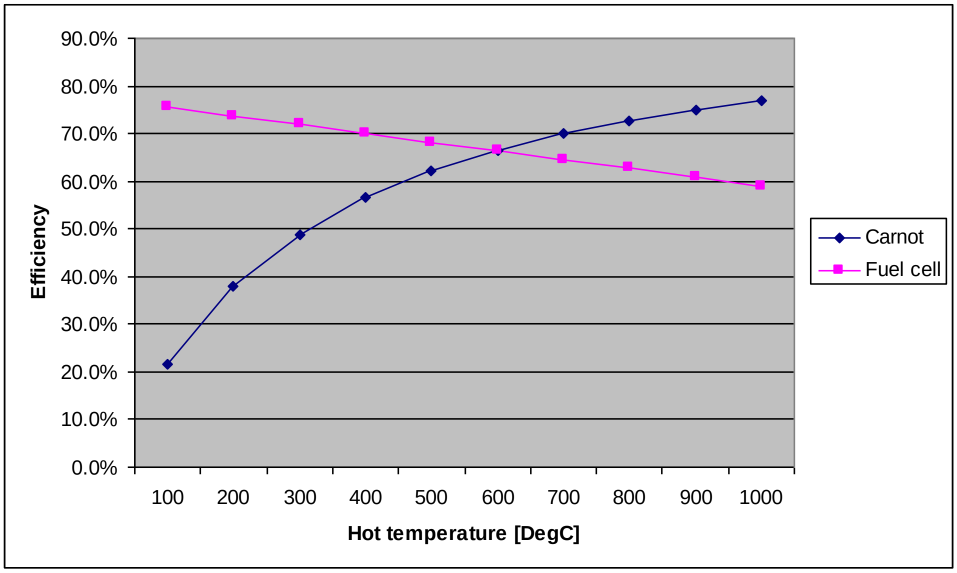
$$\eta = \frac{\Delta T}{T} = \frac{T - T_0}{T}$$

At breakeven, the two will be equal, which we can rearrange to solve for T:

$$\frac{T - T_0}{T} = (-0.0001837)T + 0.824$$

$$(-0.0001837)T^2 + (0.824 - 1)T + T_0 = 0$$

Solving for T gives 871.4 K or 598.4 ºC = ~600 ºC



Problem 15-8.

Compare the following two luxury cars: a Tesla Model S (sport sedan) with 60 kWh battery capacity that has an estimated distance per unit of charge of 2.86 mi/kWh, and a BMW 750i that has an overall nominal fuel economy of 19 mi/h, both according to the USEPA. One kWh is equivalent to 3412 Btu, in terms of energy content, and a gallon of gasoline has an energy content of 115,400 Btu.

a. What is the nominal equivalent fuel economy of the Tesla in mpgge (miles per gallon of gasoline equivalent)?
b. What is the ratio of nominal mpg for the Tesla compared to the BMW (in other words, the ratio of the Tesla value to the BMW value)?
c. Now consider the effect of upstream energy losses: the electricity for the Tesla is generated by an upstream process that is 40% efficient in turning energy in fossil fuel in the ground into electric output from the generating plant, and the grid is 90% efficient in delivering the current to the battery system inside the car. The gasoline delivery system has an 88% well-to-tank efficiency. Calculate the effective mpg for each vehicle, and the ratio of Tesla to BMW.

Solution.

Part (a): Convert to mpgge as follows:

$$(2.86 mi / kWh)\left(\frac{1 kWh}{3412 Btu}\right)\left(\frac{115{,}400 Btu}{1 gal}\right) = 96.7 mpgge$$

Part (b): Ratio is therefore (96.7)/(19) = 5.1, so the ratio is 5.1:1.

Part (c) Effective fuel economy for the Tesla and BMW:

$$FE = (96.7)(0.4)(0.9) = 34.8 mpg$$
$$FE = (19)(0.88) = 16.7 mpg$$

Thus the new ratio is (34.8)/(16.7) = 2.1, or 2.1:1.

Problem 15-9.

Repeat Problem 15-6, but with electricity generated using the 2010 Oregon state average generation mix, as given in table below. Use the following figures: distance per year of 10,000 mi; for ICEVs, 22 mpg average fuel economy; for EVs, 274 Wh of electricity per mile; for the grid, average 6% losses in electricity transmission and 10% in distribution; for the ICEVs, well-to-tank (WTT) efficiency of 88% for estimating upstream CO_2 emissions.

Oregon electricity mix and emissions, 2010:

Source	$kgCO_2/MWh$	Share
Natural gas	325	16.24%
Coal	850	35.46%
Petroleum	661	0.17%
Hydropower	0	38.74%
Geothermal	0	0.12%
Landfill gas	0	0.04%
Waste-to-energy	0	0.34%
Wind	0	4.31%
Biomass	0	0.77%
Other renewables	0	0.14%

Solution.

For the ICEV, 10,000 mi/year @ 22 mpg requires 455 gal of gasoline consumption per year. Therefore, using a figure of 19.5 $lbCO_2$ per gallon of gasoline consumed gives:

$$(455 \text{gal/yr})(19.5 \text{lbCO}_2/\text{gal})\frac{1}{0.88} = 10072 \text{lbsCO}_2/\text{yr}$$

For the EV, from the electricity production mix data, the following average emissions value can be calculated:

Source	$kgCO_2/MWh$	Share	Emit × shr
Natural gas	325	16.24%	52.78
Coal	850	35.46%	301.41

Petroleum	661	0.17%	1.1237
Hydropower	0	38.74%	0
Geothermal	0	0.12%	0
Landfill gas	0	0.04%	0
Waste-to-energy	0	0.34%	0
Wind	0	4.31%	0
Biomass	0	0.77%	0
Other RE*	0	0.14%	0
TOTAL			355.31

In other words, the overall emissions are a weighted average across sources *i*:

$$\text{Emit rate} = \sum_i s_i \mu_i = 355.3 \frac{\text{kg}}{\text{MWh}}$$

This amount converts to 782 lb/MWh or 0.782 lb/kWh. The total emissions per year taking into account 94% transmission and 90% distribution efficiency are then:

$$\left(10{,}000\frac{mi}{yr}\right)\left(0.274\frac{kWh}{mi}\right)\left(0.782\frac{lb}{kWh}\right)\left(\frac{1}{0.94}\right)\left(\frac{1}{0.9}\right) = 2532 lb/yr$$

Conclusion: Since EV emissions per year are lower, the operator would be encouraged to shift to EVs, other factors equal.

Problem 15-10.

You are to compare two vehicles in terms of their total cost per mile, one conventional ICE vehicle, and one electric vehicle. The purchase cost of the respective vehicles is \$20K and \$33K respectively, the higher price for the EV due to the high cost of the battery pack.

Since the comparison involves an EV, the assumed miles driven per year is 6000 mi, lower than that of a typical ICEV (~9500 mi/year). Gasoline for the ICEV costs \$4/gal, and the consumer is willing to pay a premium price for 100% CO_2-free electricity from renewables of \$0.24/kWh.

In terms of energy consumption, the ICEV gets 22-mpg city, 32-mpg highway, and 70% of the miles are driven in city conditions. The EV travels 4 mi/kWh of charge consumed. For costs other than purchase an energy (maintenance, insurance, parking, etc.) there is an additional charge of \$0.20/mi and \$0.18/mi for the ICEV and EV respectively; the lower cost per mile for the EV is due to the mechanically simpler drivetrain requiring fewer repairs, no oil changes, etc.

The investment lifetime for the two vehicles is 10 years, at which point they each retain 40% of their original value, since they have not been driven all that far over the 10-year lifetime. You will need to use discounting to annualize both the purchase cost and salvage value. Since the consumer in question favors the greener technology, they are willing to adopt a low discount rate of 4%.

a. What is the energy cost per mile for the two vehicles?
b. What is the total cost per mile for the two vehicles, including capital, energy, and other costs?
c. Now suppose a carbon tax is introduced to incorporate the externality cost of emitting CO_2 to the atmosphere. What level of tax must be set in dollars per ton of carbon per year (use U.S. customary tons, i.e., 2000 lb per ton) in order for the two vehicles to have equal total cost?Ignore upstream (also known as well-to-tank) CO_2 emissions. Hint: you can use the figure of 19.6 lbCO_2 per gallon of gasoline for this calculation.

Solution.

Part (a): Cost per mile for the ICEV takes into account the mix of city and highway driving can be calculated in two ways, one of which is as follows:

$$MPG = (22)[0.7] + (32)(0.3) = 25mpg$$
$$Cost = \frac{1mi}{25mpg}(\$4/gal) = \$0.16/mi$$

There is an alternative solution that is also acceptable: Suppose the car drives 10,000 mi. According to the ratio, 7000 would be city and the rest highway. Total fuel consumption would therefore be:

$$\frac{7000}{22} + \frac{3000}{32} = 411.9gal$$

Cost per mile is then:

$$(411.9gal)(\$4/gal)\left(\frac{1yr1}{10000mi}\right) = \$0.1648/mi$$

Thus either \$0.16/mi or \$0.1648/mi could be accepted, although \$0.16 is used hereafter. Energy cost for EV:

$$Cost = \frac{1kWh}{4mi/kWh}(\$0.24/kWh) = \$0.06/kWh$$

Part (b): Since the vehicles retain 40% of their original value after 10 years, the remaining value is \$8000 and \$13,200, respectively. The annualized cost is therefore calculated as follows:

$$ICEV = (20K)(A/P,4\%,10) - (8K)(A/F.4\%,10) = \$1799.49$$
$$EV = (33K)(A/P,4\%,10) - (13.2K)(A/F.4\%,10) = \$2969.16$$

The fuel and non-fuel operating cost can then be calculated by multiplying cost per mile by 6000 mi driven per year:

$$ICEV = (\$0.16 + \$0.20)(6000) = \$2160/yr$$
$$EV = (\$0.06 + \$0.18)(6000) = \$1440/yr$$

Overall cost per mile is then calculated by dividing total annual cost by total miles:

$$ICEV = \frac{\$1799.49 + \$2160}{6000} = \$0.66 / mi$$
$$EV = \frac{\$2969.16 + \$1440}{6000} = \$0.73 / mi$$

Part (c): Since the ICEV uses 0.04 gal/mi and emits (0.04 gal/mi)(19.6 lb/gal) = 0.784 $lbCO_2$ per mile, and the cost difference is \$0.07 per mile, the markup should be:

$$\frac{\$0.07}{0.784lb} = \$0.0893 / lbCO_2$$
$$\frac{\$0.0893}{lbCO_2}\left(\frac{2000lb}{ton}\right) =\sim \$179 / tonCO_2$$

Problem 15-11.

An electric vehicle with the following characteristics is proposed for use in a V2G system: battery capacity of 30 kWh, maximum depth of discharge of 90%, expected life of 150,000 1-kWh cycles, and a capital cost of \$330/kWh battery capacity. The vehicle also requires a capital investment of \$1800 for all equipment necessary to participate in V2G. Both pieces of capital equipment should be valued annually using a 10-year lifetime and a 7% discount rate. The grid pays the vehicle owner \$0.14/kWh for energy transmitted to the grid but production in the vehicle costs \$0.14/kWh for purchase from the grid, plus the added value of conversion losses, plus degradation of the battery system. Conversion from DC current out of the battery to AC current into the grid is 80% efficient. The owner is paid for standby capacity at the rate of \$0.04/kWh for maximum power flow of 18 kW and time duration of 18 hours per day. Actual power dispatched to the grid averages 12 kWh/day. Calculate expected annual net revenue.

Solution.

The required discounting factor used in the calculations is (A/P, 7%, 10) = \$0.1424. The capital cost of the battery system is (\$330)(30) = \$9900. Therefore, the two capital costs for battery and control system are:

$$(\$9900)(0.1424) = \$1410$$
$$(\$1800)(0.1627) = \$256$$

Assuming the battery lasts 10 years, on average 15,000 cycles could be expected per year, so the cost per cycle of wearing out the battery is (\$1410)/(15000) = \$0.094/kWh. The capital cost of the control system of \$256/year is incurred regardless of the number of kWh sold to V2G.

The earnings from standby capacity due to V2G participation are based on a maximum capacity of 18 kW and time of 18 hours per day:

$$(18kW)(18h / day)(365days / yr)(\$0.04 / kWh) = \$4730$$

Earnings from electricity sold amount to:

$$(12 kWh/day)(365 day/yr)(\$0.14/kWh) = \$613$$

Thus the gross revenue per year is $4730 + $613 = $5343. Total annual cost is based on (12 kWh/day)(365 day/year) = 4380 kWh cycled through the battery and sold to V2G. The cost of this energy is $613/year per the above calculation, plus the value of losses:

$$E_{in} = \frac{E_{out}}{\eta} = \frac{4380}{0.8} = 5475 kWh$$
$$C_{in} = (5475)(\$0.14/kWh) = \$766$$

The total cost per year for degradation of the battery is then:

$$(5475)(\$0.094/kWh) = \$515$$

Combining and including the annual capital cost of the V2G equipment gives:

$$\$766 + \$515 + \$256 = \$1537$$

The difference is then the net revenue from V2G participation of $5343 − $1537 = $3806. Thus V2G is a net financial gain for the vehicle owner.

Problem 15-12.

Note to instructor: Problems 15-12 and 15-13 require additional background that can be obtained from Kempton and Tomic (2005a and 2005b, see references in Chap.15). In particular, the student should become familiar with the following variables: R_{d-c}, dispatch to contract ratio; t_{plug}, time that vehicle is plugged in to the V2G network; p_{cap}, price paid for capacity; p_{el}, price paid for electricity; P, power in kW; η_{conv}, conversion efficiency of inverter, c_{pe}, cost of purchased electricity; c_{deg}, cost of degradation of battery system in \$/kWh transmitted, E_{disp}, energy dispatched in kWh; c_{ac}, annualized capital cost of ancillary equipment required to participate in V2G, such as inverter and charge controller for dispatching electricity to the grid.

A vehicle owner is considering improving the financial return on their PHEV by making it available to the V2G system, which involves upgrading the electrical supply at their home and workplace to accommodate 16 kW of energy flux in current to/from the PHEV, and entering into a contract to provide both regulation and energy services to the independent system operator. For all electricity sales to/from the vehicle, assume a unit cost of $0.16/kWh. Assume the inverter for converting charge from the PHEV battery to AC current for the grid is 93% efficient.

The terms of the contract specify that, on the regulation side, the vehicle will be available 16 hours per day. For regulation services, the price paid for 1 kW of regulation capacity for 1 hour is $0.05/kWh. On the energy sales side, the dispatch-to-contract ratio R_{d-c} is 0.13.

The annualized capital cost of all additional equipment needed to join V2G is $356 per year. The round-trip efficiency of taking charge from the grid and returning it to the grid is 76%. The degradation charge for using the batteries is $0.04 per kWh of electricity stored.

Calculate the net revenue or cost, and indicate whether the V2G option is attractive or not to the owner.

Solution.

On the revenue side, the vehicle is connected 16 hours per day for 365 days per year or 5840 h/year, so that the total revenue is:

$$
\begin{aligned}
r &= Pt_{plug}\left(p_{cap} + p_{el}R_{d-c}\right) \\
&= (16kW)(5840h)(\$0.05 + \$0.16 \cdot 0.13) \\
&= \$6616
\end{aligned}
$$

On the cost side, the total amount of electricity is the power of 16 kW multiplied by the time of 5840 h/year and the dispatch to contract ratio of 13%, or 12,147 kWh/year. The total cost payment per year is therefore:

$$
\begin{aligned}
c &= \left(\frac{c_{pe}}{\eta_{conv}} + c_{\deg}\right)E_{disp} + c_{ac} \\
&= \left(\frac{0.16}{0.76} + 0.04\right)12147 + 356 \\
&= \$3399 / yr
\end{aligned}
$$

The net revenue per year is the difference between total revenue and total cost, or \$6616 − \$3399 = \$3216. Therefore, the V2G arrangement would be attractive to the PHEV owner.

Problem 15-13.

Note to instructor: See exercise 15-12.

A Nissan Leaf with rate of energy consumption of 2.94 mi/kWh of charge enters into a V2G contract with a maximum power of 18 kW. The vehicle has a capacity of 24 kWh (but 20% of the capacity cannot be accessed because of limits on depth of discharge, which has a maximum of 80%), and a typical daily driving distance of 22 mi. On average, it is expected to be plugged in 16 hours per day, thanks to charge points at both home and workplace. The inverter efficiency is 93%. The owner requires a reserve driving capability of 12 mi. The price paid for capacity is \$0.05/kWh, and the price of electricity both into and out of the vehicle is \$0.14/kWh. The capital cost for upgrading to V2G is \$2100, to be discounted over 10 years at 10%. The battery system costs \$11,000 and is expected to store and discharge 142,000 kWh over its lifetime. The round-trip efficiency of the system is 76% and the dispatch-to-contract ratio is 0.08. Calculate (a) the number of minutes that the system can discharge from full charge before reaching the minimum amount of charge it must retain to meet customer requirements, (b) the annual revenues, (c) the annual cost, and (d) the net revenue.

Solution.

Part (a): The length of time that the vehicle can provide power at the contract rate P_{contr} under expected conditions regarding the total available storage E_s, the distance already driven d_d, and the desired range reserve d_{rb}. Since the vehicle might on average be expected to have driven half its daily average mileage, or 11 mi, t is:

$$t = \frac{\left(E_s - \frac{d_d + d_{rb}}{\eta_{veh}}\right)\eta_{inv}}{P_{contr}} = \frac{\left(19.2 - \frac{11+12}{2.94}\right)0.93}{18} = 0.59hr =\sim 35\min$$

Part (b): Since the vehicle is available 16 hours per day, the total hours per year is

$$t_{plug} = (16h/d)(365d/yr) = 5840h/yr$$

Next, revenue is calculated based on capacity and energy components:

$$r = Pt_{plug}\left(p_{cap} + p_{el}R_{d-c}\right)$$
$$= (18kW)(5840h)[\$0.05 + (\$0.14)0.08] = \$6433$$

Part (c): Annual cost: the battery lasts for 142,000 cycles and costs \$11,000, so the discharge cost per kWh is:

$$c_d = \frac{c_{bat}}{L_{ET}} = \frac{\$11000}{142000} = \$0.0775/kWh$$

Incorporating the electricity cost along with the battery cost gives:

$$c_{en} = \frac{c_{pe}}{\eta_{conv}} + c_d = \frac{\$0.14}{0.76} + \$0.0775 = \$0.262/kWh$$

Turning to capital cost, c_c = \$2100, so c_{ac} is calculated as:

$$c_{ac} = c_c(A/P, i\%, N) = (\$2100)(A/P, 10\%, 10) = (\$2100)(0.1627) = \$342$$

The annual electricity dispatched E_{disp} is a function of the dispatch-to-contract ratio $R_{d\text{-}c}$, the contracted maximum power P_{contr}, and the contracted time per year in hours t_{contr}:

$$E_{disp} = R_{d-c}P_{contr}t_{contr} = (0.08)(18)(5840) = 8410kWh/yr$$

Combining operating and capital cost:

$$c = c_{en}E_{disp} + c_{ac} = (0.262)(8410) + 342 = \$2542$$

Part (d): Net revenue = \$6433 – \$2542 = \$3891/year.

Problem 15-14.

An example of a tradeoff in electric generation sources is between low and high capital cost natural gas fired power plants. In this problem, you are to supply an electricity market with two types of gas fired plant technology. The description of the load duration curve for this market is the following: the curve has a peak portion where the highest demand hour per year has a demand of 15 GW and a curve of form $y = -0.003\ x + 15$, where x is the hour of the year and y is demand in GW. The curve then has a lower demand portion with a curve of form $y = -0.0004427\ x + 8.628$. Hint: in the lowest demand or 8760th hour per year, the demand according to this curve is approximately 4.75 GW. The values for fixed cost as a function of capacity in kW and variable cost per kWh are given in the table below.

a. Draw the load duration curve for this electricity market. What is the number of the hour in the load duration curve where the "peak" and "lower demand" curves intersect?
b. What is the combination of low and high capital cost plants that can meet the demand for electricity at minimum cost?
c. What is the total fixed cost paid for this capacity? Note: you can ignore the variable cost payments for purposes of this calculation.

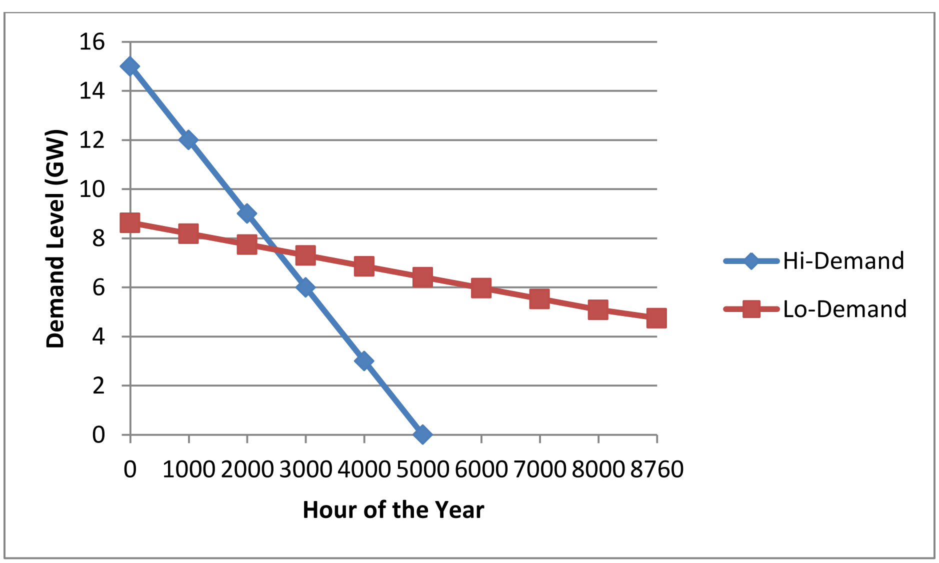
	Hi-cap	Lo-cap
Fixed [$/kW]	140	120
Variable [$/kWh]	0.0238	0.0336

Solution.

Part (a): Figure is shown below. The point of intersection is found by setting the two curves equal to each other and solving for X:

$$-0.003X + 15 = -0.0004427X + 8.628$$
$$X = 2492$$

Therefore, the overall load duration curve is piecewise linear, with the formula for the high-demand curve for hour values between 0 and 2492, and the low- demand curve for hour values between 2492 and 8760. The following curve results:



Part (b): The break-even number of hours between high- and low capital cost is also found by setting the two formuli equal to each other and finding the number of hours *H* where combined cost is equal:

$$140 + 0.0238H = 120 + 0.0336H$$
$$H = 2041$$

For levels of demand below 2041 hours per year, the capacity should be provided using the low capital cost technology, and for the remainder, the high levels. The demand level at *H* = 2041 hours is found using the high-demand portion of the low duration curve.

$$H = 2041$$
$$Demand(2041) = -0.003(2041) + 15 = 8.88GW$$

Since the first 8.88 GW should be provided using the hi-cost technology, and a total of 15 GW are required, the remaining capacity of 6.12 GW should be provided using the low-cost technology.

Part (c): Total fixed cost is the cost per kW multiplied by the number of kW required. Note that units should be converted to kW:

$$TotCost = (\$140/kW)(8.88\times10^6 kW) + (\$120/kW)(6.12\times10^6 kW)$$
$$= \$1.24B + \$730M = \$1.98B$$

Problem 15-15.

Suppose the EV in exercise 15-3 is in the maximum-discharged state (i.e., minimum number of kWh still remaining in the battery system). The EV is plugged in to a 20 A, 110-volt circuit, i.e., one that can delivery 2200 volt-amps or 2.2 kW. The ICEV typically requires 4 minutes of pumping at the filling station to refill the gas tank. Assuming it recharges at a constant rate, how many minutes will the EV require to reach full charge, and how many more minutes does it require than the ICEV?

Solution.

The time required is the desired energy divided by the rate and the charging efficiency:

$$\frac{25kWh}{2.2kW(0.9)} = 12.63h = 758m$$

Thus the additional time is 754 minutes. This comparison indicates the strong advantage maintained by ICEVs in terms of speed of refueling (!).

Problem 15-16.

A large commercial property owner is considering the economic cost and benefit of installing an EV charging station at their facility (e.g., in dedicated parking spaces) and recouping their investment by charging EV owners to charge in the space. Assume that the charging station will average two charges per day over a 365-day year, the average requirement per charge is 16 kWh, and the cost of electricity is \$0.18/kWh. If the total installed cost of the station is \$10,000 and the discount rate is 15%, what must the owner charge per visit from EV owners in order to breakeven in 5 years? Hint: you can treat the revenue from charges as all being incurred at the end of each year, so that there are five annuities over the life of the project. Also, public utility law generally does not allow private facilities to charge per kWh for electricity in a region where the grid operator is already doing this, so this obstacle would need to be overcome to make the proposed scheme a reality.

Solution.

Electricity cost per year is the following:

$$(2chg/d)(16kWh)(\$0.18/kWh)(365d/yr) = \$2102/yr$$

The annualized cost of the facility is:

$$(\$10{,}000)(-PMT(15\%,5,\$1)) = (\$10{,}000)(\$0.2983) = \$2983$$

Thus, the operation requires \$2102 + \$2983 = \$5085 per year. The fee per visit is therefore \$5085/730 visits = ~\$6.97.

Problem 15-17.

An EV, similar in characteristics to a Nissan Leaf, weighs 1330 kg not including the weight of the batteries. The battery pack weighs 270 kg and each kilogram of battery has an effective storage capacity of 85 Wh. The EV consumes 0.092 Wh/kg of total vehicle weight moved 1 km. The maximum depth of discharge is 90%. Questions:

a. How many kWh does the battery pack store when it goes from completely discharged to completely charged?
b. What is the range of the vehicle on a full charge, in kilometers?

Solution.

Part (a): The storage capacity of the battery system is the following:

$$(270kg)(85Wh/kg)(1000Wh/kWh) = 22.95kWh$$

Taking into account the maximum depth of discharge gives (22.95)(0.9) = 20.65 kWh.

Part (b): The range equation can be simplified to the following based on the information already given:

$$R = \frac{20.65kWh}{0.092Wh/kg-km(1330kg + 270kg)} =\sim 140km$$

Problem 15-18.

Short answer, half page maximum. A typical midsize ICEV (think Chevy Impala, Toyota Camry, etc.) might cost \$20 to 25K, travel 300 mi on the highway between refueling, and have a top speed of 100 mi/h or more. The major automakers were not able to market an EV that could achieve these characteristics up to the present time. However, prior to the launch of the current generation of EVs (including the Tesla Roadstar and Model S, or the Nissan Leaf), some makers such as Ford or Chrysler (Think! and GEM brands) were able to market LUVs with a maximum speed of 35 mi/h and range of 40 to 50 mi for local driving, for about $10K. Why could they get LUVs to market but not a full-capability EV? You do not need to do any calculations, just explain qualitatively.

Solution.

The tractive power requirement equation can be used to show that the maximum power requirement for speeds under 35 mi/h is low compared to highway speed requirements. Then the combined low maximum power requirement and low range could be used to show that the maximum energy storage requirement in the battery system will also be relatively small, compared to a highway-capable EV like the Nissan Leaf. Using representative figures for the cost per kWh of energy storage would show that the cost of the battery system would be relatively small, and furthermore because the system is small and does not weigh as much, the rest of the vehicle can also be smaller and lighter weight. All of this leads to lower overall curb weight and lower cost.

Chapter 16 Bioenergy Resources and Systems

Problem 16-1.

As an exercise in exploring the impact of energy input in ethanol production on NEB, as discussed in the body of this chapter, consider the following process, in which each liter of ethanol requires 2.69 kg of corn as a feedstock. The corn must first be grown on arable land; per hectare (100m × 100m), the corn field has the following energy input requirements:

Input	Energy [GJ]
Machinery (embodied)	4.97
Energy products (diesel, gasoline, electricity)	6.04
Nitrogen	10.25
Balance of inputs	12.72

The hectare yields approximately 9000 kg of corn. The corn is then processed in an ethanol plant to make almost pure (99.5%) ethanol. Per 1000 L of ethanol, the ethanol production process requires the energy inputs shown in the table below, in addition to the corn. Assume that a liter of ethanol contains 21.3 MJ net. For simplicity, ignore the energy impact of by-products resulting from corn ethanol production.

Input	Energy [GJ]
Transportation	1.35
Distillation steam	10.66
Electricity	4.87
Balance of inputs	0.59

Calculate the ratio of energy available in the resulting ethanol to the total energy input. Does the ethanol provide more energy than it consumes?

<u>Solution.</u>

First sum up total energy per hectare:

$$4.97 + 6.04 + 10.25 + 12.72 = 33.98 \text{ GJ/ha}$$

The 33.98 GJ/ha must then be assigned to the amount of corn required for ethanol production. Since each hectare produces 9000 kg and 2.69 kg of corn is required as feedstock, the conversion can be made as follows:

$$\left(33.98\frac{\text{GJ}}{\text{ha}}\right)\left(\frac{1\,\text{ha}}{9000\,\text{kg}}\right)\left(\frac{1000\,\text{MJ}}{\text{GJ}}\right)\left(\frac{2.69\,\text{kg}}{1\text{ L}}\right) = 10.16\,\text{MJ/L}$$

The 10.16 MJ/L will then be added to the total consumption in production per liter, which after converting from energy per 1000 L in GJ to energy per liter in MJ is summed as follows:

10.16 + 1.35 + 10.66 + 4.87 + 0.59 = 27.63 MJ/L

Using the given figure for the energy content per liter of ethanol of 21.3 MJ gives the following NEB ratio value:

$$NEBRatio = \frac{E_{out}}{E_{in}} = \frac{21.3}{27.63} = 0.763$$

Note that there is a net loss of energy in creation of ethanol for use as a transportation fuel in this case.

Problem 16-2.

Repeat exercise 16-1, but this time for biodiesel production from soy. The energy requirement per hectare for soy is the following:

Component	Energy [GJ]
Fuels	2.67
Fertilizer	1.60
Embodied energy	0.53
All Other	0.53

Each hectare yields 508 kg of soy, and each liter of soy biodiesel requires 0.925 kg of soy as input. A liter of biodiesel contains 32.6 MJ of net energy content. Production energy requirements per 1000 kg of soy beans processed are in the table below. Calculate ratio of output to input, ignoring by-product energy impact.

Component	Energy [GJ]
Process heat and electricity	4.14
Embodied energy	1.38
Transportation	0.69
All other	0.69

Solution.

First sum up total energy per hectare:

2.67 + 1.6 + 0.53 + 0.53= 5.33 GJ/ha

The 5.33 GJ/ha must then be assigned to the amount of soy produced. Since each hectare produces 508 kg, the energy consumption per pound is:

$$\left(5.33\frac{GJ}{ha}\right)\left(\frac{1ha}{508kg}\right) = 0.0105GJ/kg = 10.5MJ/kg$$

It therefore follows that 1000 kg will require 10.5 GJ of energy. Adding this amount to the energy required per 1000 kg of processing gives:

4.14 + 1.38 + 0.69 + 0.69 + 10.5 = 17.4 GJ

This amount is equivalent to 17.4 MJ consumed per kg of soy grown and processed. Converting the energy used per kg to an equivalent quantity of energy per unit of biodiesel produced gives:

$$\left(17.4\frac{MJ}{kg}\right)\left(0.925\frac{kg}{L}\right) = 16.1 MJ/L$$

Using the given figure for the energy content of 32.6 MJ per liter of biodiesel gives the following NEB ratio value:

$$NEB\,ratio = \frac{E_{out}}{E_{in}} = \frac{32.6}{16.1} = 2.03$$

Problem 16-3.

Repeat the calculation of exercise 16-1 for corn ethanol for the advanced process with higher efficiency discussed in the chapter. Suppose that the net energy input in agriculture per hectare is cut by 50% thanks to advances in practices for growing corn. Suppose also that the "transportation" and "balance of inputs" figures for energy consumption remain fixed. What must the combined energy value per 1000 L of production for distillation and electricity be to achieve the net energy balance ratio of 2:1, which is the average of the reported current values discussed in the chapter?

Solution.

Since the energy content of the ethanol is 21.3 MJ/L, the available budget for production is 50% in order to achieve the desired NEB ratio, or 10.65 MJ/L. The total current agricultural consumption is 10.16 MJ/L, so the future value is 50% of this amount, or 5.08 MJ/L. The remaining budget for production is therefore 10.65 – 5.08 = 5.57 MJ. The combined transportation and balance energy value is 1.35 + 0.59 = 1.94 MJ/L. Therefore remaining amount of energy consumption in electricity and distillation is 5.57 – 1.94 = 3.63 MJ/L. This amount is equivalent to 3.63 GJ per 1000 L.

Problem 16-4.

As an exercise in exploring the impact of energy input in ethanol production on NEB, as discussed in the body of this chapter, consider the following process, in which each gallon of ethanol requires 22.4 lb of corn as a feedstock. The corn must first be grown on arable land; per acre, the corn field has the following energy input requirements measured in millions of Btus (MMBtu):

Input	Energy [MMBtu]
Machinery (embodied)	1.906
Energy products (diesel, gasoline, electricity)	2.317

Nitrogen	3.932
Balance of inputs	4.879

The acre yields approximately 8013 lb of corn. The corn is then processed in an ethanol plant to make almost pure (99.5%) ethanol. Per 1000 gal of ethanol, the ethanol production process requires the energy inputs shown in the table below, in addition to the corn. Assume that a gallon of ethanol contains 75.7 MBtu (1 MBtu = 1000 Btu). For simplicity, ignore the energy impact of by-products resulting from corn ethanol production.

Input	Energy [MMBtu]
Transportation	4.84
Distillation steam	38.24
Electricity	17.47
Balance of inputs	2.12

Calculate the ratio of energy available in the resulting ethanol to the total energy input. Does the ethanol provide more energy than it consumes?

Solution.

First sum up total energy per acre:

1.906 + 2.317 + 3.932 + 4.879 = 13.034 MMBtu/acre

The 13.034 MMBtu/acre must then be assigned to the amount of corn required for ethanol production. Since each acre produces 8013 lb corn and 22.4 lb of corn is required as a feedstock per gallon, the conversion can be made as follows:

$$\frac{13.034 MMBtu}{8013 lb} = 1.627 MBtu/lb$$
$$(1.627 MBtu/lb)(22.4 lb/gal) = 36.44 MBtu/gal$$

The 36.44 MBtu/gal will then be added to the total consumption in production per gallon. Figures above are given in units of MMBtu per 1000 gal, which is the same as units of MBtu/gal. Adding production and agricultural energy gives:

4.84 + 38.24 + 17.47 +2.12 + 36.44 = 99.11 MBtu/gal

Using the given figure for the energy content per gallon of ethanol of 75.7MBtu/gal gives the following NEB ratio value:

$$NEBRatio = \frac{E_{out}}{E_{in}} = \frac{75.7}{99.1} = 0.763$$

Note that there is a net loss of energy in creation of ethanol for use as a transportation fuel in this case.

Problem 16-5.

Repeat exercise 16-4, but this time for biodiesel production from soy. The energy requirement per acre for soy is the following:

Component	MBtu
Fuels	1025
Fertilizer	615
Embodied energy	205
All other	205

Each acre yields 452 lb of soy, and each gallon of soy biodiesel requires 7.7 lb of soy as input. A gallon of biodiesel contains 117,000 Btu of net energy content. Production energy requirements per 1000 lb of soy beans processed are in the table below. Calculate the ratio of output to input, again ignoring by-product energy impact.

Component	1000 Btu
Process heat and electricity	1784
Embodied energy	595
Transportation	297
All other	297

<u>Solution.</u>

First sum up total energy per acre:

1.025 + 0.615 + 0.205 + 0.205 = 2.05 MMBtu/acre

The 2.05 MMBtu/acre must then be assigned to the amount of soy produced. Since each hectare produces 452 lb, the energy consumption per pound is:

$$\left(2.05\frac{MMbtu}{acre}\right)\left(\frac{1acre}{452lb}\right) = 4.54MBtu/lb$$

It therefore follows that 1000 lb will require 4.54 MMBtu. Adding this amount to the energy required per 1000 lb of processing gives:

4.54 + 1.784 + 0.595 + 0.297 + 0.297 = 7.5 MMBtu

This amount is equivalent to 7.51 MBtu consumed per pound of soy grown and processed. Converting the energy used per pounds to an equivalent quantity of energy per unit of biodiesel produced gives:

$$\left(7.51\frac{MBtu}{lb}\right)\left(7.7\frac{lb}{gal}\right) = 57.81MBtu/gal$$

Using the given figure for the energy content of 117 MBtu per gallon of biodiesel gives the following NEB ratio value:

$$NEB\,ratio = \frac{E_{out}}{E_{in}} = \frac{117}{57.81} = 2.03$$

Problem 16-6.

Repeat the calculation of exercise 16-4 for corn ethanol for the advanced process with higher efficiency discussed in the chapter. Suppose that the net energy input in agriculture per acre is cut by 50% thanks to advances in practices for growing corn. Suppose also that the "transportation" and "balance of inputs" figures for energy consumption remain fixed. What must the combined energy value per 1000 gal of production for distillation and electricity be to achieve the net energy balance ratio of 2.1, which is the average of the reported current values discussed in the chapter?

Solution.

Since the energy content of the ethanol is 75.7 MBtu/gal, the available budget for production is 50% in order to achieve the desired NEB ratio, or 37.85 MBtu/gal. The total current agricultural consumption is 8.013 MMBtu/acre or 357.7 MBtu/gal, so the future value is 50% of this amount, or 18.22 MBtu/gal. The remaining budget for production is therefore 37.85 – 18.22 = 24.82 MBtu/gal. The combined transportation and balance energy value is 4.84 + 2.12 = 6.96 MBtu/gal. Therefore. remaining amount of energy consumption in electricity and distillation is 24.82 – 6.96 = 17.86 MBtu/gal. This amount is equivalent to 17.86 MMBtu per 1000 gal.

Chapter 17 Conclusion: Toward Sustainable Transportation Systems

Problem 17-1.

Consider the CO_2 reduction pathways for passenger and freight transportation presented in Figs.17-6 and 17-7. (a) Reconstruct the emissions shown in the figures using population values given in exercise 1-2, the passenger-km and tonne-km per capita figures given in Chap.1, the CO_2 emissions factors given in this chapter, and the triangle function. (b) Use the model from part (a) to develop alternative pathways for emissions based on changes to the assumptions about each of the four country groups. Discuss the reasoning behind your changed assumptions, and the implications for the resulting total CO_2 emissions over the course of the 21st century.

Solution.

Part (a): Note that for both passenger and freight scenarios the following population projection holds, in millions of persons:

Year	Hi-Inc	Hi-Inc	Mid-Inc	Lo-Inc	Total
	Hi-Vol	Lo-Vol			
2000	500	800	2500	2200	6000
2010	500	800	3032	2668	7000
2020	500	800	3457	3043	7800
2030	500	800	3777	3323	8400
2040	500	800	3989	3511	8800
2050	500	800	4069	3581	8950
2060	500	800	4096	3604	9000
2070	500	800	4096	3604	9000
2080	500	800	4096	3604	9000
2090	500	800	4096	3604	9000
2100	500	800	4096	3604	9000

Looking first at the passenger side, we take mid-income in the year 2010 as an example for calculation, and then present the final results:

$$PKM.CAP_{2010} = 2\left(\frac{t-a}{b-a}\right)^2 \left(x_{final} - x_{init}\right)$$

$$= 2\left(\frac{t-a}{b-a}\right)^2 \left(15K\frac{pkm}{cap} - 5K\frac{pkm}{cap}\right) = 5800\, pkm/cap$$

$$PKM_{2010} = \left(Pop_{2010}\right)\left(PKM.CAP_{2010}\right)$$

$$= \left(3.032 \times 10^9\, pers\right)\left(5{,}800\, pkm/cap\right) =$$

= 17.59 tril. pkm

Declining CO_2 per passenger-km are also modeled using the triangle function:

$$CO_2.PKM_{2010} = 2\left(\frac{t-a}{b-a}\right)^2 \left(x_f - x_i\right)$$

$$= 2\left(\frac{2010-2000}{2100-2000}\right)^2 \left(0.162-0\right) = 0.158 kgCO_2 / pkm$$

$$CO2_{2010} = PKM_{2010} \cdot CO_2.PKM_{2010}$$

$$= 17.59 \times 10^{12} \left(0.158 kgCO_2 / pkm\right) = 2.79 bil.tonne.CO_2$$

Repeating these steps for all four country groups and all years in the study gives the following tables of pkm/cap, $kgCO_2$ per pkm, and total CO_2 for 2000 to 2100:

1000 pkm per cap	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc
2000	25	15	5	2
2010	25	15	5.8	2.26
2020	25	15	8.2	3.04
2030	25	15	11.8	4.34
2040	25	15	14.2	6.16
2050	25	15	15	8.5
2060	25	15	15	10.84
2070	25	15	15	12.66
2080	25	15	15	13.96
2090	25	15	15	14.74
2100	25	15	15	15

$kgCO_2$ per pkm	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc
2000	0.162	0.162	0.162	0.162
2010	0.149	0.149	0.158	0.158
2020	0.110	0.110	0.149	0.149
2030	0.052	0.052	0.133	0.133
2040	0.013	0.013	0.110	0.110
2050	0.000	0.000	0.081	0.081
2060	0.000	0.000	0.052	0.052
2070	0.000	0.000	0.029	0.029
2080	0.000	0.000	0.013	0.013
2090	0.000	0.000	0.003	0.003
2100	0.000	0.000	0.000	0.000

Bil.tonne CO2	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc	Total
2000	2.02	1.94	2.02	0.71	6.69
2010	1.86	1.79	2.79	0.96	7.39

2020	1.37	1.32	4.22	1.38	8.29
2030	0.65	0.62	5.91	1.91	9.09
2040	0.16	0.16	6.23	2.38	8.92
2050	0.00	0.00	4.93	2.46	7.40
2060	0.00	0.00	3.18	2.02	5.20
2070	0.00	0.00	1.79	1.33	3.12
2080	0.00	0.00	0.79	0.65	1.45
2090	0.00	0.00	0.20	0.17	0.37
2100	0.00	0.00	0.00	0.00	0.00

Similar steps are repeated here on the freight side:

$$TKM.CAP_{2010} = 2\left(\frac{t-a}{b-a}\right)^2 \left(x_{final} - x_{init}\right)$$

$$= 2\left(\frac{2010-2000}{2100-2000}\right)^2 \left(8K\frac{tkm}{cap} - 4K\frac{tkm}{cap}\right) = 4320tkm/cap$$

$$TKM_{2010} = \left(Pop_{2010}\right)\left(TKM.CAP_{2010}\right)$$

$$= \left(3.032\times10^9\ pers\right)\left(4320\,pkm/cap\right) = 13.1\times10^{12}tkm$$

= 13.1 tril. pkm

$$CO_2.TKM_{2010} = 2\left(\frac{t-a}{b-a}\right)^2 \left(x_f - x_i\right)$$

$$= 2\left(\frac{2010-2000}{2100-2000}\right)^2 \left(0.099-0\right) = 0.097kgCO_2/tkm$$

$$CO2_{2010} = PKM_{2010} \cdot CO_2.PKM_{2010}$$

$$= 13.1\times10^{12}\left(0.097kgCO_2/tkm\right) = 1.27bil.tonne.CO_2$$

Repeating these steps for all four country groups and all years in the study gives the following tables of tkm/cap, $kgCO_2$ per pkm, and total CO_2 from freight for 2000 to 2100:

1000 tkm per capita	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc
2000	17	8	4	2
2010	17	8	4.32	2.12
2020	17	8	5.28	2.48
2030	17	8	6.72	3.08
2040	17	8	7.68	3.92
2050	17	8	8	5
2060	17	8	8	6.08
2070	17	8	8	6.92
2080	17	8	8	7.52

2090	17	8	8	7.88
2100	17	8	8	8

$kgCO_2$ per tkm	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc
2000	0.099	0.099	0.099	0.099
2010	0.091	0.091	0.097	0.097
2020	0.067	0.067	0.091	0.091
2030	0.032	0.032	0.081	0.081
2040	0.008	0.008	0.067	0.067
2050	0.000	0.000	0.049	0.049
2060	0.000	0.000	0.032	0.032
2070	0.000	0.000	0.018	0.018
2080	0.000	0.000	0.008	0.008
2090	0.000	0.000	0.002	0.002
2100	0.000	0.000	0.000	0.000

Bil.tonne CO_2	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc	Total
2000	0.84	0.63	0.99	0.43	2.90
2010	0.77	0.58	1.27	0.55	3.17
2020	0.57	0.43	1.66	0.69	3.35
2030	0.27	0.20	2.06	0.83	3.36
2040	0.07	0.05	2.06	0.92	3.10
2050	0.00	0.00	1.61	0.88	2.49
2060	0.00	0.00	1.04	0.69	1.73
2070	0.00	0.00	0.58	0.44	1.03
2080	0.00	0.00	0.26	0.21	0.47
2090	0.00	0.00	0.06	0.06	0.12
2100	0.00	0.00	0.00	0.00	0.00

The above calculation thus reproduces Figs.17-6 and 17-7.

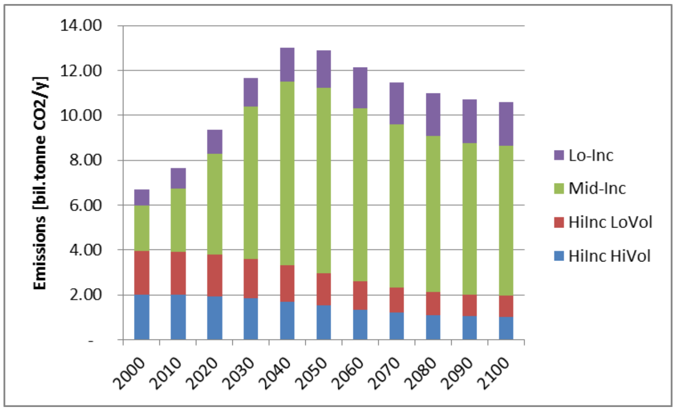
Part (b): Many alternative scenarios are possible, but the one that is discussed here is that for both passenger and freight the following conditions hold. First, mid-income countries achieve passenger and freight parity with the high-income low-volume countries, at 15,000 pkm/cap and 8000 tkm/cap, which is unchanged from the base case. The low-income countries, however, continue to lag, so that in the year 2100 they have only increased to 5000 pkm/cap and 5000 tkm/cap. Second, transportation systems continue to depend on fossil fuels without any form of carbon capture and sequestration, so that reductions in CO_2 emission rates come strictly from improvements in technology. These are assumed to achieve 50% reduction by 2100 for the high-income countries (since they are wealthier and more advanced) and 33% reduction by 2100 for the other two groups, both phased in using a triangle function.

It is assumed that the student or instructor can generate the pkm and tkm pathway for the low-income countries, so the table of values is not presented here. For the emission rate pathway, the values are the following:

Year	Hi Income	Other	Hi Income	Other
	CO_2/pkm	CO_2/pkm	CO_2/tkm	CO_2/tkm
2000	0.162	0.162	0.099	0.099
2010	0.160	0.161	0.098	0.098
2020	0.155	0.157	0.095	0.096
2030	0.147	0.152	0.090	0.093
2040	0.136	0.145	0.083	0.088
2050	0.121	0.135	0.074	0.083
2060	0.107	0.125	0.065	0.077
2070	0.095	0.118	0.058	0.072
2080	0.087	0.113	0.053	0.069
2090	0.082	0.109	0.050	0.067
2100	0.081	0.108	0.049	0.066

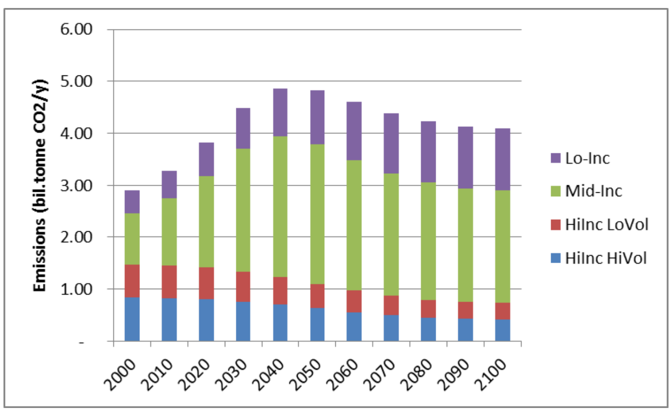
The resulting table of data and accompanying graphical figure is then compared with the original figures. For passenger transportation demand:

Year	Bil.t CO_2	Bil.t CO_2	Bil.t CO_2	Bil.t CO_2	Total
	Hi-Inc Hi-Vol	Hi-Inc Lo-Vol	Mid-Inc	Lo-Inc	Bil.t CO_2
2000	2.02	1.94	2.02	0.71	6.69
2010	2.00	1.92	2.82	0.88	7.63
2020	1.94	1.86	4.46	1.07	9.34
2030	1.84	1.77	6.78	1.28	11.67
2040	1.70	1.63	8.19	1.50	13.02
2050	1.52	1.46	8.24	1.69	12.90
2060	1.33	1.28	7.71	1.83	12.15
2070	1.19	1.14	7.25	1.90	11.48
2080	1.09	1.05	6.92	1.93	10.99
2090	1.03	0.99	6.72	1.95	10.69
2100	1.01	0.97	6.66	1.95	10.59



For freight transportation the table and figure are the following:

Year	Bil.t CO_2 Hi-Inc Hi-Vol	Bil.t CO_2 Hi-Inc Lo-Vol	Bil.t CO_2 Mid-Inc	Bil.t CO_2 Lo-Inc	Total Bil.t CO_2
2000	0.84	0.63	0.99	0.43	2.90
2010	0.83	0.63	1.29	0.54	3.28
2020	0.81	0.61	1.76	0.66	3.83
2030	0.76	0.58	2.36	0.78	4.48
2040	0.71	0.53	2.71	0.92	4.86
2050	0.63	0.47	2.69	1.03	4.82
2060	0.55	0.42	2.51	1.12	4.60
2070	0.50	0.37	2.36	1.16	4.39
2080	0.45	0.34	2.25	1.18	4.23
2090	0.43	0.32	2.19	1.19	4.13
2100	0.42	0.32	2.17	1.19	4.10



Discussion: Comparing both the passenger and freight pathways in the above figures to those in Figs.17-6 and 17-7, in both cases there is growth into the 2020 to 2030 period, but after that the paths diverge. In the former, rapid growth in passenger-km and tonne-km in the middle-income countries outweigh reductions in intensity, so that the emissions from this group of countries rise to 2050 and then decline slightly to 2100. Low-income countries rise slightly from 2000 to 2100, and high-income countries decline, but the rise in middle-income countries leads to an overall large increase. Thus not only do annual emissions not decrease under these two scenarios (unlike Figs.17-6 and 17-7) but they actually rise.

Another alternative scenario would be one where no growth in total emissions occurs from 2020 onward, thanks to a slower rate of overall pkm and tkm growth, coupled with more aggressive reductions in emission rate. This scenario would be difficult to implement because it involves a sharp change of direction, but is of interest since it explores hypothetically what combination of demand and emission rates would be required. Although it could be developed using the same approach, it has not been done here.

Problem 17-2.

Consider the two CO_2 reduction pathways presented in Figs.17-1 and 17-2. (a) Use the given data and the triangle function to reconstruct the pathways shown. (b) The discussion in the chapter notes that alternative 2 (as presented in Fig.17-2) exceeds the 80% target because it uses the emission factor targets from alternative 1 but also substantially reduces total passenger-miles (pmi). Develop an alternative set of emission targets that exactly meet the 80% target when implemented according to the triangle function. Explain the reasoning behind your assumed targets.

Solution.

Part (a): The first step in the problem is to reconstruct the BAU case. According to this case, demand measured in pmi grows by 10% every decade, or, since the total in 2010 sums to 4538.3 billion pmi, 453.8 billion pmi every decade. We can then create a table by mode of growth per decade, total growth, and total emissions in the years 2010 to 2050, as seen in Figs. 17-1 and 17-2:

Mode	Demand	Emit rate	10-y growth	2050 demand	2050 emits
	(bil.pmi)	(lb/pmi)	(bil.pmi)	(bil.pmi)	(mil.tons)
Air	3645.4	0.650	364.54	5103.5	1658.6
Bus	564.8	0.553	56.48	790.7	218.7
Rail	292.3	0.429	29.23	409.2	87.8
Redux	35.8	0.462	3.58	50.1	11.6
Total	4538.3		453.83	6353.6	1976.7

Emissions by mode 2010 to 2050, in million tons of CO_2:

Mode	2010	2020	2030	2040	2050
Car + LT	1184.7	1303.2	1421.6	1540.1	1658.6
Air	156.2	171.8	187.4	203.1	218.7
Bus	62.7	69.0	75.3	81.6	87.8
Rail	8.3	9.1	9.9	10.7	11.6
Total	1411.9	1553.1	1694.3	1835.5	1976.7

The emission values in the "Total" row are the values that appear as the "BAU" curve in the figure. As an example of the calculation, the value for 2050 emissions for car and light truck is the following:

$$\left(3.645\times 10^{12}\, pmi\right)\left(0.650\frac{lbCO_2}{pmi}\right)\left(\frac{1ton}{2000lb}\right)=1184.7mil.ton$$

Other values are calculated in a similar way.

We now turn to the two reduction pathways in the figures, using the triangle function. For alternative 1, the approach is to calculate the emissions rate reduction that will lead to an 80% reduction by 2050 despite a 40% increase in p-mi, and then to phase in this reduction using the triangle function. Detailed calculations are shown here for the "Car+LT" mode and for the year 2010. It is first found that for the emission reduction to be shared proportionally among the four modes, the emission rate must be reduced by 85.7%. Applying this fraction to the initial 2010 rate of 0.65 lb/pmi gives a reduction of 0.557 lb/pmi. The emissions rate in 2010 is therefore:

$$\Delta\mu = 2\left(\frac{2020-2010}{2050-2010}\right)^2\left(-0.557\right) = -0.07lbCO_2\,/\,pmi$$
$$\mu_{2020} = 0.65-0.07 = 0.58lbCO_2\,/\,pmi$$

Repeating this process to calculate emission rates for each mode in each year, and multiplying by demand in each year, gives the following table of emission values (which also appear in Fig.17-1):

Mode:	2010	2020	2030	2040	2050
Car + LT	1185	1164	812	385	237
Air	156	153	107	51	31
Bus	63	62	43	20	13
Rail	8.3	8.1	5.7	2.7	1.7
Total	1412	1387	968	459	282

In alternative 2, both the demand levels and emission rates change, with Car+LT and air modes having declining modal share and bus and rail having theirs increasing. Therefore, the demand levels by mode and year are presented here:

Mode	2010	2020	2030	2040	2050
Car + LT	3645	3374	2559	1745	1474
Air	565	551	508	466	452
Bus	292	370	602	834	912
Rail	36	145	474	802	912
TOTAL	4538	4440	4144	3848	3750

It is useful to observe the emission rates for calculating total emissions, given here in units of lbCO_2 per pmi:

Mode	2010	2020	2030	2040	2050
Car + LT	0.650	0.580	0.371	0.162	0.093
Air	0.553	0.494	0.316	0.138	0.079
Bus	0.429	0.383	0.245	0.107	0.061
Rail	0.462	0.412	0.264	0.115	0.066

Multiplying pmi by emission rate and converting from lb to tons gives the following units of millions of tons of CO_2:

Mode:	2010	2020	2030	2040	2050
Car + LT	1185	979	475	142	68
Air	156	136	80	32	18
Bus	63	71	74	45	28
Rail	8	30	62	46	30
Total	1412	1216	692	265	144

These figures appear as bar graphs in Fig.17-2. Note that the CO_2 reduction increases to 89.8% from 80% by combining modal shifting with emission rate reduction.

Part (b): Multiple solutions are possible to this problem, but in this case we use the 2050 demand levels from alternative 2 and adjust the emission rates by a constant amount (factor of 1.95 larger) so that the 80% reduction is exactly achieved. The 2050 rates are therefore:

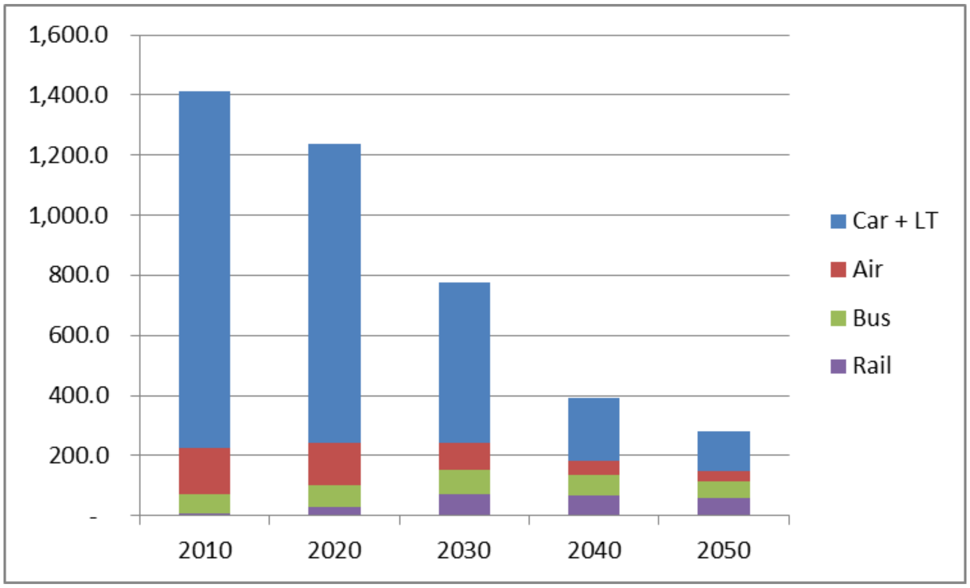
Mode	lbCO_2/pmi
Car + LT	0.182
Air	0.155
Bus	0.120
Rail	0.129

The emissions rates in intermediate years are then calculated using a triangle function. It is assumed that the calculation can be completed in a spreadsheet so the details are not given. The resulting emission rates by mode and year are as follows:

Mode	2010	2020	2030	2040	2050
Car + LT	0.650	0.591	0.416	0.240	0.182
Air	0.553	0.503	0.354	0.204	0.155
Bus	0.429	0.391	0.275	0.159	0.120
Rail	0.462	0.420	0.295	0.171	0.129

Lastly, multiplying by alternative 2 demand levels gives the following table and figure for year on year emissions:

Mode	2010	2020	2030	2040	2050
Car + LT	1184.7	997.7	532.2	209.6	133.9
Air	156.2	138.6	90.0	47.7	35.0
Bus	62.7	72.2	82.7	66.2	54.7
Rail	8.3	30.5	70.0	68.4	58.8
TOTAL	1411.9	1239.0	774.8	391.9	282.4



The resulting figure resembles the total emissions from Fig.17-1 but the modal split in emissions from Fig.17-2. Other variations are possible, such as a more modest reduction in CO_2 per passenger-mile from aviation, because of the challenges with creating aviation fuels from renewable energy sources (this variation was not attempted here, however).

Appendix C Engineering Economic Tools

Question 1. A transportation system operator is considering an investment in increased fuel efficiency of their system that costs \$1.2 million initially and lasts for 10 years, during which time it will save \$250,000 per year. The investment has no salvage value. The operator has set an MARR of 13%. Is the investment viable?

Solution.

The problem is solved by converting the annual savings into its equivalent present value. The factor needed is (P/A, 13%, 10 years) = 5.43, which can be solved either with the formula or using a built-in spreadsheet or software function. Accordingly, the NPV of the investment is:

$$-\$1.2M + \$250K(5.43) = \$157{,}500$$

Thus, the investment is viable, although fairly close to the break-even point.

Question 2. Calculate the B/C ratio for exercise 1.

Solution.

Calculate the benefit *B* in terms of present worth and then the ratio of the two. There is no operating cost to consider, so there is no distinction between conventional and modified ratio.

$$B = (P/A,10,13\%)(\$250K) = (5.43)(\$250K) = \$1{,}357{,}500$$

$$B/C = \frac{\$1{,}357{,}500}{\$1{,}200{,}000} = 1.13 > 1$$

Therefore the result is the same as in exercise 1, the investment is viable.

Question 3. A transportation project requires an investment in year 0 of \$8 million, and has an investment horizon of 10 years and an MARR of 7%. If the annuities in years 1 through 10 are as shown in the table, what is the NPV?

Year	Annuity
1	\$1,480,271
2	\$1,165,194
3	\$1,190,591
4	\$1,286,144
5	\$1,318,457

6	$973,862
7	$1,108,239
8	$1,468,544
9	$1,105,048
10	$851,322

Solution.

Create a spreadsheet table from the above data that includes the discounted value from each year, and the cumulative discounted value:

Year	Cap	Annuity	Disc. Value	Cumulative Value
0	$ (8,000,000)	$ -	$ (8,000,000)	$ (8,000,000)
1	$ -	$ 1,480,271	$ 1,383,431	$ (6,616,569)
2	$ -	$ 1,165,194	$ 1,017,726	$ (5,598,843)
3	$ -	$ 1,190,591	$ 971,877	$ (4,626,966)
4	$ -	$ 1,286,144	$ 981,193	$ (3,645,773)
5	$ -	$ 1,318,457	$ 940,042	$ (2,705,731)
6	$ -	$ 973,862	$ 648,925	$ (2,056,806)
7	$ -	$ 1,108,239	$ 690,155	$ (1,366,651)
8	$ -	$ 1,468,544	$ 854,706	$ (511,945)
9	$ -	$ 1,105,048	$ 601,073	$ 89,128
10	$ -	$ 851,322	$ 432,769	$ 521,897

The solution is then the cumulative value at the end of year 10, or $521,897.

Appendix D Geometric Design

Problem d-1.

Compute the minimum length of vertical curve to provide passing sight distance for a design speed of 100 km/h at the intersection of a +1.40% grade with a –0.60% grade.

Assume $S \le L$

$$h_1 = 1.070\,\text{m}$$

$$h_2 = 1.300\,\text{m}$$

$$g_1 = +1.40$$

$$g_2 = -0.60$$

$$A = g_1 - g_2 = 1.40 - (-0.60) = 2.00$$

$S = 670\,\text{m}$ (Table 3.4)

$$L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} = \frac{(2.00)\left(670^2\right)}{200\left(\sqrt{1.070} + \sqrt{1.300}\right)^2} = 751.5\text{m}$$

751.5 m < 670 m, so $S > L$ n.g.

Try $S < L$

$$L_{\min} = 2S - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A} = 2(670) - \frac{200\left(\sqrt{1.070} + \sqrt{1.300}\right)^2}{2.00} = 742.7\text{m}$$

$L_{MIN} = 472.7\,\text{m}$ Round to 750 m

Problem d-2.

Compute the minimum length of vertical curve that will provide 190-m stopping sight distance for a design speed of 100 km/h at the intersection of a +2.60% grade and a –2.40% grade.

Assume $S \leq L$

$$h_1 = 1.070\,\text{m}$$

$$h_2 = 0.150\,\text{m}$$

$$g_1 = +2.60$$

$$g_2 = -2.40$$

$$A = g_1 - g_2 = 2.60 - (-2.40) = 5.00$$

$$S = 190\,\text{m}$$

$$L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} = \frac{(5.00)(190^2)}{200\left(\sqrt{1.070} + \sqrt{0.150}\right)^2} = 446.5\text{m}$$

446.5 m > 190 m, so $S < L$ O.K.

$L_{MIN} = 446.5\,\text{m}$ Round to 450 m

Problem d-3.

Compute the minimum length of vertical curve that will provide 220-m stopping sight distance for a design speed of 110 km/h at the intersection of a +3.50% grade and a –2.70% grade.

Assume $S \leq L$

$$h_1 = 1.070\,\text{m}$$

$$h_2 = 0.150\,\text{m}$$

$$g_1 = +3.50$$

$$g_2 = -2.70$$

$$A = g_1 - g_2 = 3.50 - (-2.70) = 6.20$$

$$S = 220\,\text{m}$$

$$L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} = \frac{(6.20)(220^2)}{200\left(\sqrt{1.070} + \sqrt{0.150}\right)^2} = 742.3\text{m}$$

742.3 m > 220 m, so $S < L$ O.K.

$L_{MIN} = 742.3\,\text{m}$ Round to 750 m

Problem d-4.

Compute the minimum length of vertical curve that will provide 130-m stopping sight distance for a design speed of 80 km/h at the intersection of a +2.30% grade and a –4.80% grade.

Assume $S \le L$

$h_1 = 1.070\,\text{m}$

$h_2 = 0.150\,\text{m}$

$g_1 = +2.30$

$g_2 = -4.80$

$A = g_1 - g_2 = 2.30 - (-4.80) = 7.10$

$S = 130\,\text{m}$

$$L = \frac{AS^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2} = \frac{(7.10)(130^2)}{200\left(\sqrt{1.070} + \sqrt{0.150}\right)^2} = 296.8\text{m}$$

296.8 m > 130 m, so $S < L$ O.K.

$L_{MIN} = 296.8\,\text{m}$ Round to 300 m

Problem d-5.

Compute the minimum length of vertical curve that will provide 190-m stopping sight distance for a design speed of 100 km/h at the intersection of a –2.60% grade and a +2.40% grade.

Assume $S \le L$

$g_1 = -2.60$

$g_2 = +2.40$

$A = g_2 - g_1 = 2.40 - (-2.6) = 5.00$

$S = 190\,\text{m}$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{(5.00)(190^2)}{120 + (3.5)(190)} = 229.94\text{m}$$

190 < 229.94, so $S < L$ O.K.

$L_{min} = 229.94\,\text{m}$ Round to 230 m.

Problem d-6.

Compute the minimum length of vertical curve that will provide 220-m stopping sight distance for a design speed of 110 km/h at the intersection of a –3.50% grade and a +2.70% grade.

Assume $S \le L$

$g_1 = -3.50$

$g_2 = +2.70$

$A = g_2 - g_1 = 2.70 - (-3.50) = 6.20$

$S = 220\,\text{m}$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{(6.20)(220^2)}{120 + (3.5)(220)} = 337.17\text{m}$$

220 < 337.17, so $S < L$ O.K.

$L_{min} = 337.17\,\text{m}$ Round to 340 m.

Problem d-7.

Compute the minimum length of vertical curve that will provide 130-m stopping sight distance for a design speed of 80 km/h at the intersection of a –2.30% grade and a +4.80% grade.

Assume $S \le L$

$g_1 = -2.30$

$g_2 = +4.80$

$A = g_2 - g_1 = 4.80 - (-2.30) = 7.10$

$S = 130\,\text{m}$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{(7.10)(130^2)}{120 + (3.5)(130)} = 208.68\text{m}$$

130 < 208.68, so $S < L$ O.K.

$L_{min} = 208.68\,\text{m}$ Round to 210 m.

Problem d-8.

(*a*) Compute curve elevations and offsets from tangents at full stations and +25 points for a 350-m vertical curve joining a +2.70% grade with a −1.50% grade. Assume the PI is at station 150+00 and elevation 25.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.

(*b*) Plot the profile for the curve data in part *a*.

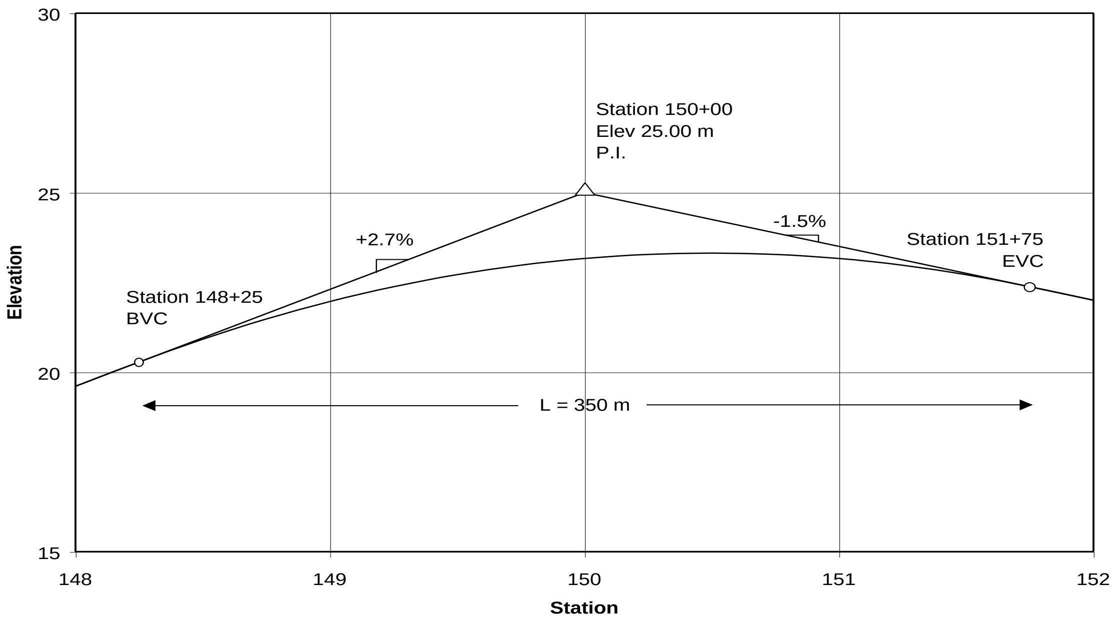
Solution.

(*a*) Curve elevations

$$r = \frac{g_2 - g_1}{L} = \frac{-1.50\% - (+2.70\%)}{3.5\text{sta}} = -1.2\%/\text{Sta}$$

Station		Tangent Elevation	Offset	Curve Elevation, m
148+00		19.600	0.000	19.600
+25	BVC	20.275	0.000	20.275
+50		20.950	–0.038	20.912
+75		21.625	–0.150	21.475
149+00		22.300	–0.338	21.962
+25		22.975	–0.600	22.375
+50		23.650	–0.938	22.712
+75		24.325	–1.350	22.975
150+00	PI	25.000	–1.838	23.162
+25		24.625	–1.350	23.275
+50		24.250	–0.938	23.312
+75		23.875	–0.600	23.275
151+00		23.500	–0.338	23.162
+25		23.125	–0.150	22.975
+50		22.750	–0.038	22.712
+75	EVC	22.375	0.000	22.375
152+00		22.000	0.000	22.000

(*b*) Profile



Problem d-9.

(*a*) Compute curve elevations and offsets from tangents at full stations and +25 points for a 250-m vertical curve joining a +2.60% grade with a –2.40% grade. Assume the PI is at station 200+00 and elevation 30.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.

(*b*) Plot the profile for the curve data in part *a*.

Solution.

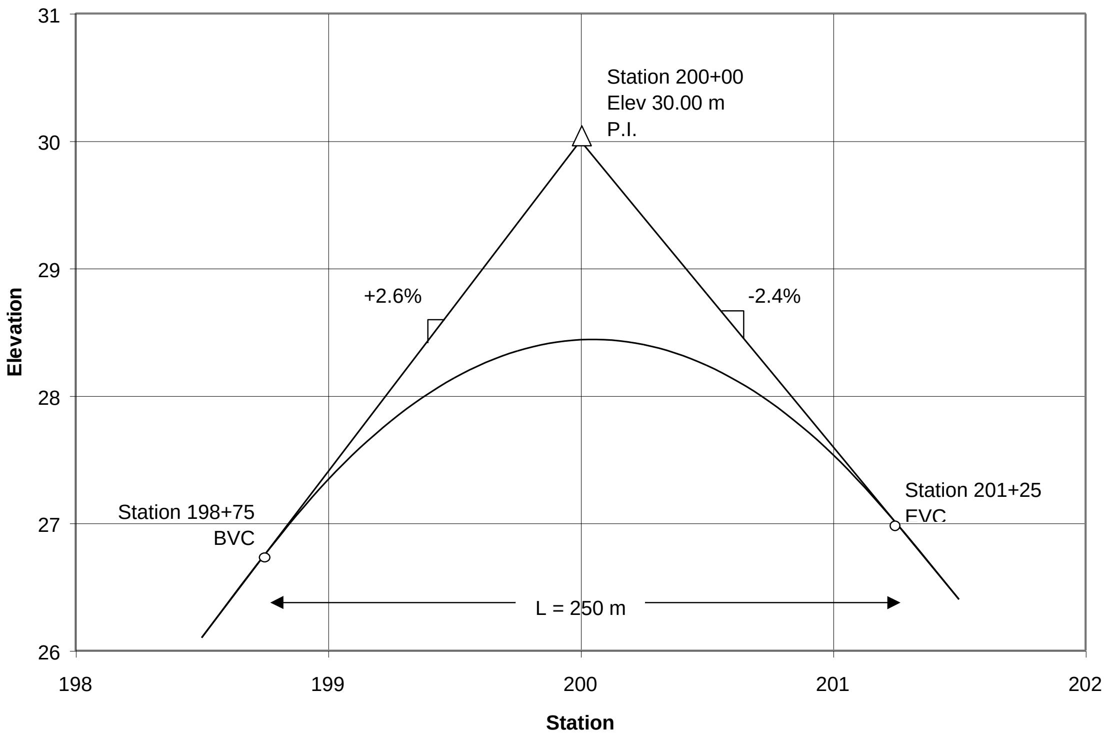
(*a*) Curve elevations

$$r = \frac{g_2 - g_1}{L} = \frac{-2.40\% - (+2.60\%)}{2.5\text{sta}} = -2.0\%/\text{Sta}$$

Station		Tangent Elevation	Offset	Curve Elevation, m
198+50		26.100	0.000	26.100
+75	BVC	26.750	0.000	26.750
199+00		27.400	–0.063	27.337
+25		28.050	–0.250	27.800
+50		28.700	–0.563	28.137
+75		29.350	–1.000	28.350
200+00	PI	30.000	–1.563	28.437
+25		29.400	–1.000	28.400
+50		28.800	–0.563	28.237

+75		28.200	–0.250	27.950
201+00		27.600	–0.063	27.537
+25	EVC	27.000	0.000	27.000
+50		26.400	0.000	26.400

(*b*) Profile



Problem d-10.

(*a*) Compute curve elevations and offsets from tangents at full stations and +25 points for a 300m vertical curve joining a +1.50% grade with a –3.30% grade. Assume the PI is at station 100+00 and elevation 60.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.

(*b*) Plot the profile for the curve data in part *a*.

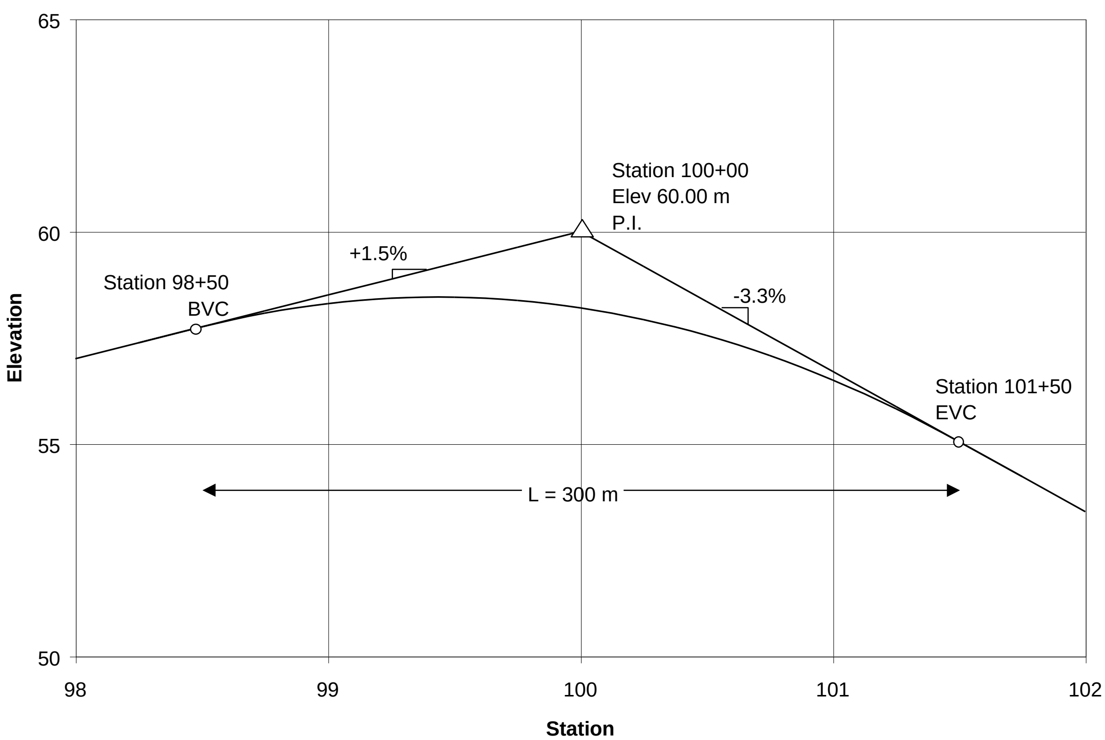
<u>Solution.</u>

(*a*) Curve elevations

$$r = \frac{g_2 - g_1}{L} = \frac{-3.30 - (1.50)}{3.0} = -1.6\%/\text{Sta}$$

Station		Tangent Elevation	Offset	Curve Elevation, m
99+25		57.375	0.000	57.375
+50	BVC	57.750	0.000	57.750
+75		58.125	–0.050	58.075
99+00		58.500	–0.200	58.300
+25		58.875	–0.450	58.425
+50		59.250	–0.800	58.450
+75		59.625	–1.250	58.375
100+00	PI	60.000	–1.800	58.200
+25		59.175	–1.250	57.925
+50		58.350	–0.800	57.550
+75		57.525	–0.450	57.075
101+00		56.700	–0.200	56.500
+25		55.875	–0.050	55.825
+50	EVC	55.050	0.000	55.050
101+75		54.225	0.000	54.225

(*b*) Profile



Problem d-11.

A 350-m vertical curve connects a +3.00% grade with a –2.00% grade. If the station of the BVC is 150+00, what is the station of the highest point on the curve?

Rate of change of grade:

$$r = \frac{g_2 - g_1}{L} = \frac{-2.00\% - (+3.00\%)}{3.5\text{sta}} = -1.429\%/\text{sta}$$

Station of the high point

At high point, $g = 0$

$$g = g_1 + rx = 0$$

or

$$x = \frac{-g_1}{r} = \frac{-3.00}{-1.429} = 2.10 = 2+10\ \text{sta}$$

Station of high point = (150+00) + (2+10) = 152+10

Problem d-12.

A 400-m vertical curve connects a –2.00% grade to a +4.00% grade. The PI is located at station 150+00 and elevation 60.00 m above sea level. A pipe is to be located at the low point on the vertical curve. The roadway at this point consists of two 3.6-m lanes with a normal crown slope of 2%. If the lowest point on the surface of the roadway must clear the pipe by 0.75 m, what is the station and maximum elevation of the pipe?

Rate of change of grade

$$r = \frac{g_2 - g_1}{L} = \frac{4.00\% - (-2.00\%)}{4.00\text{sta}} = 1.50\%/\text{sta}$$

Station of the low point

At low point, $g = 0$

$$g = g_1 + rx = 0$$

or

$$x = -\frac{g_1}{r} = -\frac{-2.00}{1.50} = 1.33\text{sta}$$

Station of BVC = (150+00) – (2+00) = 148+00

Station of low point (and pipe) = (148+00) + (1+33) = 149+33

Elevation of BVC

$$y_0 = 60.00\text{m} - (2\text{sta})(-2.00\%) = 64.00\text{m}$$

Elevation of low point

$$y = y_0 + g_1 x + \frac{rx^2}{2}$$

$$y = 64.00\text{m} + (-2.00\%)(1.33\text{sta}) + \frac{(1.50\%/\text{sta})(1.33\text{sta})^2}{2} = 62.67\text{m}$$

Elevation of top of pipe

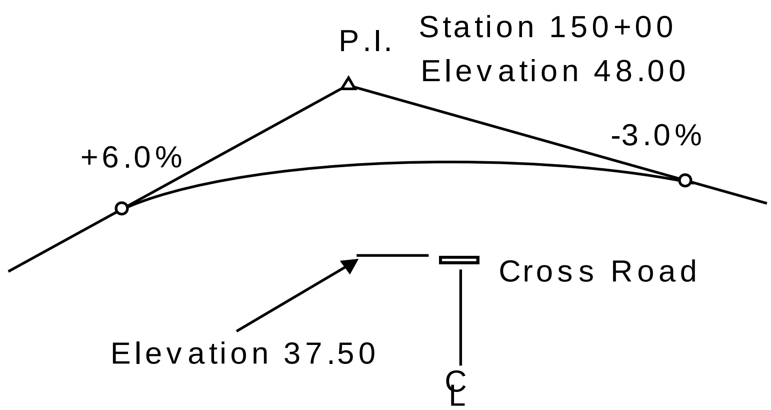
62.67 m – 0.02(3.6 m) – 0.75 m = 61.85 m

Problem d-13.

Given the profile below, determine:

(*a*) The length of vertical curve needed to make the highest point on the vertical curve come out exactly over the centerline of the cross road at station 150+70.

(*b*) The vertical clearance between the profile grade on the vertical curve and the centerline of the cross road.



Solution.

(a) Vertical curve length

$$x = L/2 + 0.70\,\text{sta}$$
$$r = \frac{g_2 - g_1}{L}$$
At high point, g = 0

$$g = g_1 + rx = 0$$

or

$$x = -g_1/L$$
$$L/2 + 0.70 = \frac{-g_1}{(g_2 - g_1)/L} = \frac{-6.00\%}{(-3.00\% - 6.00\%)/L}$$
$$L/2 + 0.70 = (2/3)L$$

$$L/6 = 0.70$$

L = 4.20 sta = 420 m

(b) Clearance
$$y = y_0 + g_1 x + \frac{rx^2}{2}$$
BVC elevation
$$y_0 = 48.00\text{m} - 6.0\%(2.1\text{sta}) = 35.4\text{m}$$
$$x = \frac{4.20}{2} + 0.70 = 2.80\,\text{sta}$$
$$r = \frac{-3.0\% - 6.0\%}{4.20\,\text{sta}} = -2.14\%/\text{sta}$$
$$y = 35.4 + 6.0\%(2.80\text{sta}) + \frac{2.14\%/\text{sta}(2.80\text{sta})^2}{2} = 43.8\text{m}$$
Clearance
Clearance = 43.8 m – 37.5 m = 6.3 m

Problem d-14.
A vertical curve joins a –0.5% grade to a +1.0% grade. The PI of the vertical curve is at station 200+00 and elevation 150.00 m above sea level. The centerline of the roadway must clear a pipe located at station 200+70 by 0.75 m. The elevation of the top of the pipe is 150.40 m above sea level. What is the minimum length of vertical curve that can be used?

Determine z

z = (200+70) − (200+00) = 0.70 sta

Determine y'

Elevation of tangent = 150.00 m − (0.5%)(0.70 sta) = 149.65 m

Elevation of roadway = 150.40 m + 0.75 m = 151.15 m

$y' = 151.15\,\text{m} - 149.65\,\text{m} = 1.50\,\text{m}$

Determine *w*

$A = g_2 - g_1 = 1.00\% - (-0.50\%) = 1.50\%$

$$w = \frac{y'}{A} = \frac{1.50}{1.50} = 1.00$$

Determine *L*

$L = 4w - 2z + 4\sqrt{w^2 - wz}$

$L = 4(1.00) - 2(0.70) + 4\sqrt{1.00^2 - (1.00)(0.70)} = 4.79\text{sta} = 479\text{m}$

Check y'

$$x = \frac{4.79}{2} + 0.70 = 3.095\,\text{sta}$$

$$r = \frac{1.50}{4.79} = 0.3132\,\%/\text{sta}$$

$$y' = \frac{rx^2}{2} = \frac{(0.3132)(3.095^2)}{2} = 1.50\text{m}\ \underline{\text{Check}}$$

Problem d-15.

A vertical curve joins a –2.0% grade to a +0.5% grade. The PI of the vertical curve is at station 100+00 and elevation 69.50 m above sea level. The centerline of the roadway must clear an overhead structure located at station 99+20 by 5.67 m. The elevation of the bottom of the structure is 77.45 m above sea level. What is the maximum length of vertical curve that can be used?

Determine z

z = (99+20) – (100+00) = –0.80 sta

Determine y'

Elevation of tangent = 69.50 m – (–2.0%)(0.80 sta) = 71.10 m

Elevation of roadway = 77.45 m – 5.67 m = 71.78 m

$$y' = 71.78\,\text{m} - 71.10\,\text{m} = 0.68\,\text{m}$$

Determine w

$$A = g_2 - g_1 = 0.50\% - (-2.00\%) = 2.50\%$$

$$w = \frac{y'}{A} = \frac{0.68}{2.50} = 0.272$$

Determine L

$$L = 4w - 2z + 4\sqrt{w^2 - wz}$$

$$L = 4(0.272) - 2(-0.80) + 4\sqrt{0.272^2 - (0.272)(-0.80)} = 4.85\text{sta} = 485\text{m}$$

Check y'

$$x = \frac{4.85}{2} - 0.80 = 1.625\text{sta}$$

$$r = \frac{2.50}{4.85} = 0.5155\%/\text{sta}$$

$$y' = \frac{rx^2}{2} = \frac{(0.5155)(1.625^2)}{2} = 0.68\text{m}\ \underline{\text{Check}}$$

Problem d-16.

Compute the minimum radius of a circular curve for a highway designed for 110 km/h. The maximum superelevation rate is 12%.

$$R = \frac{V^2}{127(f + e)}$$

$V = 110$ km/h

$f = 0.11$

$e = 0.12$

$$R = \frac{110^2}{127(0.11 + 0.12)} = 414.24\text{m}$$

Problem d-17.

Compute the minimum radius of a circular curve for a highway designed for 80 km/h. Because snow and ice are present, the maximum superelevation rate is 8%.

$$R = \frac{V^2}{127(f + e)}$$

$V = 80$ km/h

$f = 0.14$

$e = 0.08$

$$R = \frac{80^2}{127(0.14 + 0.08)} = 229.06\text{m}$$

Problem d-18.

Compute the minimum radius of a circular curve for a highway designed for 100 km/h. The maximum superelevation rate is 12%.

$$R = \frac{V^2}{127(f + e)}$$

$V = 100$ km/h

$f = 0.12$

$e = 0.12$

$$R = \frac{100^2}{127(0.12 + 0.12)} = 328.08\text{m}$$

Problem d-19.

(*a*) A two-lane highway (one 3.6-m lane in each direction) goes from normal crown with 2% cross-slopes to 10% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to 1/200. Round up to the next largest 20-m interval.

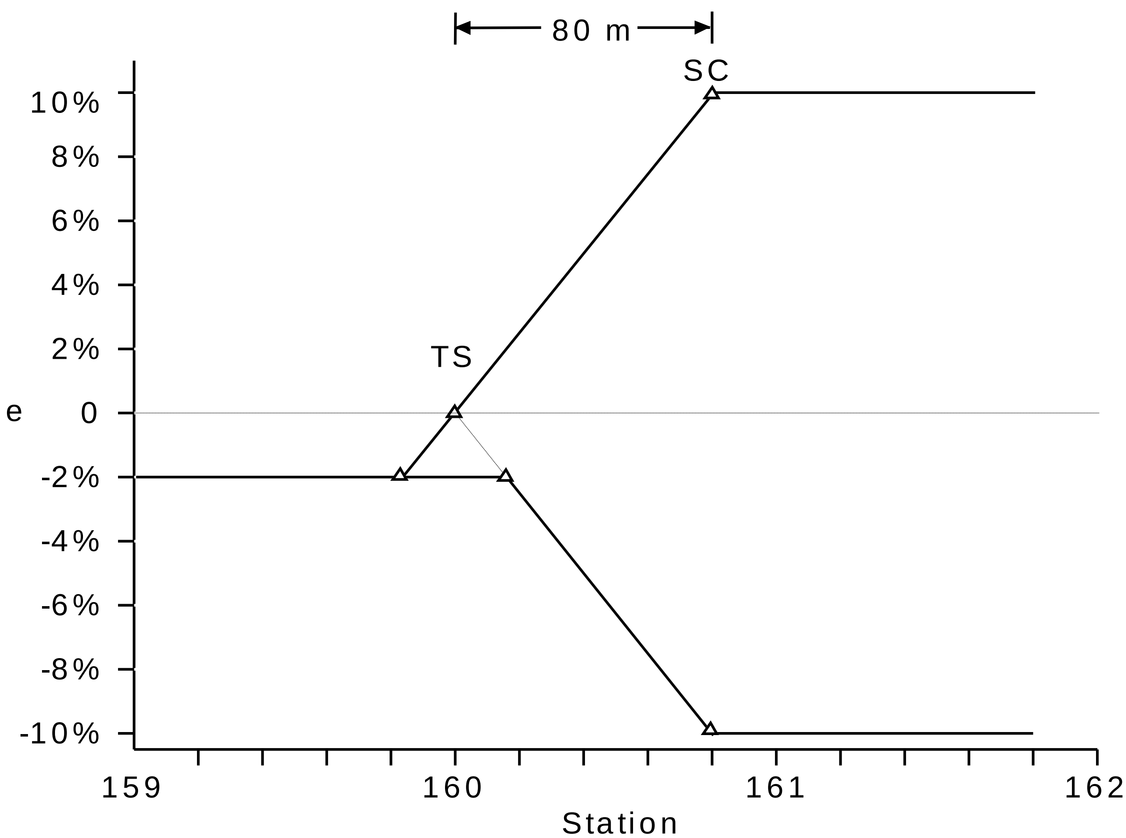
(*b*) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is 160+00.

Solution.

(*a*) Length of transition

$$L \geq 200De \geq 200(3.6\text{m})(0.10) \geq 72\text{m} \quad \text{Round to 80 m}$$

(*b*) Superelevation diagram



Problem d-20.

(*a*) A two-lane highway (one 3.6-m lane in each direction) goes from normal crown with 2% cross-slopes to 8% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to 1/200. Round up to the next largest 20-m interval.

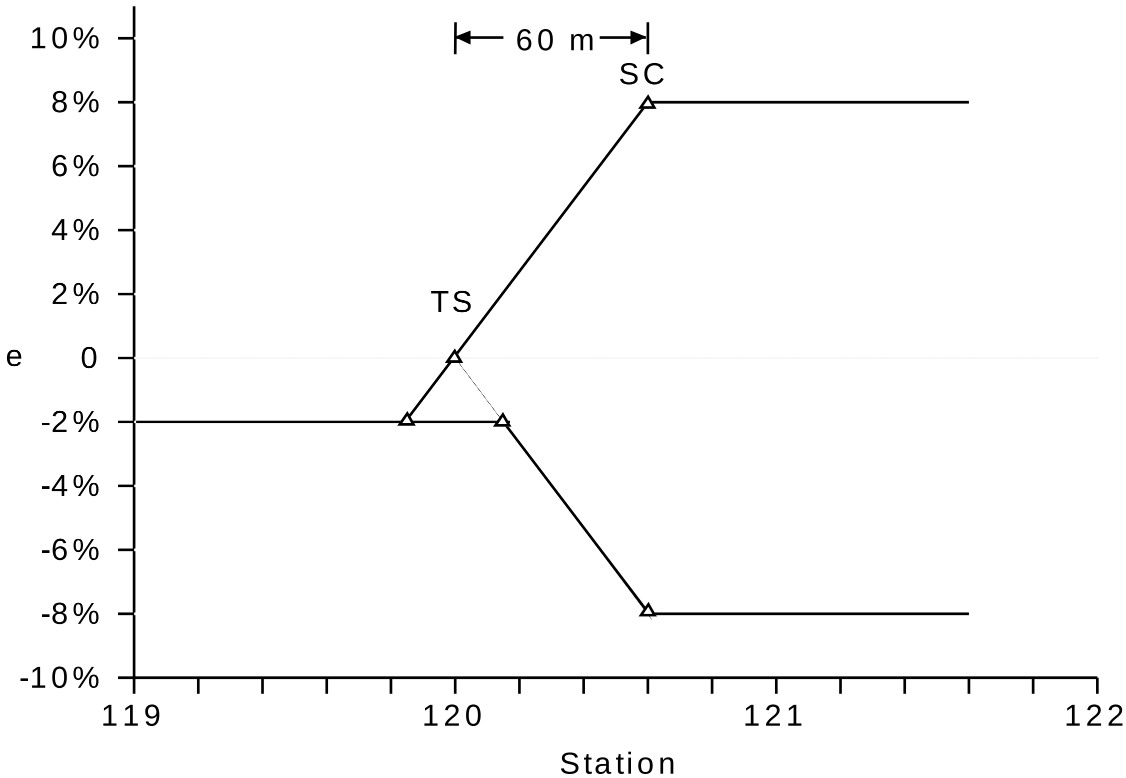
(*b*) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is 120+00.

Solution.

(a) Length of transition

$$L \geq 200De \geq 200(3.6\text{m})(0.08) \geq 57.6\text{m} \text{ Round to 60 m}$$

(b) Superelevation diagram



Problem d-21.

(*a*) A two-lane highway (one 3.6-m lane in each direction) goes from normal crown with 2% cross-slopes to 6% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to 1/200. Round up to the next largest 20-m interval.

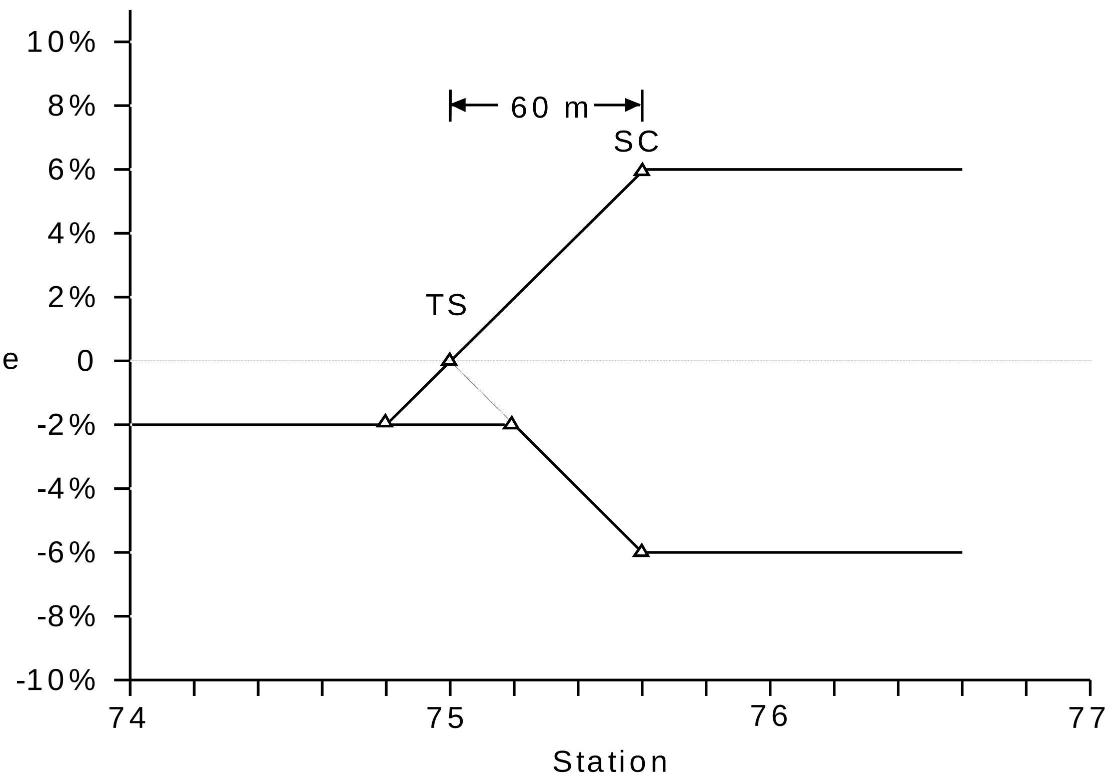
(*b*) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is 75+00.

Solution.

(*a*) Length of transition

$$L \geq 200De \geq 200(3.6\text{m})(0.06) \geq 43.2\text{m} \text{ Round to 60 m}$$

(*b*) Superelevation diagram



Problem d-22.

Prepare a table giving chords and deflection angles for staking out a 450-m-radius circular curve with a total deflection angle of 17°. The TC point is at station 22+40. Give deflection angles and chords for full stations and +20 points.

Radius = 450 m					
			Deflection Angle		
	Station	*x*, meters	Radians	Degrees	Chord, meters
TC	22+40	0.0	0.0000	0°00’00”	0.000
	22+60	20.0	0.0222	1°16’24”	19.998
	22+80	40.0	0.0444	2°32’47”	39.987
	23+00	60.0	0.0666	3°49’11”	59.956
	23+20	80.0	0.0888	5°05’35”	79.895
	23+40	100.0	0.1111	6°21’58”	99.794
	23+60	120.0	0.1333	7°38’22”	119.645
CT	23+73.5	133.5	0.1483	8°29’56”	133.011

Problem d-23.

Prepare a table giving chords and deflection angles for staking out a 650-m-radius circular curve with a total deflection angle of 13°. The TC point is at station 13+25. Give deflection angles and chords for full stations and +20 points.

Radius = 650 m					
			Deflection Angle		
	Station	*x*, meters	Radians	Degrees	Chord, meters
TC	13+25	0.0	0.0000	0°00’00”	0.000
	13+40	15.0	0.0115	0°39’40”	15.000
	13+60	35.0	0.0269	1°32’33”	34.996
	13+80	55.0	0.0423	2°25’27”	54.984
	14+00	75.0	0.0577	3°18’20”	74.958
	14+20	95.0	0.0731	4°11’13”	94.915
	14+40	115.0	0.0885	5°04’06”	114.850
	14+60	135.0	0.1038	5°57’00”	134.757
CT	14+72.5	147.5	0.1135	6°30’03”	147.184

Problem d-24.

Prepare a table giving chords and deflection angles for staking out a 480-m-radius circular curve with a total deflection angle of 22°. The TC point is at station 10+32. Give deflection angles and chords for full stations and +20 points.

Radius = 480 m					
			Deflection Angle		
	Station	*x*, meters	Radians	Degrees	Chord, meters
TC	10+32	0.0	0.0000	0°00’00”	0.000
	10+40	8.0	0.0083	0°28’39”	8.000
	10+60	28.0	0.0292	1°40’16”	27.996
	10+80	48.0	0.0500	2°51’53”	47.980
	11+00	68.0	0.0708	4°03’30”	67.943
	11+20	88.0	0.0917	5°15’08”	87.877
	11+40	108.0	0.1125	6°26’45”	107.772
	11+60	128.0	0.1333	7°38’22”	127.621
	11+80	148.0	0.1542	8°49’59”	147.414
	12+00	168.0	0.1750	10°01’36”	167.144
CT	12+16.3	184.3	0.1920	10°59’59”	183.170

Problem d-25

(*a*) A roadway goes from tangent alignment to a 250-m circular curve by means of a 80-m-long spiral transition curve. The deflection angle between the tangents is 45°. Use formulas to compute X_s, Y_s, *p*, and *k*. Assume that the station of the PI, measured along the back tangent, is 250+00, and compute the stations of the TS, SC, CS, and ST.

(*b*) Prepare a table giving coordinates, spiral angles, deflection angles, and chords (from the TS) for full stations and +20 points.

Solution.

(*a*) Spiral data

Determine spiral angle and coordinates of SC point

$$R_c = 250\,\text{m}$$

$$L_s = 80\,\text{m}$$

$$\theta_s = \frac{L_s}{2R_c} = \frac{80}{2(250)} = 0.160\text{rad}$$

$$A = \sqrt{L_s R_c} = \sqrt{(80)(250)} = 141.42$$

$$X_s = L_s - \frac{L_s^5}{40A^4} + \frac{L_s^9}{3{,}456A^8} = 80 - \frac{80^5}{40(141.42^4)} + \frac{80^9}{3{,}456(141.42^8)}$$

$$= 80 - 0.205 + 0.0002 = 79.795\text{ m}$$

$$Y_s = \frac{L_s}{6A^2} - \frac{L_s^7}{336A^6} + \frac{L_s^{11}}{42{,}240A^{10}}$$

$$= \frac{80^3}{6(141.42^2)} - \frac{80^7}{336(141.42^6)} + \frac{80^{11}}{42{,}240(141.42^{10})}$$

$$= 4.267 - 0.008 + 0.000006 = 4.259\text{ m}$$

Determine *p*, *k*, T', and L_c

$$p = Y_s - R(1 - \cos\theta_s) = 4.259 - 250[1 - \cos(0.160\text{rad})] = 1.066\text{m}$$

$$k = X_s - R_c \sin\theta_s = 79.795 - 250\sin(0.160\text{rad}) = 39.965\text{m}$$

$$T' = (R_c + p)\tan\left(\frac{\Delta}{2}\right) = (250 + 1.066)\tan\left(\frac{45°}{2}\right) = 103.995\text{m}$$

$$L_c = R_c\Delta_{rad} - L_s = 250(0.785) - 80 = 116.250\text{m}$$

Determine stations of critical points

TS station = PI station – (T' +k)

= (250+00) – [(1+04) + (0+40)] = 248+56

SC station = TS station + L_s = (248+56) + (0+80) = 249+36

CS station = SC station + L_c = (249+56) + (1+16) = 250+52

ST station = CS station + L_s = (250+52) + (0+80) = 251+32

(*b*) Table of coordinates, spiral angles, deflection angles, and chords

					Spiral angle, θ		Deflection angle, d		
	Station	L	X	Y	rad.	deg.	rad.	deg.	c
TS	248+56	0	0.000	0.000	0.0000	0°00'00"	0.0000	0°00'00"	0.000
	248+60	4	4.000	0.001	0.0004	0°01'23"	0.0001	0°00'28"	4.000
	248+80	24	24.000	0.115	0.0144	0°49'30"	0.0048	0°16'30"	24.000
	249+00	44	43.990	0.710	0.0484	2°46'23"	0.0161	0°55'28"	43.995
	249+20	64	63.933	2.183	0.1024	5°52'02"	0.0341	1°57'20"	63.970
SC	249+36	80	79.795	4.259	0.1600	9°10'02"	0.0533	3°03'18"	79.909

Problem d-26.

(*a*) A roadway goes from tangent alignment to a 275-m circular curve by means of a 100-m-long spiral transition curve. The deflection angle between the tangents is 60°. Use formulas to compute X_s, Y_s, *p*, and *k*. Assume that the station of the PI, measured along the back tangent, is 200+00, and compute the stations of the TS, SC, CS, and ST.

(*b*) Prepare a table giving coordinates, spiral angles, deflection angles, and chords (from the TS) for full stations and +20 points.

Solution.

(*a*) Spiral data

Determine spiral angle and coordinates of SC point

$$R_c = 275\text{m}$$

$$L_s = 100\,\text{m}$$

$$\theta_s = \frac{L_s}{2R_c} = \frac{100}{2(275)} = 0.182\text{rad}$$

$$A = \sqrt{L_s R_c} = \sqrt{(100)(275)} = 165.83$$

$$X_s = L_s - \frac{L_s^5}{40A^4} + \frac{L_s^9}{3{,}456A^8} = 100 - \frac{100^5}{40(165.83^4)} + \frac{100^9}{3{,}456(165.83^8)}$$

= 100 − 0.331 + 0.0005 = 99.670 m

$$Y_s = \frac{L_s}{6A^2} - \frac{L_s^7}{336A^6} + \frac{L_s^{11}}{42240A^{10}}$$

$$= \frac{100^3}{6(165.83^2)} - \frac{100^7}{336(165.83^6)} + \frac{100^{11}}{42240(165.83^{10})}$$

= 6.061 − 0.014 + 0.00002 = 6.047 m

Determine p, k, T', and L_c

$$p = Y_s - R(1 - \cos\theta_s) = 6.047 - 275[1 - \cos(0.182\text{rad})] = 1.505\text{m}$$

$$k = X_s - R_c \sin\theta_s = 99.670 - 275\sin(0.182\text{rad}) = 49.896\text{m}$$

$$T' = (R_c + p)\tan\left(\frac{\Delta}{2}\right) = (275 + 1.505)\tan\left(\frac{60°}{2}\right) = 159.640\text{m}$$

$$L_c = R_c \Delta_{rad} - L_s = 275(1.047) - 100 = 187.825\text{m}$$

Determine stations of critical points

TS station = PI station − (T' + k)

= (200+00) − [(1+60)+ (0+50)] =197+90

SC station = TS station + L_s = (197+90) + (1+00) = 198+90

CS station = SC station + L_c = (198+90) + (1+88) = 200+78

ST station = CS station + L_s = (200+78) + (1+00) = 201+78 (*b*) Table of coordinates, spiral angles, deflection angles, and chords

	Station	L	X	Y	Spiral angle, θ		Deflection angle, d		
					rad.	deg.	rad.	deg.	c
TS	197+90	0	0.000	0.000	0.0000	0°00'00"	0.0000	0°00'00"	0.000
	198+00	10	10.000	0.006	0.0018	0°06'15"	0.0006	0°02'05"	10.000
	198+20	30	29.999	0.164	0.0164	0°56'15"	0.0055	0°18'45"	30.000
	198+40	50	49.990	0.757	0.0455	2°36'16"	0.0152	0°52'05"	49.995
	198+60	70	69.944	2.078	0.0891	5°06'16"	0.0297	1°42'05"	69.975
	198+80	90	89.805	4.411	0.1473	8°26'17"	0.0491	2°48'44"	89.913
SC	198+90	100	99.670	6.046	0.1818	10° 25'03"	0.0606	3°28'17"	99.853

Problem d-27.

(*a*) A roadway goes from tangent alignment to a 380-m circular curve by means of a 60-m-long spiral transition curve. The deflection angle between the tangents is 40°. Use formulas to compute X_s, Y_s, *p*, and *k*. Assume that the station of the PI, measured along the back tangent, is 100+00, and compute the stations of the TS, SC, CS, and ST.

(*b*) Prepare a table giving coordinates, spiral angles, deflection angles, and chords (from the TS) for full stations and +20 points.

Solution.

(*a*) Spiral data

Determine spiral angle and coordinates of SC point

$$R_c = 380\,\text{m}$$

$$L_s = 60\,\text{m}$$

$$\theta_s = \frac{L_s}{2R_c} = \frac{60}{2(380)} = 0.079\text{rad}$$

$$A = \sqrt{L_s R_c} = \sqrt{(60)(380)} = 151.00$$

$$X_s = L_s - \frac{L_s^5}{40A^4} + \frac{L_s^9}{3456A^8} = 60 - \frac{60^5}{40(151.00^4)} + \frac{60^9}{3456(151.00^8)}$$

$= 60 - 0.037 + 0.00001 = 59.963$ m

$$Y_s = \frac{L_s}{6A^2} - \frac{L_s^7}{336A^6} + \frac{L_s^{11}}{42240A^{10}}$$

$$= \frac{60^3}{6(151.00^2)} - \frac{60^7}{336(151.00^6)} + \frac{60^{11}}{42240(151.00^{10})}$$

$= 1.579 - 0.001 + 0.000 = 1.578$ m

Determine p, k, T', and L_c

$$p = Y_s - R(1 - \cos\theta_s) = 1.578 - 380[1 - \cos(0.079\text{rad})] = 0.393\text{m}$$

$$k = X_s - R_c \sin\theta_s = 59.963 - 380\sin(0.079\text{rad}) = 29.974\text{m}$$

$$T' = (R_c + p)\tan\left(\frac{\Delta}{2}\right) = (380 + 0.393)\tan\left(\frac{40°}{2}\right) = 138.452\text{m}$$

$$L_c = R_c\Delta_{rad} - L_s = 380(0.698) - 60 = 205.290\text{m}$$

Determine stations of critical points

TS station = P. I. station – (T' + k)

= (100+00) – [(1+38) + (0+30)] = 98+32

SC station = TS station + L_s = (98+32) + (0+60) = 98+92

CS station = SC station + L_c = (98+92) + (1+49) = 100+41

ST station = CS station + L_s = (100+41) + (0+60) = 101+01

(*b*) Table of coordinates, spiral angles, deflection angles, and chords

					Spiral angle, θ		Deflection angle, d		
	Station	L	X	Y	rad.	deg.	rad.	deg.	c
TS	98+32	0	0.000	0.000	0.0000	0°00’00”	0.0000	0°00’00”	0.000
	98+40	8	8.000	0.004	0.0014	0°04’49”	0.0005	0°01’36”	8.000
	98+60	28	27.999	0.160	0.0172	0°59’06”	0.0057	0°19’42”	28.000
	98+80	48	47.988	0.808	0.0505	2°53’42”	0.0168	0°57’54”	47.995

SC	98+92	60	59.693	1.578	0.0789	4°31’24”	0.0263	1°30’28”	59.983

Problem d-28.

A circular curve with a radius of 350 m is connected by 60-m spiral transition curves to tangents with a deflection angle of 0.349 rad. If the station of the TS is 105+40, determine the station of the ST.

$$\theta_s = \frac{L_s}{2R_c} = \frac{60}{2(350)} = 0.0857\text{rad}$$

$$L_c = R_c\Delta_{rad} - L_s = 350(0.349) - 60 = 62.150\text{m}$$

SC station = TS station + L_s = (105+40) + (0+60) = 106+00

CS station = SC station + L_c = (106+00) + (0+62) = 106+62

ST station = CS station + L_s = (106+62) + (0+60) = 107+22

Problem d-29.

A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.26 rad. Stations of critical points are as follows: TS, 105+00; SC, 105+80; CS, 107+50; ST, 108+30. The roadway is a two-lane highway with one 3.6-m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly 1/200, at what speed can the curve be taken with no side friction?

$$L_c = (107+50) - (105+80) = 1+70 = 170\text{m}$$

$$L_s = (105+80) - (105+00) = 0+80 = 80\text{m}$$

$$L_c = R_c\Delta - L_s$$

$$R_c\Delta = L_c + L_s$$

$$R_c = \frac{L_c + L_s}{\Delta} = \frac{170+80}{0.26} = 961.538\,\text{m}$$

$$L_s = 200De$$

$$e = \frac{L_s}{200D} = \frac{80}{200(3.6)} = 0.11$$

$$R_c = \frac{V^2}{127(f+e)}$$

$$V^2 = 127(f+e)R_c$$

$$V = \sqrt{127(f+e)R_c} = \sqrt{127(0.00+0.11)(961.538)} = 116\text{km/h}$$

Problem d-30.

A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.30 rad. Stations of critical points are as follows: TS, 308+00; SC, 308+40; CS, 310+40; ST, 310+80. The roadway is a two-lane highway with one 3.6-m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly 1/200, what is the maximum speed that can be maintained on the curve if the side friction is limited to 0.10?

$$L_c = (310+40)-(308+40) = 2+00 = 200\text{m}$$

$$L_s = (308+40)-(308+00) = 0+40 = 40\text{m}$$

$$L_c = R_c\Delta - L_s$$

$$R_c\Delta = L_c + L_s$$

$$R_c = \frac{L_c + L_s}{\Delta} = \frac{200+40}{0.30} = 800\,\text{m}$$

$$L_s = 200De$$

$$e = \frac{L_s}{200D} = \frac{40}{200(3.6)} = 0.056$$

$$R_c = \frac{V^2}{127(f+e)}$$

$$V^2 = 127(f+e)R_c$$

$$V = \sqrt{127(f+e)R_c} = \sqrt{127(0.10+0.056)(800)} = 126\text{km/h}$$

Problem d-31.

The allowable side friction factor for horizontal curves with a design speed of 100 km/h is 0.12.

(*a*) What superelevation rate would you use for a curve with a design speed of 100 km/h and a radius of 420 m? Round to the nearest whole percent.

(*b*) A spiral transition curve is used to go from a normal crown slope with 2% cross-slopes to full superelevation for the curve described above. If the maximum difference in grade between the centerline and the edge is 1/200 and the roadway consists of two 3.6-m lanes, what is the minimum length of spiral? Round up to the next integral multiple of 20 m.

(*c*) The station of the TS is 501+00. Draw the superelevation diagram.

Solution.

(*a*) Superelevation rate

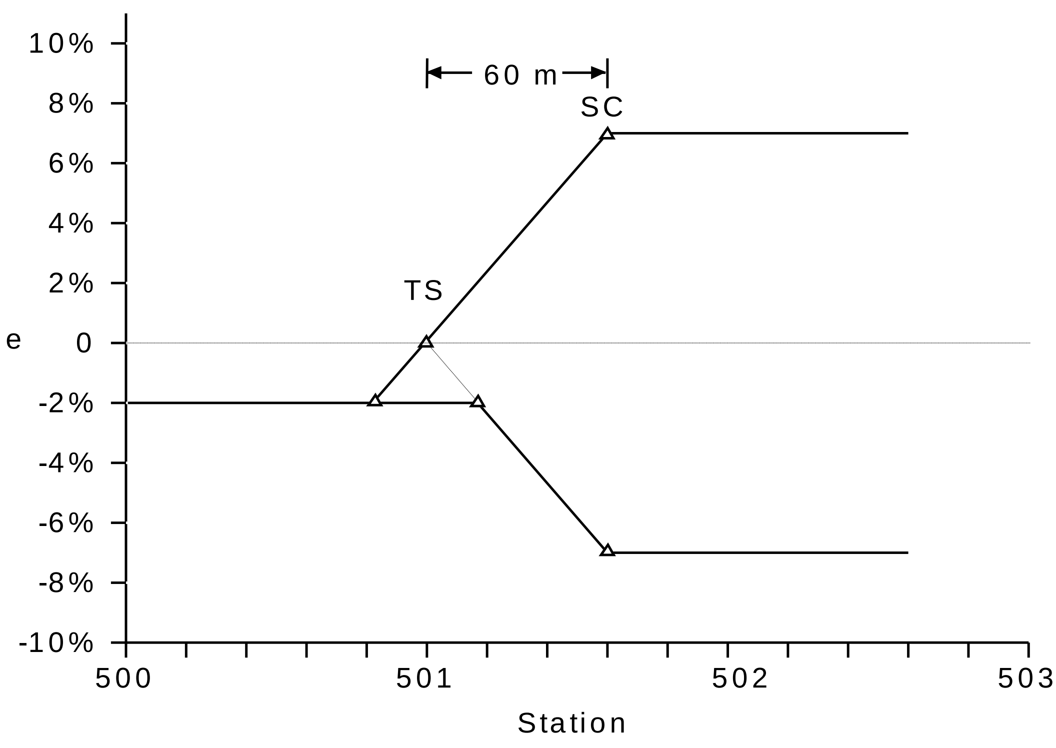
$$R = \frac{V^2}{127(f+e)}$$

$$e = \frac{V^2}{127R} - f = \frac{100^2}{127(420)} - 0.12 = 0.07$$

(*b*) Length of spiral

$$L_s \geq 200De \geq 200(3.6)(0.07) \geq 50.4\text{m} \text{ Round up to 60 m}$$

(c) Superelevation diagram



COMPUTER EXERCISES

d-1.

Programming. Write a computer program to compute the characteristics of vertical curves. The program should be interactive. The user is to enter the grades and the design speed. From this the program calculates the minimum length of vertical curve according to AASHTO standards for stopping sight distance and the California appearance criteria for minimum length of vertical curve. Use stopping sight distance values halfway between the extremes given in Table 3-3. The user then enters the station and elevation of the PI of the vertical curve and a length which must be at least as great as that calculated by the program. The program then calculates a table giving the station and elevation of points on the vertical curve at 20-m intervals. Output should also document the design speed; sight distance; minimum vertical curve length; grades; station and elevation of the PI, BVC, and EVC; and the actual length of the vertical curve.

Source code

```
C*********************************************************************
C*                                                   *
C*          PROGRAMMING EXERCISE EXERCISE 4-1                    *
C*                   VERTICAL CURVE                       *
C*                                                   *
C*********************************************************************

C*********************************************************************
C*                                                   *
C*                  DECLARE VARIABLES                      *
C*                                                   *
C*********************************************************************


    REAL S(11),H1,H2,V,LMIN,L,PISTA,PIELEV,BVCSTA,BVCELV,EVCSTA
    REAL EVCELV,X,ELEV,G1,G2,A,LSTA,LAPP
    INTEGER I,ISTA1,ISTA2
    CHARACTER RPLY*1
    DATA ITERM /5/,ITERMO/6/,INFL/10/,IOUT/11/,H1/0.150/,H2/1.070/

C********************************************************************
C*                                                  *
C*           OPEN FILES, READ INPUT FILE                    *
C*                                                  *
C********************************************************************
C*                                                  *
C*                 VARIABLES                         *
C*                                                  *
C*   ITERM = UNIT NUMBER FOR KEYBOARD                          *
C*   ITERMO = UNIT NUMBER FOR SCREEN                          *
```

```
C*    INFL = UNIT NUMBER FOR INPUT FILE                              *
C*    IOUT = UNIT NUMBER FOR OUTPUT FILE                             *
C*    S(I) = STOPPING SIGHT DISTANCE                                 *
C*                                                                   *
C********************************************************************

      OPEN(INFL,FILE='SSD1.DAT')
      OPEN(IOUT,FILE='VCOUT.DAT',FORM='FORMATTED')
      READ(INFL,*)(S(I),I=1,11)


C********************************************************************
C*                                                                   *
C*                ENTER DATA FROM KEYBOARD                           *
C*                                                                   *
C********************************************************************
C*                                                                   *
C*                   VARIABLES                                       *
C*                                                                   *
C*    H1 = HEIGHT OF OBJECT, METERS                                  *
C*    H2 = HEIGTH OF EYE, METERS                                     *
C*    G1 = TANGENT GRADE BEFORE P.I.                                 *
C*    G2 = TANGENT GRADE AFTER P.I.                                  *
C*    V = DESIGN SPEED KM/H                                          *
C*    RPLY = 'Y' OR 'N' -- REPLY ENTERED BY USER                     *
C*                                                                   *
C********************************************************************

   10 WRITE(ITERMO,*) 'ENTER G1 IN PER CENT'
      READ(ITERM,*)G1
      WRITE(ITERMO,*) 'ENTER G2 IN PER CENT'
      READ(ITERM,*)G2
   11 WRITE(ITERMO,*) 'ENTER DESIGN SPEED IN KM/H'
      READ(ITERM,*)V
      IF (V.LT.30.OR.V.GT.130) THEN
        WRITE(ITERMO,*) 'SPEED MUST BE BETWEEN 30 KM/H AND 130 KM/H'
        WRITE (ITERMO,*) 'RETRY? [Y] OR [N]'
        READ (ITERM,100) RPLY
  100   FORMAT(A1)
        IF(RPLY.NE.'Y') THEN
          STOP
        ELSE
          GO TO 11
        ENDIF
      ENDIF

C********************************************************************
C*                                                                   *
C*             CALCULATE MINIMUM CURVE LENGTH                        *
C*                                                                   *
C********************************************************************
```

```
C*                                                     *
C*                  VARIABLES                          *
C*                                                     *
C*   ISPD = INDEX FOR STOPPING SIGHT DISTANCE ARAAY                *
C*   A = DIFFERENCE IN GRADE                                 *
C*   LMIN = MINIMUM LENGTH OF VERTICAL CURVE                       *
C*   LAPP = MINIMUM LENGTH OF VERTICAL CURVE, BASED ON APPEARANCE    *
C*                                                     *
C*******************************************************************

      ISPD=((V-30)/10)+1
      IF(G1.GT.G2) THEN
        A=G1-G2
        LMIN=(A*S(ISPD)**2)/(200*(SQRT(H1)+SQRT(H2))**2)
        IF(S(ISPD).GT.LMIN) THEN
          LMIN=2*S(ISPD)-((200*(SQRT(H1)+SQRT(H2))**2)/A)
        ENDIF
      ELSE
        A=G2-G1
        LMIN=(A*S(ISPD)**2)/(120+3.5*S(ISPD))
        IF (S(ISPD).GT.LMIN) THEN
          LMIN=2*S(ISPD) - (120+3.5*S(ISPD))/A
        END IF
      END IF
      IF(V.GE.60.AND.ABS(A).GE.2)THEN
        LAPP=2*V
      ELSE
        LAPP=60
      ENDIF
      IF(LAPP.GT.LMIN)LMIN=LAPP
      WRITE (ITERMO,101)LMIN
      WRITE (IOUT,101)LMIN
  101 FORMAT (1X, 'MINIMUM LENGTH OF VERTICAL CURVE IS ',F8.2,/)

C*******************************************************************
C*                                                     *
C*               ENTER DATA FOR CURVE TABLE                  *
C*                                                     *
C*******************************************************************
C*                                                     *
C*                  VARIABLES                          *
C*                                                     *
C*   L = VERTICAL CURVE LENGTH (ENTERED BY USER)                 *
C*   PISTA = P.I. STATION                                  *
C*   PIELEV = P.I. ELEVTION                                  *
C*                                                     *
C*******************************************************************

   12 WRITE (ITERMO,*) 'ENTER ACTUAL VERTICAL CURVE LENGTH'
      READ (ITERM,*)L
      IF (L.LT.LMIN) THEN
```

```
      WRITE(ITERMO,*) 'VERTICAL CURVE LENGTH LESS THAN MIMIMUM. RETRY'
      GO TO 12
     END IF
     WRITE (ITERMO,*) 'ENTER PI STATION IN DECIMAL FORM'
     READ (ITERM, *)PISTA
     WRITE (ITERMO,*)'ENTER PI ELEVATION'
     READ (ITERM,*)PIELEV

C*********************************************************************
C*                                    *
C*           WRITE COLUMN HEADINGS FOR TABLE           *
C*                                    *
C*********************************************************************

     WRITE(ITERMO,102)
  102 FORMAT(5X,'STATION',10X,'ELEVATION',)
     WRITE(IOUT,102)




C*********************************************************************
C*                                    *
C*             CALCULATE CURVE DATA              *
C*                                    *
C*********************************************************************
C*                                    *
C*              VARIABLES                 *
C*                                    *
C*   LSTA = CURVE LENGTH IN STATIONS                  *
C*   BVCELV = BVC ELEVATION                      *
C*   BVCSTA = BVC STATION                      *
C*   EVCELV = EVC ELEVATION                      *
C*   EVCSTA = EVC STATION                      *
C*   N = NUMBER OF POINTS IN CURVE TABLE               *
C*                                    *
C*********************************************************************

     LSTA=L/100
     BVCELV=PIELEV-(G1*LSTA/2)
     BVCSTA=PISTA-LSTA/2
     EVCSTA=PISTA+LSTA/2
     EVCELV=PIELEV+(G2*LSTA/2)
     N=INT(L/20)

C*********************************************************************
C*                                    *
C*            CONSTRUCT CURVE TABLE              *
C*                                    *
C*********************************************************************
C*                                    *
```

```
C*                      VARIABLES                                *
C*                                                          *
C*    X = DISTANCE FROM BVC TO POINT ON CURVE                         *
C*    XSTA = STATION AT DISTANCE X FROM BVC                           *
C*    R = RATE OF CHANGE OF GRADE                                 *
C*    ELEV = CURVE ELEVATION                                    *
C*    ISTA1 = PORTION OF STATION NUMBER LEFT OF + SIGN                *
C*    ISTA2 = PORTION OF STATION NUMBER RIGHT OF + SIGN               *
C*                                                          *
C**********************************************************************

      DO 20 I=0,N
        X=I*.2
        XSTA=BVCSTA+X
        R=(G2-G1)/LSTA
        ELEV=BVCELV+G1*X+(R*X**2/2)
        ISTA1=INT(XSTA)
        ISTA2=NINT(100*(XSTA-ISTA1))
        WRITE(ITERMO,103)ISTA1,ISTA2,ELEV
        WRITE(IOUT,103)ISTA1,ISTA2,ELEV
  103   FORMAT(5X,I4,'+',I2.2,10X,F9.2)
   20 CONTINUE



C**********************************************************************
C*                                                          *
C*                      OUTPUT CURVE DATA                          *
C*                                                          *
C**********************************************************************

      ISTA1=INT(PISTA)
      ISTA2=NINT(100*(PISTA-ISTA1))
      WRITE(ITERMO,104)ISTA1,ISTA2,PIELEV
      WRITE(IOUT,104)ISTA1,ISTA2,PIELEV
  104 FORMAT(/,1X,'PI STATION = ',I5,'+',I2.2,10X,
     2       'PI ELEVATION = ',F8.2)
      ISTA1=INT(BVCSTA)
      ISTA2=NINT(100*(BVCSTA-ISTA1))
      WRITE(ITERMO,105)ISTA1,ISTA2,BVCELV
      WRITE(IOUT,105)ISTA1,ISTA2,BVCELV
  105 FORMAT(1X,'BVC STATION = ',I4,'+',I2.2,10X,
     2       'BVC ELEVATION = ',F7.2)
      ISTA1=INT(EVCSTA)
      ISTA2=NINT(100*(EVCSTA-ISTA1))
      WRITE(ITERMO,106)ISTA1,ISTA2,EVCELV
      WRITE(IOUT,106)ISTA1,ISTA2,EVCELV
  106 FORMAT(1X,'EVC STATION = ',I4,'+',I2.2,10X,
     2       'EVC ELEVATION = ',F7.2)
      WRITE(ITERMO,107)G1,G2,L
```

```
      WRITE(IOUT,107)G1,G2,L
  107 FORMAT(/,1X,'G1 = ',F5.2,' %',5X,'G2 = ',F5.2,' %',5X,
     2      'L = ',F8.2,' M',/////)
      WRITE(ITERMO,*)'DO YOU WISH TO CALCULATE DATA FOR ANOTHER V. C.?'
      WRITE(ITERMO,*)'ENTER [Y] OR [N]'
      READ(ITERM,100)RPLY
      IF(RPLY.EQ.'Y') GO TO 10
      STOP
      END
```

Input file

```
30
50
65
85
105
130
160
190
220
255
290
```

Output file

```
 MINIMUM LENGTH OF VERTICAL CURVE IS   607.25

    STATION          ELEVATION
      6+90             75.08
      7+10             75.70
      7+30             76.27
      7+50             76.80
      7+70             77.29
      7+90             77.73
      8+10             78.13
      8+30             78.49
      8+50             78.80
      8+70             79.06
      8+90             79.29
      9+10             79.47
      9+30             79.60
      9+50             79.69
      9+70             79.74
      9+90             79.74
     10+10             79.70
     10+30             79.62
     10+50             79.49
     10+70             79.32
     10+90             79.11
```

11+10	78.85
11+30	78.54
11+50	78.20
11+70	77.81
11+90	77.37
12+10	76.89
12+30	76.37
12+50	75.80
12+70	75.19
12+90	74.54
13+10	73.84

PI STATION = 10+00 PI ELEVATION = 85.00
BVC STATION = 6+90 BVC ELEVATION = 75.08
EVC STATION = 13+10 EVC ELEVATION = 73.84

G1 = 3.20 % G2 = −3.60 % L = 620.00 M

MINIMUM LENGTH OF VERTICAL CURVE IS 60.00

STATION	ELEVATION
7+70	50.36
7+90	50.08
8+10	49.71
8+30	49.25

PI STATION = 8+00 PI ELEVATION = 50.00
BVC STATION = 7+70 BVC ELEVATION = 50.36
EVC STATION = 8+30 EVC ELEVATION = 49.25

G1 = −1.20 % G2 = −2.50 % L = 60.00 M

MINIMUM LENGTH OF VERTICAL CURVE IS 180.00

STATION	ELEVATION
11+10	68.65
11+30	68.92
11+50	69.14
11+70	69.30
11+90	69.41
12+10	69.46
12+30	69.45
12+50	69.39
12+70	69.27
12+90	69.10

PI STATION = 12+00 PI ELEVATION = 70.00
BVC STATION = 11+10 BVC ELEVATION = 68.65

EVC STATION = 12+90 EVC ELEVATION = 69.10

G1 = 1.50 % G2 = −1.00 % L = 180.00 M

MINIMUM LENGTH OF VERTICAL CURVE IS 160.00

STATION	ELEVATION
14+20	102.40
14+40	101.83
14+60	101.30
14+80	100.83
15+00	100.40
15+20	100.03
15+40	99.70
15+60	99.43
15+80	99.20

PI STATION = 15+00 PI ELEVATION = 100.00
BVC STATION = 14+20 BVC ELEVATION = 102.40
EVC STATION = 15+80 EVC ELEVATION = 99.20

G1 = −3.00 % G2 = −1.00 % L = 160.00 M

MINIMUM LENGTH OF VERTICAL CURVE IS 200.00

STATION	ELEVATION
19+00	122.00
19+20	121.64
19+40	121.34
19+60	121.11
19+80	120.96
20+00	120.88
20+20	120.86
20+40	120.92
20+60	121.04
20+80	121.24
21+00	121.50

PI STATION = 20+00 PI ELEVATION = 120.00
BVC STATION = 19+00 BVC ELEVATION = 122.00
EVC STATION = 21+00 EVC ELEVATION = 121.50

G1 = −2.00 % G2 = 1.50% L = 200.00 M

MINIMUM LENGTH OF VERTICAL CURVE IS 286.12

STATION	ELEVATION
12+50	115.40
12+70	114.73
12+90	114.16
13+10	113.70
13+30	113.33
13+50	113.07
13+70	112.90
13+90	112.84
14+10	112.88
14+30	113.02
14+50	113.27
14+70	113.61
14+90	114.06
15+10	114.60
15+30	115.25
15+50	116.00

PI STATION = 14+00 PI ELEVATION = 110.00
BVC STATION = 12+50 BVC ELEVATION = 115.40
EVC STATION = 15+50 EVC ELEVATION = 116.00

G1 = −3.60 % G2 = 4.00 % L = 300.00 M

d-2.

Programming. Write a computer program to design superelevation transitions and spiral transition curves. The program should be interactive. The user is to enter the design speed, the radius of curvature for the horizontal curve, the deflection angle for the horizontal tangents, the distance from the axis of rotation to the edge of traveled way, and the imaginary station of the PI. From this, the program calculates the superelevation rate by means of the following formula:

$$R = \frac{V^2}{127(f+e)}$$

Superelevation rates may range from 0.02 to 0.10 and should be rounded up to the next highest multiple of 0.01. If the calculated superelevation rate exceeds 0.10, the program should display a message indicating that the radius of curvature is too small for the design speed and prompt entry of a new curve radius. Once a suitable superelevation rate is determined, the program should calculate the length of the superelevation transitions (and the spirals) based on the criterion that the difference in grade between the axis of rotation and the edge of traveled way should be no greater than 0.005. Transition lengths should be rounded up to the next 20-m interval, with a minimum transition length of 40 m. The program should then calculate the coordinates, spiral angles, deflection angles, and chords at 20-m intervals along the spirals and the stations of the TS, SC, CS, and ST. Output should document in suitable form: the design speed, radius, deflection angle, distance from axis of rotation to edge of traveled way, superelevation rate, and length of the circular portion of the curve; the length of the spirals as well as the values of *p* and *k* for the spirals; the spiral coordinates, spiral angles, deflection angles, and chords for 20-m intervals; and the stations of the TS, SC, CS, and ST.

Source code

```
C*******************************************************************
C*                                                                 *
C*                  COMPUTER EXERCISE 4.2                          *
C*               SPIRAL TRANSITION CURVE DATA                      *
C*                                                                 *
C*******************************************************************

C*******************************************************************
C*                                                                 *
C*                    DECLARE VARIABLES                            *
C*                                                                 *
C*******************************************************************

      REAL V,RC,DELTA,D,F(10),E,E1,EREM,LS,A,L,X,Y,THETA,XS,YS,THETAS
      REAL P,K,T,LC,TS,SC,CS,ST
      INTEGER I,N,ITERM,ITERMO,IOUT,IIN,IFF
      DATA ITERM/5/,ITERMO/6/,IIN/10/,IOUT/11/

C*******************************************************************
C*                                                                 *
C*               OPEN FILES AND READ INPUT FILE                    *
C*                                                                 *
C*******************************************************************
C*                                                                 *
C*                       VARIABLES                                 *
C*                                                                 *
C*   ITERM = UNIT NUMBER FOR KEYBOARD                              *
C*   ITERMO = UNIT NUMBER FOR SCREEN                               *
C*   IIN = UNIT NUMBER FOR INPUT FILE                              *
C*   IOUT = UNIT NUMBER FOR OUTPUT FILE                            *
C*   F(I) = FRICTION FACTORS                                       *
C*                                                                 *
C*******************************************************************

      OPEN(IIN,FILE='FRCFAC.DAT')
      OPEN(IOUT,FILE='SPIRAL.DAT')
      READ(IIN,*)(F(I),I=1,10)
```

```
C*****************************************************************
C*                                          *
C*        ENTER DATA FROM KEYBOARD AND ECHO TO OUTPUT FILE       *
C*                                          *
C*****************************************************************
C*                                          *
C*                  VARIABLES                        *
C*                                          *
C*    V = DESIGN SPEED                              *
C*    RC = CIRCULAR CURVE RADIUS                          *
C*    DELTA = DEFLECTION ANGLE                            *
C*    D = DISTANCE FROM AXIS OF ROTATION OT EDGE                 *
C*    PISTA = STATION OF P.I.                          *
C*    IFF = INDEX FOR FRICTION FACTOR ARRAY                  *
C*                                          *
C*****************************************************************

      WRITE(ITERMO,*)'ENTER DESIGN SPEED IN KM/H'
      READ(ITERM,*)V
      WRITE(IOUT,500)V
  500 FORMAT(1X,'DESIGN SPEED = ',F14.2,' KM/H')
      WRITE(ITERMO,*)'ENTER CURVE RADIUS IN METERS'
      READ(ITERM,*)RC
      WRITE(ITERMO,*)'ENTER DEFLECTION ANGLE IN DEGREES'
      READ(ITERM,*)DELTA
      WRITE(IOUT,502)DELTA
  502 FORMAT(1X,'DEFLECTION ANGLE = ',F10.2,' DEGREES')
      DELTA = .01745*DELTA
      WRITE(ITERMO,*)'ENTER DISTANCE FROM AXIS TO EDGE IN METERS'
      READ(ITERM,*)D
      WRITE(IOUT,503)D
  503 FORMAT(1X,'AXIS TO EDGE = ',F14.2,' METERS')
      WRITE(ITERMO,*)'ENTER IMAGINARY STATION OF P.I. IN DECIMAL FORM'
      READ(ITERM,*)PISTA
      IFF=INT(V/10-2)

C*****************************************************************
C*                                          *
C*             CALCULATE SUPERELEVATION RATE                 *
C*                                          *
C*****************************************************************
C*                                          *
C*                  VARIABLES                        *
C*                                          *
C*    E = SUPERELEVATION RATE                           *
C*    E1 = SUPERELEVATION RATE IN PERCENT                    *
C*    EREM = PORTION OF % SUPERELEVATION RATE RIGHT OF DECIMAL POINT  *
C*                                          *
C*****************************************************************

   10 E=V**2/(127*RC)-F(IFF)
```

```
      IF(E.LT.0.02)E=0.02
      IF(E.GT.0.10)THEN
        WRITE(ITERMO,*)'RADIUS ',RC,' IS TOO SMALL FOR DESIGN SPEED'
        WRITE(ITERMO,*)'ENTER ANOTHER RADIUS'
        READ(ITERM,*)RC
        GO TO 10
      END IF
      WRITE(IOUT,501)RC
  501 FORMAT(1X,'CURVE RADIUS = ',F14.2,' METERS')
      E1=100*E
      EREM=E1-INT(E1)
      IF(EREM.GT.0)THEN
        E=INT(E1+1)/100.
      ELSE
        E=E1/100
      END IF
      WRITE(IOUT,504)E
  504 FORMAT(1X,'SUPERELEVATION RATE = ',F7.2)

C*********************************************************************
C*                                                                   *
C*                   DETERMINE SPIRAL LENGTH                         *
C*                                                                   *
C*********************************************************************
C*                                                                   *
C*                        VARIABLES                                  *
C*                                                                   *
C*    LS = SPIRAL LENGTH                                             *
C*    XTEMP = TEMPORARY VARIABLE                                     *
C*                                                                   *
C*********************************************************************

      LS=200*D*E
      IF(LS.LT.40)LS=40
      IF(MOD(LS,20).NE.0)THEN
        LS=(INT(LS/20)+1)*20
      ENDIF
      WRITE(ITERMO,*)'LS = ',LS,'OVERRIDE? (ENTER POSITIVE NUMBER)'
      READ(ITERM,*)XTEMP
      IF(XTEMP.GT.0)LS=XTEMP
      WRITE(IOUT,505)LS
  505 FORMAT(1X,'LENGTH OF SPIRAL = ',F10.2,' METERS')

C*********************************************************************
C*                                                                   *
C*                   CALCULATE SPIRAL DATA                           *
C*                                                                   *
C*********************************************************************
C*                                                                   *
C*                        VARIABLES                                  *
C*                                                                   *
```

```
C*    A = FACTOR IN SPIRAL FORMULAS                                  *
C*    XS = X COORDINATE AT SC POINT                                  *
C*    YS = Y COORDINATE AT SC POINT                                  *
C*    THETAS = SPIRAL ANGLE AT SC POINT                              *
C*    P = SHIFT OF SPIRAL                                            *
C*    K = DISTANCE FROM TS TO VIRTUAL TC                             *
C*    T = DISTANCE FROM TS TO P.I.                                   *
C*    LC = LENGTH OF CIRCULAR PORTION OF CURVE                       *
C*                                                                   *
C*********************************************************************

      A=SQRT(RC*LS)
      CALL COORD(LS,RC,A,XS,YS,THETAS)
      P=YS-RC*(1-COS(THETAS))
      WRITE(IOUT,506)P
  506 FORMAT(10X,'P = ',F16.3,' METERS')
      K=XS-RC*SIN(THETAS)
      WRITE(IOUT,507)K
  507 FORMAT(10X,'K = ',F16.2,' METERS')
      T=(RC+P)*TAN(DELTA/2)
      WRITE(ITERMO,*)'T = ',T
      LC=RC*DELTA-LS
      WRITE(IOUT,508)LC
  508 FORMAT(1X,'LENGTH OF CURVE = ',F11.2,' METERS')

C*********************************************************************
C*                                                                   *
C*        CONSTRUCT TABLE OF SPIRAL COORDINATES AND ANGLES           *
C*                                                                   *
C*********************************************************************
C*                                                                   *
C*                        VARIABLES                                  *
C*                                                                   *
C*    N = NUMBER OF POINTS IN TABLE                                  *
C*    L = LENGTH OF SPIRAL TO GIVEN POINT                            *
C*    R = RADIUS OF SPIRAL AT L                                      *
C*    X = X COORDINATE OF SPIRAL AT L                                *
C*    Y = Y COORDINATE OF SPIRAL AT L                                *
C*    THETA = SPIRAL ANGLE AT L                                      *
C*                                                                   *
C*********************************************************************

      WRITE(IOUT,509)
  509 FORMAT(//,10X,'SPIRAL COORDINATES',//,9X,'L',9X,'X',9X,'Y',5X,
     2       'THETA',/)
      N=INT(LS/20)
      DO 20 I=1,N
        L=20*I
        R=(LS*RC)/L
        CALL COORD(L,R,A,X,Y,THETA)
        WRITE(IOUT,510)L,X,Y,THETA
```

```
  510   FORMAT(2F10.2,F10.3,F10.4)
   20 CONTINUE

C**********************************************************************
C*                                                      *
C*                 CALCULATE SPIRAL STATIONING                      *
C*                                                      *
C**********************************************************************
C*                                                      *
C*                      VARIABLES                        *
C*                                                      *
C*    TS = STATION OF TANGENT TO SPIRAL POINT                       *
C*    SC = STATION OF SPIRAL TO CURVE POINT                       *
C*    CS = STATION OF CURVE TO SPIRAL POINT                       *
C*    ST = STATION OF SPIRAL TO TANGENT POINT                       *
C*                                                      *
C**********************************************************************

      WRITE(IOUT,511)
  511 FORMAT(//,10X,'STATIONING',/)
      TS=PISTA-((T+K)/100)
      WRITE(ITERMO,*)'TS = ',TS
      CALL STATION(TS,IOUT,'TS')
      SC=TS+LS/100
      CALL STATION(SC,IOUT,'SC')
      CS=SC+LC/100
      CALL STATION(CS,IOUT,'CS')
      ST=CS+LS/100
      CALL STATION(ST,IOUT,'SC')
      STOP
      END

C**********************************************************************
C*                                                      *
C*                     SUBROUTINE COORD                      *
C*     CALCULATE COORDINATES AND SPIRAL ANGLE OF POINT ON SPIRAL      *
C*                                                      *
C**********************************************************************

      SUBROUTINE COORD(L,R,A,X,Y,THETA)
      REAL L,R,A,X,Y,THETA
      X=L-(L**5/(40*A**4))+(L**9/(3456*A**8))
      Y=(L**3/(6*A**2))-(L**7/(336*A**6))+(L**11/(42240*A**10))
      THETA=L/(2*R)
      RETURN
      END

C**********************************************************************
C*                                                      *
C*                  SUBROUTINE STATION                       *
C*       WRITE OUT STATION NUMBERS FOR CRITICAL POINTS              *
```

```
C*                                          *
C*********************************************************************
C*                                          *
C*              VARIABLES                   *
C*                                          *
C*  STA=STATION NUMBER IN DECIMAL FORM                 *
C*  ISTA = PORTION OF STATION NUMBER LEFT OF + SIGN        *
C*  PLUS = PORTION OF STATION NUMBER RIGHT OF + SIGN       *
C*  POINT = CRITICAL POINT ON SPIRAL (E.G. 'TS')        *
C*                                          *
C*********************************************************************

      SUBROUTINE STATION(STA,IOUT,POINT)
      REAL STA
      INTEGER ISTA,PLUS
      CHARACTER POINT*2
      ISTA=INT(STA)
      PLUS=NINT(100*(STA-ISTA))
      WRITE(IOUT,512)POINT,ISTA,PLUS
  512 FORMAT(10X,A2,I6,'+',I2.2)
      RETURN
      END
```

Input file

```
0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.12 0.11 0.09
```

Output file

```
 DESIGN SPEED =          110.00 KM/H
 DEFLECTION ANGLE =      28.00 DEGREES
 AXIS TO EDGE =           3.60 METERS
 CURVE RADIUS =          500.00 METERS
 SUPERELEVATION RATE =     .09
 LENGTH OF SPIRAL =      80.00 METERS
         P =             .533 METERS
         K =            39.99 METERS
 LENGTH OF CURVE =      164.30 METERS


         SPIRAL COORDINATES

        L         X         Y     THETA

     20.00     20.00      .033     .0050
     40.00     40.00      .267     .0200
     60.00     59.99      .900     .0450
     80.00     79.95     2.132     .0800
```

STATIONING

TS 13+35
SC 14+15
CS 15+80
SC 16+60

d-3.

Programming. Write a computer program to calculate the minimum length of a vertical curve connecting two known grades g_1 and g_2, which must clear either an existing elevated structure (in the case of a crest curve) or an underground utility (in case of a sag curve) by a known distance *C*. The user should supply the program with stations and elevations of the PI and the critical point, the two grades, and the clearance *C*. Test your program using data from Problem 4-14.

Source code

```
C*********************************************************************
C*                                                  *
C*              COMPUTER EXERCISE 4.3                 *
C*              VERTICAL CURVE LENGTH                   *
C*            WITH CONSTRAINED ELEVATION                  *
C*                                                  *
C*********************************************************************

      REAL PISTA,PIELEV,G1,G2,CRSTA,CRELEV,CLEAR,Z,TELEV,YPRIME,
     2    A,W,L
      INTEGER ITERMO,ITERM,IOUT
      DATA ITERMO/6/,ITERM/5/,IOUT/10/



C*********************************************************************
C*                                                  *
C*                 OPEN OUTPUT FILE                *
C*                                                  *
C*********************************************************************
C*                                                  *
C*                   VARIABLES                  *
C*                                                  *
C*   ITERM = UNIT NUMBER FOR KEYBOARD                      *
C*   ITERMO = UNIT NUMBER FOR SCREEN                      *
C*   IOUT = UNIT NUMBER FOR OUTPUT FILE                    *
C*                                                  *
C*********************************************************************

      OPEN(IOUT,FILE='CSTVC.OUT')

C*********************************************************************
C*                                                  *
```

```
C*          ENTER DATA AND ECHO TO OUTPUT FILE                    *
C*                                                                *
C*****************************************************************
C*                                                                *
C*                    VARIABLES                                   *
C*                                                                *
C*   PISTA  = STATION OF THE P.I.                                 *
C*   PIELEV = ELEVATION OF THE P.I.                               *
C*   G1, G2 = TANGENT GRADES                                      *
C*   CRSTA  = STATION OF THE CRITICAL POINT                       *
C*   CRELEV = ELEVATION OF THE CRITICAL POINT                     *
C*   CLEAR  = CLEARANCE BETWEEN ROADWAY AND CRITICAL POINT        *
C*                                                                *
C*****************************************************************

      WRITE(ITERMO,*)'ENTER P.I. STATION IN DECIMAL FORM'
      READ(ITERM,*)PISTA
      ISTA=INT(PISTA)
      IPLUS=NINT(100*(PISTA-ISTA))
      WRITE(IOUT,100)ISTA,IPLUS
  100 FORMAT(19X,'P.I. STATION = ',I7,'+'I2.2)
      WRITE(ITERMO,*)'ENTER P.I. ELEVATION IN METERS'
      READ(ITERM,*)PIELEV
      WRITE(IOUT,101)PIELEV
  101 FORMAT(17X,'P.I. ELEVATION = ',F10.2)
      WRITE(ITERMO,*)'ENTER GRADES G1 AND G2 IN PERCENT'
      READ(ITERM,*)G1,G2
      WRITE(IOUT,102)G1
      WRITE(IOUT,103)G2
  102 FORMAT(29X,'G1 = ',F10.2)
  103 FORMAT(29X,'G2 = ',F10.2)
      WRITE(ITERMO,*)'ENTER STATION OF CRITICAL POINT IN DECIMAL FORM'
      READ(ITERM,*)CRSTA
      ISTA=INT(CRSTA)
      IPLUS=NINT(100*(CRSTA-ISTA))
      WRITE(IOUT,104)ISTA,IPLUS
  104 FORMAT(6X,'STATION OF CRITICAL POINT = ',I7,'+'I2.2)
      WRITE(ITERMO,*)'ENTER ELEVATION OF CRITICAL POINT IN METERS'
      READ(ITERM,*)CRELEV
      WRITE(IOUT,105)CRELEV
  105 FORMAT(4X,'ELEVATION OF CRITICAL POINT = ',F10.2)
      WRITE(ITERMO,*)'ENTER CLEARANCE IN METERS.'
      WRITE(ITERMO,*)'+ IF ROAD ABOVE CRITICAL POINT'
      WRITE(ITERMO,*)'- IF ROAD BELOW CRITICAL POINT'
      READ(ITERM,*)CLEAR
      WRITE(IOUT,106)CLEAR
  106 FORMAT(22X,'CLEARANCE = ',F10.2)

C*****************************************************************
C*                                                                *
C*            DETERMINE VERTICAL CURVE OFFSET                     *
```

```
C*                                                    *
C*********************************************************************
C*                                                    *
C*                  VARIABLES                                 *
C*                                                    *
C*   TELEV  = ELEVATION OF G1 TANGENT AT CRITICAL POINT            *
C*   Z      = HORIZONTAL DISTANCE FROM P.I. TO CRITICAL POINT       *
C*   YPRIME = VERTICAL CURVE OFFSET AT CRITICAL POINT              *
C*                                                    *
C*********************************************************************

      Z=CRSTA-PISTA
      TELEV = PIELEV+Z*G1
      YPRIME=CRELEV+CLEAR-TELEV



C*********************************************************************
C*                                                    *
C*     DETERMINE AND REPORT CRITICAL VERTICAL CURVE LENGTH         C
C*                                                    *
C*********************************************************************
C*                                                    *
C*                  VARIABLES                                 *
C*                                                    *
C*   A      = G2 - G1                                       *
C*   W      = YPRIME/A                                          *
C*   L      = CRITICAL VERTICAL CURVE LENGTH                     *
C*                                                    *
C*********************************************************************

      A=G2-G1
      W=YPRIME/A
      L=4*W-2*Z+4*SQRT(W**2-W*Z)
      L=100*L
      WRITE(ITERMO,200)NINT(L)
      WRITE(IOUT,200)NINT(L)
  200 FORMAT(1X,'CRITICAL VERTICAL CURVE LENGTH = ',I10,' METERS')
      STOP
      END
```

Output file

```
              P.I. STATION =     200+00
               P.I. ELEVATION =     150.00
                         G1 =       -.50
                         G2 =       1.00
     STATION OF CRITICAL POINT =     200+70
   ELEVATION OF CRITICAL POINT =     150.40
                  CLEARANCE =        .75
 CRITICAL VERTICAL CURVE LENGTH =        479 METERS
```

d-4

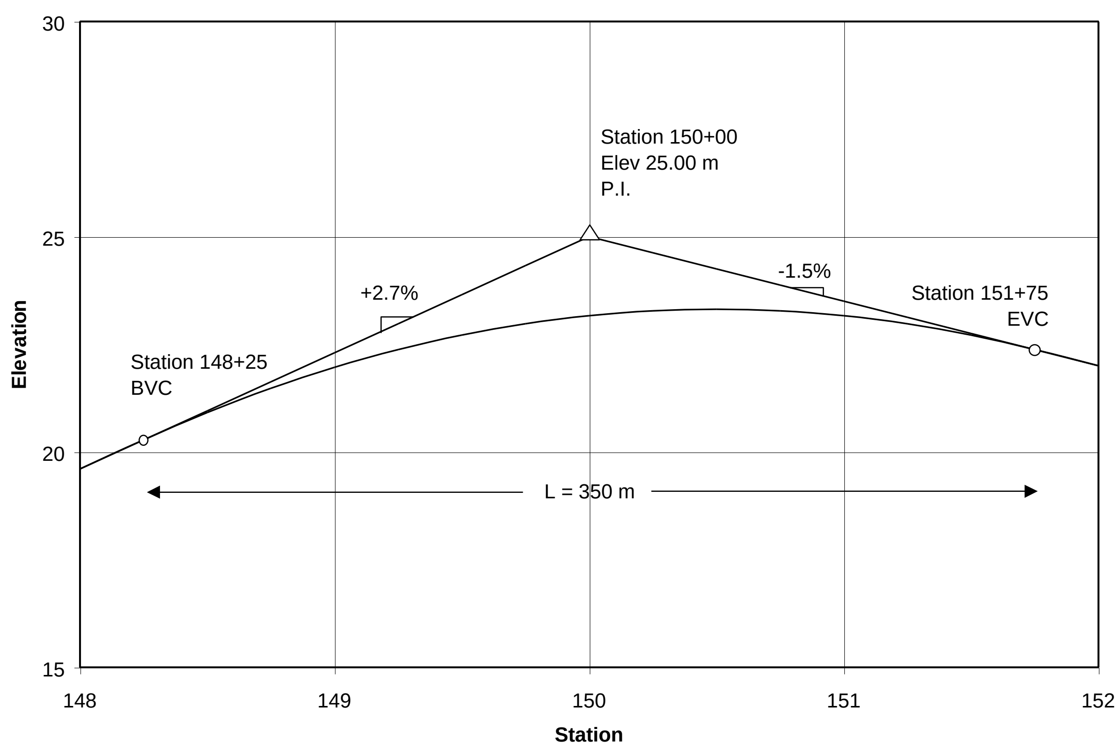
Spreadsheet. Use a spreadsheet to calculate and report the stations and elevations of the PI, BVC, and EVC of one of the curves in Problems 4-8 to 4-10 and to construct a table showing stations and roadway elevations at 25-m intervals. Also use the spreadsheet to plot the vertical curve.

Worksheet (data from Problem 4-8)

L=	350.000
g1 =	2.700
g2 =	-1.500
P.I. Sta =	150.000
P.I. Elev =	25.000
BVC Sta =	148.250
BVC Elev =	20.275
EVC Sta =	151.750
EVC Elev =	22.375
r =	-1.200

Station	x	Elev	Tan. Elev.
148.25	0.00	20.275	20.275
148.50	0.25	20.913	20.950
148.75	0.50	21.475	21.625
149.00	0.75	21.963	22.300
149.25	1.00	22.375	22.975
149.50	1.25	22.713	23.650
149.75	1.50	22.975	24.325
150.00	1.75	23.163	25.000
150.25	2.00	23.275	24.625
150.50	2.25	23.313	24.250
150.75	2.50	23.275	23.875
151.00	2.75	23.163	23.500
151.25	3.00	22.975	23.125
151.50	3.25	22.713	22.750
151.75	3.50	22.375	22.375

Chart



Notes

Cell names are

B1	L
B2	grade1
B3	grade2
B4	pista
B5	pielev
B6	bvcsta
B7	bvcelev
B8	evcsta
B9	evcelev
B10	grte

Formula

B10 =(grade1 − grade2)/(L/100)

Table is constructed as follows

Column Station is constructed by filling down
Column X (label is X) is constructed by filling down
Column Elev =bvcelev + grade1 × X+[(grte×X^2)/2]
Column Tan. Elev. =bvc × grade1 × X [from BVC to PI]
=pielev + (grade2*(X−L/200)) [from PI to EVC]

d-5.
Spreadsheet. Use a spreadsheet to determine deflection angles and chords from the TC point for full stations and +20 points for a 400-m-radius circular curve with a toal deflection angle of 20°. The station of the TC is 32+65.

Worksheet

R =	400
Delta =	20
Lc =	139.6263
TC sta =	32.65
CT sta =	34.05

		Deflection Angle		
Station	x	rad.	deg.	Chord
32+65	0	0.0000	0.0000	0.000
32+80	15	0.0188	1.0743	14.999
33+00	35	0.0438	2.5067	34.989
33+20	55	0.0688	3.9391	54.957
33+40	75	0.0938	5.3715	74.890
33+60	95	0.1188	6.8039	94.777
33+80	115	0.1438	8.2363	114.604
34+00	135	0.1688	9.6687	134.360
34+05	140	0.1750	10.0268	139.287

Notes

Cell names are

\$B\$1	radius
\$B\$2	delta
\$B\$3	length
\$B\$4	tcsta

Formulas are

\$B\$3	=radius × RADIANS(delta)
\$B\$5	=tcsta + length/100

The table is constructed as follows:

Column station	Entered as data
Column X (labelX)	Entered as data
Column Deflection angle	
rad. (label is rad)	=X/(2 x radius)
deg.	=DEGREES(rad)
Column Chord	=2 x radius x SIN(rad)

d-6.
Spreadsheet. Use a spreadsheet to calculate X_s, Y_s, p, and k for one of the spiral transition curves in Problems 4-25 to 4-27. Also use the spreadsheet to construct a table showing the *X* and *Y* coordinates, the spiral angle, the deflection angle, and the chord at 10-m intervals.

Worksheet

Ls =	80.000
Rc =	250.000
A =	141.421
Spiral Angle =	0.160
Xs =	79.795
Ys =	4.259
p =	1.066
k =	39.966

			Spiral angle		Deflection angle		
L	X	Y	rad.	deg.	rad.	deg.	chord
0	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.000
10	10.000	0.008	0.0025	0.1432	0.0008	0.0477	10.000
20	20.000	0.067	0.0100	0.5730	0.0033	0.1910	20.000
30	29.998	0.225	0.0225	1.2892	0.0075	0.4297	29.999
40	39.994	0.533	0.0400	2.2918	0.0133	0.7639	39.997
50	49.980	1.041	0.0625	3.5810	0.0208	1.1936	49.991
60	59.951	1.799	0.0900	5.1566	0.0300	1.7188	59.978
70	69.895	2.855	0.1225	7.0187	0.0408	2.3393	69.953
80	79.795	4.259	0.1600	9.1673	0.0533	3.0551	79.909

Notes

Cell names are

\$B\$1	spiral_length
\$B\$2	curve_radius
\$B\$3	A
\$B\$4	spiral_angle
\$B\$5	xs
\$B\$6	ys

Formulas are

\$B\$3 =SQRT(spiral_angle×curve_radius)
\$B\$4 =spiral_length/(2×curve_radius)
\$B\$5 =spiral_length−(spiral_length^5/40×a^4))
+(spiral_length^9/(3456×A^8)
\$B\$6 =spiral_length^3/(6×A^2) −spiral_length^7/(335A^6)
+spiral_angle^11/(42240×A^10)

The table is constructed as follows:

Column L is constructed by filling down
Column X (label is X) =L− (L^5/(40×A^4)+(L^9/3456×A^8))
Column Y (label is Y) =L^3/(6×A^2) −L^7/(336×A^6)
+L^11/(4224×A^10)
Column Spiral angle
rad. (label is sarad) =L^2/(2×A^2)
deg. =DEGREES(sarad)
Column Defelction angle
rad. (label is darad) =ATAN(X/Y)
deg. =DEGREES(darad)
Column chord =SQRT(X^2+Y^2)

DESIGN EXERCISES

Since these are design exercises, no model solution is provided. Note the following erratum: Appendix D says that you should obtain the required figures from the instructor manual, however they are available at the end of the file.

Open-Ended Class Research Problems

Note to instructor: The purpose of these problems is for each member of the class to research (usually on the internet) some topic and gather data that can be compiled into a class-wide summary for discussion and presentation in lecture.

Question 1. Topic: ITS and traffic enforcement. Identify a city anywhere in the world that uses some form of ITS (such as speed or red light cameras) to enforces traffic laws. For your city, provide (a) city and country, (b) type of ITS: speed, red light, congestion pricing, or "other," (c) if other, please state what it is, (d) year of the data to which you are referring, or if the website does not provide the year, just say "undated," and (e) web address.

Question 2. Topic: Fuel economy. For either your car or that of a friend/acquaintance/relative, calculate the delivered fuel economy for one gas tank worth of driving (note: please don't use an electric or plug-in hybrid for this exercise). State the number of miles driven, the gallons of fuel consumed, and the resulting miles per gallon. If the vehicle runs on diesel, state that as well. Then go to www.fueleconomy.gov and find out the projected fuel economy for the make, model, and year of the vehicle in question. In some cases there are multiple models with slightly different engines, transmissions, etc., in which case just do your best to choose the vehicle that is closest to your actual one.

Question 3. Topic: Estimated Ultimate Recovery. Natural gas is often considered a possible substitute for declining oil supplies. Find and report a published value of the EUR for natural gas reserves for the U.S. or another country from the internet. If possible, find a value since 2008, since estimates have been growing recently due to advancing extraction techniques. Give value in cubic feet, cubic meters, or energy content (Btu or joules) and web address.

Question 4. Topic: Bikesharing. Find a city or university in the U.S., Canada, or other country that has bikesharing and state as many of the following points as you can find: (a) what year it was formed, (b) for the most recent year available how many members it has, or (c) how many bicycles it has. Also give the website.

Question 5. Topic: Air freight. For a major world airline (but not a dedicated parcel service like FedEx or UPS), find from the internet for a recent year (1) total tonne-km carried (convert from standard if needed), (2) total revenue earned, (3) and calculate revenue per 1000 tonne-km. Provide sources, for example, for Korean Air Lines in 2009: 8.225 billion tkm, $1.94B revenue, $236 per 1000 tonne-km.

Question 6. Topic: High-speed rail (HSR). Find and report information on a planned or underconstruction HSR route, including country and end-point cities, target year of completion, total length, and anticipated cost per mile. Give a web address for your source.

Question 7. Topic: annual miles driven in private cars. Either use your own car for this problem, if you own one, or talk to a college student friend who does, if you do not own one yourself. You will need to study a car with a *working* odometer, otherwise the findings will distort the study. Compute an average miles driven per year for the car: if known, use the miles driven by the current owner during the time they have owned the car divided by the number of years of ownership. If not, use the lifetime miles divided by the difference between the current year and the year of manufacture. After the homework has been submitted, we will look at the range of data points submitted to see if the results support the hypothesis that college students on average drive less than the national average of 9000 mi per year.

Question 8. Topic: Transit ridership. Find a transit system either in the U.S. or in another country and give the total boardings in a recent year and the boardings per population. Note: for some systems, the agency may tell you boardings per capita on their website, but for others you will need to find the metropolitan area population and calculate the value yourself. (Clarification: usually a transit operator goes beyond the city limits.) As usual, provide the web address for the website used.

Question 9. Topic: Annual miles driven by long-distance trucks. In this problem you should find a source for the annual miles driven by either a single observed combination truck (a.k.a, “tractor-trailer” or “semi”) or the average from a fleet owned by a trucking company or other business. Your answer may be U.S. or international. As usual, provide the web source.

Question 10. Topic: Influx of EVs and PHEVs into the car market. Report on progress in the market for any recent (2009 or later) EV or PHEV. Include as many of the following as possible: make and model, year of launch, sales in either the U.S. or other market for as many years as possible between the launch year and current year, and a web link.

Question 11. Topic: Estimated ultimate recovery (EUR). Find and report a published value of the EUR for conventional (or combined conventional + unconventional) petroleum from the internet, for a specific country, region, or the entire world market. Give value in barrels of oil, the year in which the value was estimated, and the web address.

Question 12. Topic: Carsharing. Find a city in the U.S. or Canada that has carsharing, and state as many of the following points as possible: what year it was formed, how many members it has, and how many vehicles it has. Also give the website.

Question 13. Topic: Rail freight. For some country other than the U.S., give the total share of freight moved by rail, in terms of market share of either ton-miles (or tonne-km) or value of freight moved. Also give the year in which the market share was measured and the web address of your source.

Question 14. Topic: Energy saving technology in railroads. From the internet or other source, find one example of an energy-saving technology or strategy that railroads can use, and its estimated potential energy savings, according to your source. Include a web link.

Question 15. Topic: Evolution of battery cost per kWh. Search the internet for a quote of the cost of battery technology, for lithium ion and for NiMH (for automotive applications, i.e., large-scale batteries, not for portable electronics). Find one value for each battery technology, measured in $/kWh. For each technology, give the cost value, the year that it is quoted, and the web address.

Exam Problems and Solutions

Part 1 Problems on Motivations and Tools

Question 1.

Energy planners in a country are considering the available domestic petroleum reserves, which are thought to comprise 392 billion barrels. The cumulative consumption model with parameters $c_1 = -5.38$ and $c_2 = 0.114$ is thought to fit the cumulative consumption of the reserve. In terms of its lifetime, cumulative consumptions in year 50 and year 60 were observed at 218 bil.bbl and 322 bil.bbl, respectively. Based on these two years, what is the RMSD value of the difference between modeled and observed cumulative consumption?

Solution**.**

First, use the information about the model to calculate the projected amount of cumulative consumption in the two years in question. Accordingly:

$$f(50) = (392)\frac{e^{(-5.38+0.114\cdot 50)}}{1+e^{(-5.38+0.114\cdot 50)}} = 227.1 \text{ bil.bbl}$$

$$f(60) = (392)\frac{e^{(-5.38+0.114\cdot 60)}}{1+e^{(-5.38+0.114\cdot 60)}} = 318.1 \text{ bil.bbl}$$

The values for the RMSD calculation are then calculated by comparing the above to the observed values:

$$(218 \text{ bil} - 227.1 \text{ bil})^2 = 8.27\times 10^{19}$$

$$(322 \text{ bil} - 318.1 \text{ bil})^2 = 1.50\times 10^{19}$$

$$\text{RMSD} = \sqrt{\frac{1}{2}\left(8.27\times 10^{19} + 1.50\times 10^{19}\right)} = 6.99\times 10^{9}\, bbl$$

Thus the RMSD is 6.99 bil.bbl.

Question 2.

A natural gas resource is thought to have Hubbert-curve type lifetime exploitation characteristics, with a total estimated ultimately recoverable reserve of 33 trillion cubic feet (TCF), maximum production in year 2045, and curve width parameter of 35 years that is found to best fit historical annual production up to the present time. Suppose that observed production in 1985 and 2000 are 80 billion cubic feet (BCF) and 151 BCF, respectively. What is the RMSD between observed and estimated production, based on these two points?

Solution.

Approach: solve for estimated production in 1985 and 2000, and then compare the observed values to calculate RMSD.

$$P = \frac{Q_\infty}{S\sqrt{2\pi}} \exp\left[-(t_m - t)^2 /(2S^2)\right]$$

$$\frac{Q_\infty}{S\sqrt{2\pi}} = \frac{33{,}000BCF}{35\sqrt{2\pi}} = 376BCF$$

$$P(1985) = (376)\exp\left[-(2045-1985)^2 /(2(35)^2)\right]$$

$$= (376)\exp[-3600/2450]$$

$$= 376 \cdot 0.231 = 86.5BCF$$

$$P(2000) = (376)\exp\left[-(2045-2000)^2 /(2(35)^2)\right]$$

$$= (376)\exp[-2025/2450]$$

$$= 376 \cdot 0.437 = 164BCF$$

Thus, the estimated values for 1985 and 2000 are 86.5 and 164 BCF, respectively. Creating a table of squared error values and solving for RMSD gives:

Year	P-act	P-est	Error^2
1985	80	86.5	42.25
2000	151	164	169
Total			211.25

Then the RMSD is calculated as follows:

$$RMSD = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(P_{actual} - P_{est})^2} = \sqrt{\frac{1}{2}(211.3)} = 10.3BCF$$

Question 3.

A food processor manufactures a product at two plants (1 and 2) for three markets (A, B, C). The distances from the sources to the markets in kilometers are the following:

	A	B	C
1	1400	5300	5600
2	3800	1700	2300

Suppose 1 and 2 have a capacity of 2600 and 2400 tonnes (i.e., metric tons) per year, respectively, and the demand at 1 to 3 is 2300, 1200, and 1300 tonnes, respectively. Shipments are made 35% by rail and 65% by truck, with an energy intensity of 650 and 2250 kJ/tonne-km, respectively.

a. What is the average energy consumption in kJ per tonne-km if the 35/65 modal split holds for all possible shipments in the network?

Solution:

We must determine the energy consumption for both rail and truck and then sum them to determine overall energy consumption as follows:

$$\left(0.35 * \frac{650\ kJ}{tonne-km}\right) + \left(0.65 * \frac{2250\ kJ}{tonne-km}\right) = \frac{\mathbf{1690\ kJ}}{\mathbf{tonne-km}}$$

b. Using the transportation problem in optimization as a starting point, write out the objective function and five constraint equations (not including the non-negativity constraint) specific to the given data that are necessary to form the transportation problem for finding the minimum tonne-km/minimum energy consumption origin-destination (OD) flow pattern. Hint: There is more than one way to express the five constraint equations, but your answer should clearly define the bounds on the solution space for the problem.

Solution.

Taking into account the values for capacity and demand, the optimization problem is the following:

$$MinZ = \sum_{i,j} D_{ij} X_{ij}$$

$$s.t.$$

$$\sum_{j} X_{1,j} \leq 2600$$

$$\sum_{j} X_{2,j} \leq 2400$$

$$\sum_{i} X_{i,A} = 2300$$

$$\sum_{i} X_{i,B} = 1200$$

$$\sum_{i} X_{i,C} = 1300$$

c. Since you do not have access to Excel or Matlab describe in one or two sentences or several numbered steps a "heuristic" of your own design to find the optimal shipping pattern (different correct answers are possible).

Solution.

Different answers are possible for this problem. As an example, the following steps might be employed:

1. For each market, identify the source that is closer.
2. If sufficient supplies are available for all markets assigned to a source, fully meet the requirements of each market with that source.
3. If point #2 applies to a given source, any excess supply becomes available to meet some or all demand at more distance sources.

4. If point #2 does not apply, meet first the demand of the market that is closest to the source. Then meet the demand at the next closes. At some point the remaining supply will not be sufficient to meet all of the demand at the market that is being evaluated. In that case, meet as much of the demand as possible with the remaining supply. Then meet the remaining demand from the other source.
5. Since there are only two sources and three markets, this heuristic should work to meet all demand at all markets.

d. Find the allocation of product from sources to markets (optimal OD pattern) that minimizes total energy consumption, and show that the constraints are satisfied. Then calculate the value of total energy consumption in gigajoules [GJ] for that pattern, assuming a 35/65 rail/truck split for all shipments. You do not need to show how you arrive at the optimal OD pattern, just present it, and show the calculations after that.

Solution**.**

Pattern resulting from heuristic shown in table below:

	a	b	c	Output	Constraint
1	2300	0	100	2400	<=2600
2	0	1200	1200	2400	<=2400
Shipped tonnes	2300	1200	1300		
Constraint:	=2300	=1200	=1300		

First, we solve for the total tkm for this scenario by using the optimized tonnes shipped and the distances from the table in part (a):

$$(2300\ tonnes\ \times 1400km) + (1200\ tonnes \times 1700km) + (100\ tonnes \times 5600km) + (1300\ tonnes \times 2300km) = 8.58x10^6 tonne - km$$

Next, we determine the total energy consumption by using the modal split and energy intensity information from part (a):

$$\left(8.58x10^6 tonne - km \times 0.35 * \frac{650\ kJ}{tonne - km}\right) + \left(8.58x10^6 tonne - km 0.65 \times \frac{2250\ kJ}{tonne - km}\right) = 1.45x10^{10}\ kJ = 14{,}500\text{GJ}$$

Summary.

The resulting total tonne-km is 8.58 million, and total energy is 14.5 TJ. Note that the optimal pattern found with the solver is actually different and results in a lower value of 6.71 million tonne-km and 11,340 GJ. The computer optimal pattern is shown below. Since the purpose of the exam question is to demonstrate an understanding of the thought process for finding the best possible pattern, the result above should be acceptable.

d)	**a**	**b**	**c**	**Constraint**	**Output**
1	1200	1200	0	2600	2400
2	1100	0	1300	2400	2400
Demand (fixed constraint)	2300	1200	1300		
Total	2300	1200	1300		
Total tkm	1680000	2040000	2990000		
Summed tkm	6710000				
Total energy (kJ)	1.134E+10				

Question 4.

You are given the following data for energy consumption in passenger transportation in the U.S., divided into two categories: “passenger vehicles,” meaning privately owned cars, light trucks, etc., and “commercial,” meaning any form of transportation for which you pay for a ticket, include air travel, trains, buses, etc. The values given are approximately accurate. A passenger-kilometer [pkm] is the transportation of one passenger for a distance of 1 km.

	Energy		Passenger-kilometers	
	[EJ]	[EJ]	[trillion]	[trillion]
Year:	1990	2000	1990	2000
Psgr.Vehicles	7.82	9.32	3.38	4.22
Commercial	1.31	1.64	0.72	0.99

a. Calculate the overall energy consumption in EJ and energy intensity in kJ/pkm, both in the year 2000.

<u>Solution.</u>

Overall = 9.32 $EJ + 1.64\ EJ = 10.96\ EJ$

Since 1 EJ is equal to 1 × 10^{15} kJ, and pkm are in trillions, we have:

Energy Intensity = $\dfrac{10.96\ x\ 10^{15}\ kJ}{4.22^{12}\ pkm + 0.99^{12}\ pkm} = \dfrac{2103.65\ kJ}{pkm}$

b. Calculate the trended energy consumption in 2000 in EJ.

<u>Solution.</u>

The trended energy consumption is the consumption that would have been observed if the 2000 activity level had taken place at 1990 intensity levels. 1990 energy intensity =

$$\frac{7.82\ x\ 10^{15}\ kJ\ +\ 1.31\ x\ 10^{15}}{3.38^{12}\ pkm\ +\ 1.64^{12}\ pkm} = \frac{2226.83\ kJ}{pkm}$$

1990 overall intensity = 2.23 MJ/pkm, so 2000 trended energy = 11.6 EJ:

$$(PKM_{2000})(\mu_{1990}) = (5.21 \times 10^{12}\ pkm)(2.23 MJ / pkm) = 1.16 \times 10^{13} MJ = 11.6 EJ$$

c. Calculate the contribution of changes in structure and intensity to the difference between trended and actual energy consumption in 2000, using Divisia analysis.

<u>Solution</u>**.**

Using Divisia technique, intensity contribution = −0.621 EJ, structure contribution = −0.038 EJ. Check: 11.63 − 0.621 − 0.038 = ~10.96 EJ. The detailed steps are the following.

We first create a table of intensity values in units of kJ/pkm, showing the 1990 and 2000 values for each mode, the change from 1990 to 2000, and the average across the 2 years:

Intensity	1990	2000	Change	Avge
Psgr.vehicles	2314	2209	(105)	2261
Commercial	1819	1657	(163)	1738

We create a similar table for share of pkm in percentage points:

Share	1990	2000	Change	Avge
Psgr.vehicles	82.4%	81.0%	−1.4%	81.7%
Commercial	17.6%	19.0%	1.4%	18.3%

The terms needed for intensity and share for the Divisia analysis are the following:

$$e_k = (e_{2000} - e_{1990})\left(\frac{s_{2000} + s_{1990}}{2}\right)$$
$$e_{psgr} = (-105)(0.817) = -85.87$$
$$e_{comm} = (-163)(0.183) = -29.78$$
$$s_k = (s_{2000} - s_{1990})\left(\frac{e_{2000} + e_{1990}}{2}\right)$$
$$s_{psgr} = (-0.014)(2261) = -32.58$$
$$s_{comm} = (0.014)(1738) = 25.04$$

The multiplier for intensity and structure is then the sum of the passenger and commercial terms calculated above:

$$e = e_{psgr} + e_{comm} = -85.87 - 29.78 = -115.65$$
$$s = s_{psgr} + s_{comm} = -32.58 + 25.04 = -7.54$$

To complete the Divisia, these terms are multiplied by the activity level of A_{2000} = 5.21 trillion pkm. The accuracy is confirmed by showing that adding these two terms to the trended energy consumption of 11.60 EJ gives the actual energy consumption of 10.96 EJ:

$$\Delta E = (5.21 tril.pkm)(-115.65 kJ/pkm) + (5.21 tril.pkm)(-7.54 kJ/pkm)$$
$$= -0.603 EJ - 0.039 EJ = -0.64 EJ$$
$$E_{trended} + \Delta E = 11.60 - 0.64 = -10.96 EJ$$

(Note that the numbers calculated in an exam setting may not agree exactly due to rounding, but the calculation is still completely correct.)

Question 5.

Note to instructor: Preparation for posting this problem on an exam may require particular support for learning about transportation economics and congestion pricing. A large-scale transportation infrastructure link has a total cost function as a function of number of units Q on the link as follows, for values Q > 70 units:

$$TC(Q) = 0.2Q^3 + 4.5Q^2 + 100Q$$

Note that because of the large figures of cost involved, you should not use the analogy of tolls to individual cars on a toll expressway. The inverse demand curve for P as a function of Q is given by

$$P(Q) = 20000 - 67Q$$

What congestion price should be charged to achieve the socially optimal equilibrium, i.e., one where the price paid equals the marginal cost? Show all work. To simplify the calculation, you can round Q to the nearest integer (Hint: If you are not able to solve exactly for the equilibrium *Q* value, trial and error to find approximately the correct point is also acceptable.)

Solution.

To take the first derivative of TC(*Q*), we have $MC(Q) = 0.6Q^2 + 9Q + 100$

Let MC(*Q*) = *P*(*Q*), we have $20000 - 67Q = 0.6Q^2 + 9Q + 100$. There are two solutions for this equation: $Q \approx 129.48 \ or \ -256.15$ Since *Q* should be positive and an integer, *Q* = 130.

When *Q* = 130, *P*(130) = \$11290, MC (130) = \$11410, AC(130) = TC(Q)/Q = \$4065. So congestion price = MC(130) − AC(130) = \$7345. Answers *Q* = 129, charge = \$7236.90; *Q* = 129.48, charge = \$7288.69 also accepted.

Question 6.

A fictitious country in 2008 has a population of 300 million, a passenger transportation rate of 28,000 passenger-km per capita per year [pkm/cap], an emission rate of 85 gCO_2 per MJ of energy, and total emissions from the passenger transportation sector of 1.071×10^{15} grams CO_2 per year. Calculate the value of transportation CO_2 emissions intensity, transportation energy intensity, and transportation demand intensity, including appropriate units.

Solution.

Use the following equation that contains all three intensity terms:

$$CO_2\ Emissions = \left(\frac{Emissions}{Energy}\right)\left(\frac{Energy}{person - km}\right)\left(\frac{person - km}{Person}\right) Persons$$

Total pkm

$$(2.8E4)(3E8) = 8.4E12\,\text{pkm}$$

Total energy

$$\frac{1.071E15g}{85g/MJ} = 1.26E13\,\text{MJ}$$

Energy consumption rate

$$\frac{1.26E13MJ}{8.4E12\,pkm} = 1.5MJ/pkm$$

(b): The values for this same equation for Japan in 2008 are 71.5 gCO_2/MJ, 1.6 MJ/pkm, 10,000 pkm/cap, and 128 million. For the three factors over which countries are generally thought to have greater control, state whether, from the perspective of trying to reduce CO_2 emissions, the fictitious country is doing better or worse than Japan.

Solution.

Japan 10,000 pkm/cap versus 28,000 pkm/cap, so Japan is better.

Japan 71.5 gCO_2/MJ versus 85 gCO_2/MJ, so Japan is better.

Japan 1.6 MJ/pkm versus 1.5MJ/pkm, so Japan is worse.

Question 7.

Note to instructor: This question may require additional preparation on the topics of price elasticity, elastic versus inelastic demand, and arc elasticity of demand.

The quantity Q of seats on an airliner is found to respond to price P with the function $Q = 300 – 0.45P$. Suppose price is set such that the demand for seats is $Q = 223$.

a) [9 points] What is the price elasticity of demand for this combination of P and Q?
b) [3 points] Is demand elastic or inelastic at this point? Why? Explain in one sentence.
c) [9 points] What is the value of consumer surplus at this price and level of demand?
d) [9 points] Suppose the price is changed so that the combination of P and Q is at the unit elastic point on the demand curve. Calculate the new values of P and Q and the arc price elasticity for this change.

Solution.

a. Elasticity = −0.345

$$Q = 300 - 0.45(P) = 223 \quad => \quad P = 171.1$$

$$\boldsymbol{e_p} = \frac{dQ_i}{dP_i} \times \frac{P_i}{Q_i} = -0.45 \times \frac{171.1}{223} = -0.345$$

b. Inelastic, b/c the elasticity value is between −1 and 0

c. Consumer surplus = $55,304.

$$CS = \frac{1}{2}(P_{max} - P_i) \times Q_i = \frac{1}{2}(667 - 171) \times 223 = 55{,}304$$

Where P_{max} *is calculated by inverting demand function and setting* $Q = 0$

$$\mathrm{Q} = 300 - 0.45\mathrm{P} \;=>\; P(\mathrm{Q} = 0) = 666.7 - 2.222(0) = 667$$

d. (P,Q) = ($333,150). Arc price elasticity = −0.608.

set $e_p = -1$ and solve for P to find unit elastic point

$$e_p = \frac{dQ_i}{dP_i} \times \frac{P_i}{Q_i} = -1 = -0.45 \times \frac{P}{300 - 0.45\mathrm{P}}$$

$$-300 - 0.45P = -0.45 \times P$$

$$P = 333.3 \; and \; Q(P = 333.3) = 300 - 0.45(333.3) = 150$$

$$E_{arc} = \frac{(Q_2 - Q_1) \times (P_1 + P_2)}{(P_2 - P_1) \times (Q_1 + Q_2)}$$

$$E_{arc} = \frac{(150 - 223) \times (171.1 + 333)}{(333.3 - 171.1) \times (223 + 150)} = -0.60824$$

Question 8.

Note to instructor: This problem may require additional background on congestion pricing.

Total cost in a transportation system *TC* increases as a function of quantity *Q* of units in the system as follows:

$$TC = Q^3 - 5Q^2 - 56Q + 60$$

The relationship between demand and price is given by the following curve:

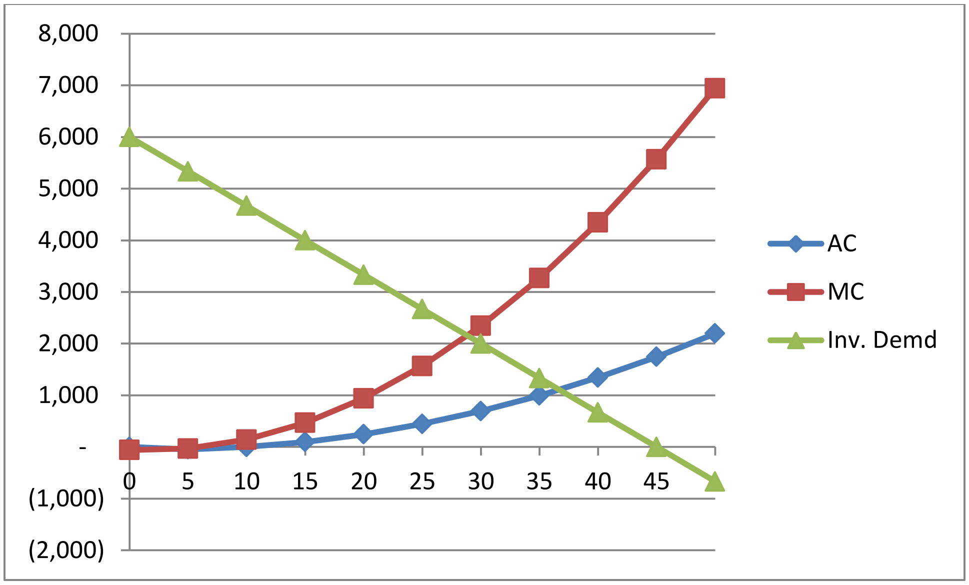
$$P = 6000 - 133.3Q$$

(Hint: This is a theoretical problem where *TC* and *P* are given in arbitrary cost units, rather than U.S. dollars.)

a. What congestion pricing user charge must be added to the average cost observed by the user in order to achieve the socially optimal equilibrium demand value?
b. If this user charge is added, by how much will the quantity *Q* be reduced compared to the situation with no user charge?
c. Sketch on a single set of axes as a function of demand level *Q*: Price corresponding to inverse demand, average cost AC, marginal cost MC. Since this is an exam setting, a rough graph not to scale is fine, but the shape should be correct.

Solution.

a. Solve for marginal cost: MC = 3*Q*^2 −10*Q* – 56. Setting equal to demand gives *Q* = 28.9 =~29, *P* = 2153. AC = *Q*^2 – 5*Q* – 56 + 60/*Q*. AC(*Q*) = 637. Charge = 2153 – 637 = 1516 units.
b. Equilibrium w/o user charge at AC(36.7) = *Q*^2 – 5*Q* – 56 + 60/*Q* = 1108 = Demand(36.7). Reduction = 36.7 – 28.9 = 7.8.
c. A version of the graph generated in Excel, although the students would create by hand with pencil and paper:



Question 9.

Total demand for travel on a corridor is Q_{IJ} = 10,000 trips per day. The choices are either private car or public transit. If inputs to a utility function for travel are X_1 = time in motion, X_2 = time spent on waiting, and X_3 = out-of-pocket cost, the equation can be written as follows:

$$U_k = a_k - 0.02X_1 - 0.035X_2 - 0.004X_3$$

The modal bias coefficient values for car and transit are 0.0 and −0.4, respectively. The inputs to the utility function are as follows:

Mode	X_1	X_2	X_3
Auto	10	0	300
Bus	30	5	150

Use a multinomial logit model to predict the number of trips per day by each mode.

Solution.

Multiplying modal values by parameters gives the following:

Mode	X_1	X_2	X_3	Sum
Auto	−0.2	0	−1.2	−1.4
Bus	−0.6	−0.175	−0.6	−1.375

Adding the calibration parameter 0 and −0.4 gives −1.4 and −1.775, respectively.

Raising *e* to the respective power and dividing by the total gives the following table of values:

Auto	0.247	0.593
Bus	0.169	0.407
Comb:	0.416	1.000

Multiplying the fractions by the total flow gives flow values of 5927 and 4073, respectively.

Question 10.

Transportation energy planners in a country are considering the available domestic petroleum reserves, which are thought to comprise 357 bil.bbl. The technological substitution model with parameters $c_1 = -5.38$ and $c_2 = 0.114$ is thought to fit the cumulative consumption of the reserve. In terms of its lifetime, cumulative consumption in year 50 and year 60 was observed at 212 bil.bbl and 281 bil.bbl. Based on these two years, what is the RMSD value of the difference between modeled and observed cumulative consumption?

Solution.

First use the information about the model to calculate the projected amount of cumulative consumption in the 2 years in question. Accordingly

$$f(50) = (357)\frac{e^{(-5.38+0.114 \cdot 50)}}{1+e^{(-5.38+0.114 \cdot 50)}} = 206.8 Bbbl$$

$$f(60) = (357)\frac{e^{(-5.38+0.114 \cdot 60)}}{1+e^{(-5.38+0.114 \cdot 60)}} = 289.7 Bbbl$$

The values for the RMSD calculation are then calculated by comparing the above to the observed values:

$$(212B - 206.8B)^2 = 2.68 \times 10^{19}$$

$$(281B - 289.7B)^2 = 7.60 \times 10^{19}$$

$$RMSD = \sqrt{\frac{1}{2}\left(2.68 \times 10^{19} + 7.60 \times 10^{19}\right)} = 7.17 \times 10^{9}$$

Question 11.

A region uses primarily two alternative passenger modes, passenger car (labeled "CAR") and public transportation (labeled "PT"). The region has levels of transportation activity and energy consumption for the two modes for years 2000 and 2005 shown in the tables below:

Transportation activity [million passenger-km per year]

Year	CAR	PT
2000	38	21

2005	41	26

Energy consumption [billion kJ per year]

Year	Car	PT
2000	83.6	35.7
2005	85.3	36.2

Use the Divisia analysis technique to calculate trended energy consumption in 2005, the contribution of energy intensity and modal structure to the change in energy consumption from 2000 to 2005, and to show that for 2005 the trended energy plus intensity and structure contribution equals the actual energy consumption.

Solution.

First compile data into a table of overall pkm, energy consumption, and energy intensity to assist with calculating trended energy consumption:

Year	pmi	kJ	kJ/pmi
2000	5.90E+07	1.19E+11	2022
2005	6.70E+07	1.21E+11	1813

Trended energy consumption in 2005 is then:

$$\left(6.7\times10^{7}\ pkm\right)\left(2022kJ/pkm\right)=1.35\times10^{11}kJ$$

Based on pkm values, the shares in 2000 and 2005 for CAR and PT modes are 64.4%/35.6% and 61.2%/38.8%, respectively. Similarly, the intensity values in 2000 and 2005 for CAR and PT modes are 2200/1700 and 2080/1392, respectively. These values are used to calculate the 2000 to 2005 transition values needed for the contributions.

Intensity contribution for CAR, PT, and combined:

$$\left(2080-2200\right)\left(\frac{0.644+0.612}{2}\right)=-75.5$$

$$\left(1392-1700\right)\left(\frac{0.356+0.388}{2}\right)=-114.5$$

$$-75.5-114.5=-190.0$$

Structure contribution for CAR, PT, and combined:

$$(0.612-0.644)\left(\frac{2080+2200}{2}\right)=-68.8$$
$$(0.388-0.356)\left(\frac{1392+1700}{2}\right)=49.7$$
$$-68.8+49.7=-19.1$$

To convert contributions to units of energy, multiply by 2005 pkm:

$$(-190)(67M)=-1.27\times10^{10}kJ$$
$$(-19.1)(67M)=-1.28\times10^{9}kJ$$

The accuracy of the Divisia analysis is then confirmed by adding intensity and structure to trended to show that the result equals actual energy in 2005:

$$135B-12.7B-1.3B=121B=1.21\times10^{11}kJ$$

Question 12.

Note to instructor: This problem is based on the same numbers as Question 9, but asks the problem in a different direction. It is a way of testing the students' comprehension of the Divisia technique.

The following values of transportation activity in a region are observed for two modes, namely private car ("CAR") or public transportation ("PT"), for the years 2000 and 2005:

Year:	CAR	PT	Total
2000	38	21	59
2005	41	26	67

The two modes are given numbers 1 = CAR and 2 = PT. The following values are observed, where *i* is the subscript for mode and *t* is the subscript for time period (either t = 1 for 2000 or t = 2 for 2005):

$$\sum_{i=1}^{2}\left[(e_t-e_{t-1})_i\frac{(s_t+s_{t-1})_i}{2}\right]=-189.9kJ/pkm$$
$$\sum_{i=1}^{n}\left[(s_t-s_{t-1})_i\frac{(e_t+e_{t-1})_i}{2}\right]=-19.1kJ/pkm$$

The observed energy intensity in 2005 for the combination of CAR and PT is 1806 kJ/pkm. If the energy consumption for CAR in 2000 is 83.6 billion kJ, what is the energy consumption for PT in 2000? Use the Divisia analysis methodology to compute your answer.

Solution.

First use summation terms to calculate intensity and structure contribution, respectively, to the difference between trended and actual energy consumption:

$$-189.9\left(6.7\times10^{7}\right)=-1.27\times10^{10}$$
$$-19.1\left(6.7\times10^{7}\right)=-1.28\times10^{9}$$

Actual energy consumption in 2005 can be obtained by multiplying pkm by energy intensity, so trended energy can be calculated by subtracting intensity and structure from actual:

$$1806\left(6.7\times10^{7}\right)=1.21\times10^{11}kJ$$
$$1.21\times10^{11}-\left(-1.27\times10^{10}\right)-\left(-1.28\times10^{9}\right)=1.35\times10^{11}kJ$$

From trended energy, it is possible to obtain 2000 overall energy intensity, and from there 2000 energy consumption:

$$\frac{1.35\times10^{11}}{6.7\times10^{7}}=2015kJ/pkm$$
$$2015\left(5.9\times10^{7}\right)=1.19\times10^{11}kJ$$

Lastly, subtract CAR energy from total 2000 energy to calculate PT energy:

$$1.19\times10^{11}-83.6\times10^{10}=35.4\times10^{10}kJ$$

Question 13.

You are given the following data for energy consumption in passenger transportation in the U.S., divided into two categories: "passenger vehicles," meaning privately owned cars, light trucks, etc., and "commercial," meaning any form of transportation for which you pay for a ticket, include air travel, trains, buses, etc. The values given are approximately accurate. A passenger-kilometer [pkm] is the transportation of one passenger for a distance of 1 km.

	Energy		Passenger-kilometers	
	[EJ]	[EJ]	[trillion]	[trillion]
Year:	1990	2000	1990	2000
Psgr.Vehicles	7.82	9.32	3.38	4.22
Commercial	1.31	1.64	0.72	0.99

a. For the year 2000, calculate the overall energy consumption in EJ and energy intensity in kJ/pkm.
b. Calculate the trended energy consumption in 2000 in EJ.
c. Calculate the contribution of changes in structure and intensity to the difference between trended and actual energy consumption in 2000, using Divisia analysis.

Solution.

(a): The combination of passenger vehicle and commercial energy consumption and demand is, respectively, 10.96 EJ and 5.21 trillion pkm. Therefore, energy intensity is:

$$\mu_{energy} = \frac{10.96EJ}{5.21tril.pkm} = 2.10EJ / tril.pkm = \sim 2100kJ / pkm$$

(b): To calculate trended energy consumption in 2000, we first need energy intensity in 1990:

$$\mu_{energy} = \frac{7.82 + 1.31EJ}{3.38 + 0.72tril.pkm} = \frac{9.13}{4.1} = 2.23EJ / tril.pkm = \sim 2230kJ / pkm$$

Multiplying gives (5.21 trillion pkm)(2230 kJ/pkm) = 11.62 EJ

(c). Based on pkm values, the shares in 1990 and 2000 for PSV and COM modes are 82.4%/17.6% and 81.0%/19.0%, respectively. Similarly, the intensity values in 1990 and 2000 for PSV are 2314 and 2209, and for COM are 1819 and 1657 kJ/pkm. These values are used to calculate the 1990 to 2000 transition values needed for the contributions.

Intensity contribution for PSV, COM, and combined:

$$(2209 - 2314)\left(\frac{0.824 + 0.810}{2}\right) = -85.87$$
$$(1657 - 1819)\left(\frac{0.176 + 0.190}{2}\right) = -29.78$$
$$-85.87 - 29.78 = -115.65$$

Structure contribution for PSV, COM, and combined:

$$(0.810 - 0.824)\left(\frac{2314 + 2209}{2}\right) = -32.58$$
$$(0.190 - 0.176)\left(\frac{1819 + 1657}{2}\right) = 25.04$$
$$-32.58 + 25.04 = -7.54$$

To convert contributions to units of energy, multiply by 2000 total volume of 5.21 trillion pkm:

$$(-115.65)(5.21T) = \sim -619PJ$$
$$(-7.54)(5.21T) = \sim -39PJ$$

The accuracy of the Divisia analysis is then confirmed by using units of PJ rather than EJ and adding intensity and structure to trended to show that the result equals actual energy in 2000. Starting with 10,960 PJ and 11,600 PJ in 1990 and 2000, respectively:

$$11{,}600 - 621 - 39 = 10{,}959 \approx 10{,}960 PJ$$

Thus ignoring the slight rounding error, the validity of the Divisia analysis is confirmed.

Part 2 Passenger Transportation Problems

Question 1. In a particular city, a prospective vehicle buyer is deciding whether to buy their own car or join a Carshare. Carsharing costs \$200 per year to join, \$6 per hour for time, and \$0.25 per mile. On every 7 mi of driving will result in 1 hour of time charge. If the buyer chooses their own car, it will cost \$16,000 and be paid off with a loan at 8% for 5 years; ignore the remaining years of the cars life and any salvage value at the end of it. (Note that this plan implies a single annual payment each year of the loan; in reality, auto loans are monthly and not yearly, so this plan is a simplification for the purposes of the exam.) Repairs are expected to cost \$500 per year, and there is an additional \$1500 per year for "overhead": insurance, inspections, registrations, parking, etc. The car averages 25 mpg and gas is expected to cost \$4/gal.

a. Calculate the break-even distance between the vehicle owning their own car and joining carsharing.
b. If the buyer expects to drive 6000 mi per year, should they buy their own car or join Carshare? (No work required, just state the correct answer.)

Solution.

(a): On the own car side, the value of the annual payment is:

$$(\$16{,}000)(A/P, 8\%, 5) = (\$16{,}000)(0.2505) = \sim \$4007$$

The total annual cost is therefore \$4007 + \$500 + \$2000 = \$6007. The cost per mile is based on fuel cost and is equivalent to \$4/25 mpg = \$0.16/mi.

On the carsharing side, the conversion of hours to mileage charge gives:

$$(\$6)\left(\frac{1h}{7mi}\right) = \$0.86/mi$$

Then the total cost per mile is the sum of hourly and per mile charges, or \$0.86 + \$0.25 = \$1.11/mi. Calculating the break-even distance gives:

$$D = \frac{FC_{own} - FC_{share}}{VC_{share} - VC_{own}} = \frac{6007 - 200}{1.11 - 0.16} = 6112mi$$

(b): Since $D_{breakeven}$ > 6000 mi, they should join carsharing.

Question 2. A Concorde supersonic aircraft travels at 2160 km/h, has a supported area A_s of 1587.2 m^2, a frontal area of 59.7 m^2, jet engines with an efficiency of 33%, a drag coefficient of 0.03, and an assumed mass when cruising of 185,000 kg. Assume an air density at cruising altitude of 0.4 kg/m^3.

a) What is the power that must be delivered by the fuel to the jet engines to maintain this altitude and speed in units of MW?

Solution.

(a): Supported area A_s can be calculated from length and width of Concorde:

$$A_s =\sim 1587 m^2$$

P_{total} is next calculated, converting 2160 km/h to 600 m/s:

$$P_{total} = 0.5\rho C_d A_p V^3 + 0.5\frac{(mg)^2}{\rho V A_s}$$

$$= 0.5(0.4)(0.03)(59.7)(600)^3 + 0.5\frac{(185{,}000 kg \cdot 9.8 m/s^2)^2}{(0.4)(600)(1587)}$$

$$= 81.7 MW$$

Energy required to power jets:

$$\frac{81.7 MW}{0.33} = 248 MW$$

One reason given for the demise of the Concorde in 2003 was the impact of sustained higher jet fuel prices on the economics of the service. Consider a flight from North America to Europe that is 5600 km in length, carried out at the conditions in part (a). Suppose the cost of jet fuel rises from $0.35/L to $0.75/L between the early 1990s and 2003, the last year of service.

b) By how much does this increase in fuel price raise the fuel cost for a one-way flight of the Concorde in U.S. dollars?

Solution.

Since the flight is 5600 km it takes 2.58 hours.

$$E_{out} = (6.04 MW)(2.58 h)(3600 MJ / MWh) = 230{,}000 MJ$$

$$\frac{230{,}000 MJ}{35.9 MJ / L} = 63{,}997 L$$

Since the change in cost is $0.40, the increase is worth ($0.40)(63,997) = $25,600

c) [2 pts, no partial credit, no questions answered] Give one example of a way in which the calculations in (a) or (b) are simplistic.

Examples:

1. Aircraft becomes lighter during flight as it consumes fuel.
2. Fuel consumption does not consider effect of takeoff or landing.
3. Fuel tanks may not be completely full at beginning of flight if fuel is not necessary.

4. Fuel consumption does not consider impact of winds or changes in speed during flight, as well as changes in elevation to avoid turbulence.
5. Flight may or may not be full.
6. Model is overly simplistic and therefore subject to inaccuracy.
7. The model of the supported area as a box is a simplification.

Question 3. Flow approaches a bottleneck in a limited access highway with a flow value of 4550 veh/h (2275 veh/lane/h); there are normally two travel lanes available for travel, but the traffic must narrow down to one lane to pass an obstacle (such as a construction zone or disabled vehicle). The flow in each lane (both in the regular travel lanes and in the bottleneck lane) can be modeled using a Greenshields model with functional form $u = 130 - 1.857k$, where *u* is in km/h and *k* is in veh/km.

What is the speed of the shock wave at the end of the queue waiting to merge and enter the bottleneck?

<u>Solution.</u>
Greenshields model:

$$\rightarrow u = u_f - \left(\frac{u_f}{k_j}\right) k$$

Therefore u_f = 130 km/h and $k_j = \frac{u_f}{1.857} = \frac{130}{1.857} = 70 v/km$

$q_{\max} = \frac{u_f k_j}{4} = \frac{(70)(130)}{4}$ = 2275 veh/lane/h.Therefore for two lanes q_{max} =(2275)(2) = 4550 veh/h.

It is therefore noted that the approaching traffic is traveling in maximum flow conditions, so that:

k_1 = 70/2 = 35 veh/km.

To calculate flow in the lanes approaching the bottleneck, we note that $q = q_{max}$ = 2275 veh/h. It therefore follows that flow will be 2275 veh/h across two lanes in the queue, or 1137.5 veh/lane/h in each lane. Note that the roots of the equation are found using the quadratic formula. Solving on the basis of flow in two lanes:

$$q = 2uk$$
$$= 260k - 3.714k^2$$
$$q = 2275 = 260k - 3.714k^2$$
$$3.714k^2 - 260k + 2275 = 0$$
$$k = \frac{-(-260) \pm \sqrt{(-260)^2 - 4(3.714)(2275)}}{2(3.714)}$$
$$k = 10.25, 59.75$$

Since the queue is waiting to enter the bottleneck, it is a high-density, low-speed situation, so we choose the larger root:

k_2 = 59.75 veh/km

We now have all *q* and *k* values needed to calculate *u*, the speed of the shock wave:

$$u = \frac{\Delta q}{\Delta k} = \frac{q_2 - q_1}{k_2 - k_1} = \frac{1137.5 - 2275}{59.75 - 35} = -45.96 \text{ km/h}$$

Question 4. Total cost of travel *TC* on a roadway is a function of number of vehicles *Q* with a functional form $TC(Q) = 0.03Q^2 + 2.2Q$. The relationship between demand *Q* and price *P* is the following:

$$Q = \frac{475}{P}$$

Note that this relationship breaks down at either low or high values of *P*, but for purposes of the problem, the relationship holds for prices $4 \le P \le 18$.

a. [7 points] What is the equilibrium flow *Q* that will be observed if flow is determined by the average cost of using the roadway? Round *Q* to the nearest integer.

Solution.

First convert demand to inverse demand, then solve for AC and set equal:

$$P = \frac{475}{Q}$$

$$AC = \frac{TC}{Q} = 0.03Q + 2.2$$

$$0.03Q + 2.2 = \frac{475}{Q}$$

$$0.03Q^2 + 2.2Q - 475 = 0$$

Then use the quadratic to solve:

$$Q = \frac{-(2.2) \pm \sqrt{(2.2)^2 - 4(0.03)(-475)}}{2(0.03)}$$

$$Q = 94.4$$

Since the other root is negative, it is ignored. Rounding gives *Q* = ~94.

b. [8 points] What congestion price should be charged so that the equilibrium flow value of *Q* will occur at the socially optimal level where the marginal cost is equal to inverse demand? Use the value of *Q* rounded to the nearest integer to calculate the congestion price.

Solution.

Solve for MC and set equal to inverse demand:

$$MC = \frac{dTC}{dQ} = 0.06Q + 2.2$$
$$0.06Q + 2.2 = \frac{475}{Q}$$
$$0.06Q^2 + 2.2Q - 475 = 0$$

Then use the quadratic to solve:

$$Q = \frac{-(2.2) \pm \sqrt{(2.2)^2 - 4(0.06)(-475)}}{2(0.06)}$$
$$Q = 72.5$$

Since the other root is negative, it is ignored. Rounding gives Q = ~73. Two approaches with slightly different values are equally valid: (1) Calculating P based on the inverse demand function, the value of AC, and then the difference gives the required congestion charge:

$$P(73) = \frac{475}{73} = 6.51$$
$$AC(73) = 0.03(73) + 2.2 = 4.39$$
$$\text{Assuming } MC(73) = P(73) = \$6.51$$
$$Chg = \$6.51 - \$4.39 = \$2.12$$

Alternatively, you might use the exact value calculated from MC(73):

$$MC(73) = 0.06(73) + 2.2 = \$6.58$$
$$Chg = MC(73) - AC(73) = \$6.58 - \$4.39 = \$2.19$$

Question 5. There is a bottleneck on an expressway where two lanes of traffic moving in one direction must narrow down to one lane. On the lanes inside of and surrounding the bottleneck, the speed-density relationship can be modeled using a Greenshields model with free-flow speed of 72 mi/h (mph) and jam density of 116 vehicles per mile (veh/mi). A queue of 2.5 mi length has formed upstream from the bottleneck. At time t = 0, the free-flow upstream from the queue drops from its previous value (which you do not need for this problem) to 1600 vehicles per hour (veh/h), or 800 veh/lane/h. The speed of the vehicles upstream from the queue is 64.5 m-/h. (Note that to save time I have applied the quadratic formula to the point on the flow-density curve upstream from the queue, so that you do not need to do it yourself.) How many minutes from time t = 0 does it take for the queue to disappear? Show all necessary calculations.

Solution in short form:

q_{max} is 2088 veh/lane/h. At this flow, density in the queue approaching the bottleneck is k = 99.0 veh/mi and speed is u = 10.5 mi/h. ΔQ is therefore 244 veh/lane/h and ΔK is 86.6 veh/h, so that u_w is $\Delta Q/\Delta K$ = 244/86.6 = 2.82 mi/h. A queue of 2.5 mi will therefore disappear in L/u_w = 2.5/2.82 = 0.89 hour or 53.2 minutes.

Detailed answer:

Greenshields model:

$\rightarrow u = u_f - (\frac{u_f}{k_j})\, k$

$\rightarrow q = ku_f - (\frac{u_f}{k_j})k^2$

To find q_{max}:

$$q = 72k - (\frac{72}{116})k^2$$

$$\frac{dq}{dk} = 72 - (\frac{144}{116})\, k = 0 \text{ or } \quad q_{max} = \frac{u_f k_j}{4} = \frac{(72)(116)}{4} = 2088 \text{ veh/lane/h}$$

$$k_{max} = k_j/2 = 116/2 = 58 \text{ veh/mi.}$$

$$q_{max} = 72\,(58) - (\frac{72}{116})(58)^2$$

$$q_{max} = 2088 \text{ veh/lane/h}$$

Across two lanes: Note that the roots of the equation are found using the quadratic formula

$$q = 144k - (\frac{144}{116})k^2 \qquad\qquad q = uk$$

$$2088 = 144k - (\frac{144}{116})k^2 \qquad\qquad u = (q/k) = (2088/99)/2$$

$$1.242k^2 - 144k + 2088 = 0 \qquad\qquad u = 10.5 \text{ mi/h (per lane)}$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

k = 98.95, ~~16.99~~

(We choose the higher value of k because we know that the flow is in a high-density, low-speed situation as it waits to enter the bottleneck.)

k = ~99 veh/mi

Upstream: The solution shows how the k and u values are derived using the quadratic formula. Note that since you are given q and u, you can simply calculate the needed k value using $k = q/u = 800/64.5 = 12.4$ veh/mi.

$800 = 72k - (\frac{72}{116})k^2$ $u = (q/k) = (800/12.4)$

$(\frac{72}{116})k^2 - 72k + 800 = 0$ $u = 64.5$ mi/h (per lane)

$k =$ ~~103.55~~, 12.4

	Upstream	Queue
q (one lane)	800	1044
q (two lane)	1600	2088
k	12.4	99
u	64.5	10.5

$u = \frac{\Delta q}{\Delta k} = \frac{1044-800}{99-12.4} = 2.82$ mi/h

Queue length = 2.5 mi, so $t = \frac{2.5\ mi}{2.82\ mi/h} = .89$ h = 53.24 min

→ $t = 53.2$ minutes for the queue to disappear

Question 6. Traffic is traveling on a two-lane rural highway (one lane of travel in each direction) at a speed of 55 km/h and a density of 25.4 veh/km. The road is winding and therefore does not allow passing. Flow on the highway can be modeled with the following equation for the relationship of speed in km/h to density in veh/km:

$$u = 90 - 1.38k$$

You can assume that the flow of vehicles is infinitely large and therefore provides a continuous stream of cars traveling with constant speed and density. Suddenly a parade of college students, celebrating a very recent victory over a collegiate rival in a sporting event, enters the roadway in front of the traffic and proceeds to travel forward at a speed of 10 km/h.

a. (5 points) What is the maximum possible flow in vehicles per hour in one direction for the roadway? You may solve by any appropriate method. *Short answer: 1467 veh/h.*
b. (5 points) What is the density of vehicles (in units of veh/km, given to the nearest 0.1 veh/km) that forms behind the students? *Short answer: 58.0 veh/km.*

c. (10 points) After 10 minutes, the students exit the road, allowing the platoon of vehicles to continue unimpeded. At that point, how long is the platoon that has formed, in km? Give your answer to the nearest tenth of a kilometer (i.e., nearest 0.1 km). *Short answer: Length = 5.8 km.*
d. (10 points) After the students have left the road, what is the speed of the shock wave at the front of the platoon that represents vehicles dispersing off the front, to the nearest 0.1 km/h? *Short answer: −34.9 km/h.*

Solution.

This is a Greenshields model. For the traffic, u_1 = 55 km/h, k_1 = 25.4 veh/km. For the students, u_2 = 10 km/h. Free-flow speed is u_f = 90 km/h. Jam density k_j = 90/1.38 = 65.2 veh/km

a. maximum possible flow $q_{\max} = k_j u_f/4 = (65.2 \times 90)/4 = 1467$ veh/h

b. Since $u = 90 - 1.38\,k$, $k = 65.22 - 0.72\,u$

$k_2 = 65.22 - 0.72\,u_2 = 65.22 - 0.72 \times 10 = 58.0$ veh/km

c. $q_1 = k_1 \times u_1 = 55 \times 25.4 = 1397$ veh/h

$q_2 = k_2 \times u_2 = 58.0 \times 10 = 580$ veh/h

$u_w = (q_2 - q_1)/(k_2 - k_1) = (580 - 1397)/(58.0 - 25.4) = -25.06$ km/h

t = 10 minutes/60 minutes = 1/6 hour

The relative speed is $u_{rel} = u_2 - u_w = 10 - (-25.06) = 35.06$ km/h

Length = $u_{rel} \times t = 35.06 \times (1/6) = 5.84$ km

d. When the parade has left, $q_3 = q_{\max} = 1467$ veh/h, and $k_3 = k_j/2 = 65.2/2 = 32.6$ veh/km

$u_w = (q_3 - q_2)/(k_3 - k_2) = (1467 - 580)/(32.6 - 58.0) = -34.9$ km/h.

Question 7. A city is considering the construction of either a subway line with one track in each direction, or a freeway whose required number of lanes depends on the required capacity. The subway has a maximum line capacity of 30,000 spaces/h in one direction at the peak. The operating speed is 35 km/h. The freeway has a capacity of 8000 spaces/lane/h in one direction. The subway costs $120 M/km for a pair of tracks (one track in each direction) and the freeway costs $50 M/km per pair of lanes (one lane in each direction). The anticipated maximum required flow is 21,500 spaces/h in one direction.

a. What is the productive capacity of the subway line?

Solution.

Productivity capacity = (operation speed)(lane capacity)

= (35 km/h)(30,000 spaces/h)

= 1,050,000 space-km/h^2

b. What is the largest possible freeway, measured in number of lanes in each direction, which the city can build such that the line capacity of that freeway in spaces/h is less than that of the subway with one track in each direction? If either the subway or freeway is 18-km long, how much construction cost (in dollars) does the city save by building the subway instead of the freeway?

Solution.

(8000 spaces/lane/h)(3 lanes) = 24,000 spaces/h > 21,500 spaces/h

→ 3 lanes/6 lanes total

Subway cost = ($120 M/km)(18 km) = $2160 M

Freeway cost = ($150 M/km)(18 km) = $2700 M

Save $30 M per km and $540 M total.

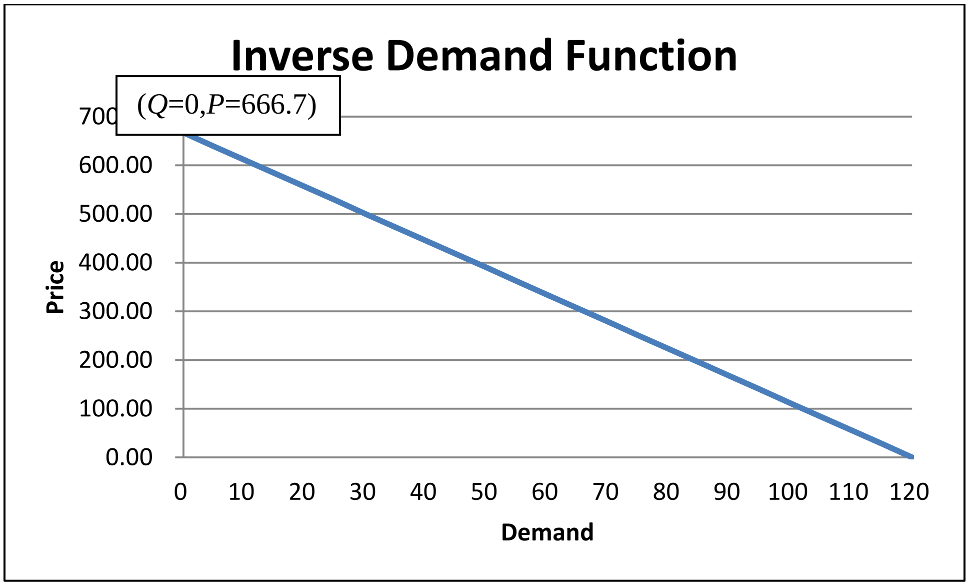
Question 8. An airline operates an aircraft with 110 seats on a route where both business and leisure (also called tourist) fares are available. The demand for seats Q responds to the prices P set by the airline as follows:

$$Q_B = 120 - 0.18P_B$$

$$Q_L = 165 - 0.65P_L$$

a.) For the business class write the inverse demand function and sketch the function on *X*-*Y* axes, labeling price and demand on the appropriate axis and giving the numerical values at the *x*- and *y*-intercepts.

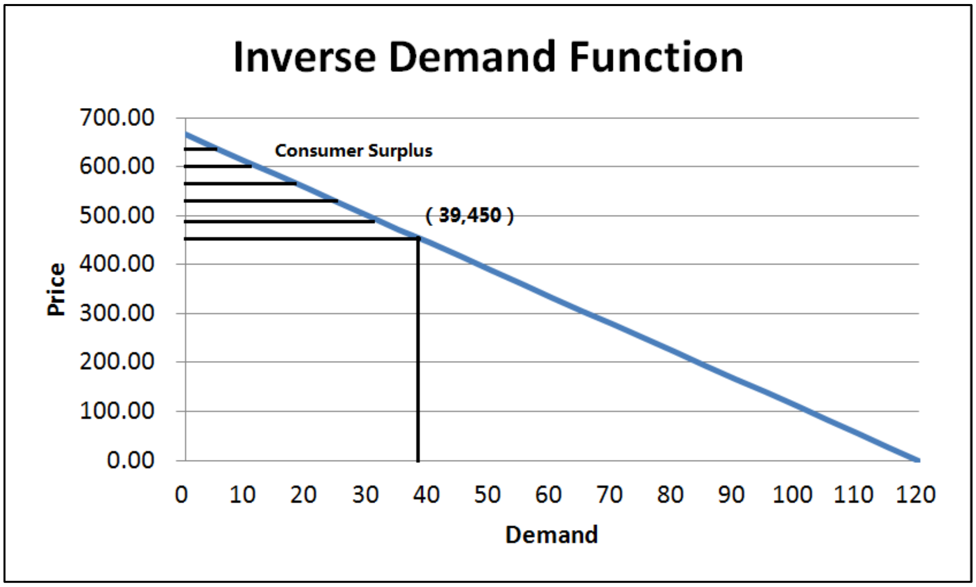
$$P_B = \frac{120}{0.18} - \frac{Q_B}{0.18} \approx 666.7 - 5.56Q_B$$



b. If the airline sets the business fare such that 39 business travelers purchase tickets, what is the value of the consumer surplus at this level of fare and demand?

Answer: Consumer surplus = $\frac{1}{2}\left(\frac{2000}{3} - 450\right) \times 39 = 4225$

Diagram to show consumer surplus visually:



c. Solve for the optimal pair of business and leisure prices per ticket that maximize revenue. Give your answer to the nearest \$0.01. Note that you only need to give the ticket prices, you do not need to state the demand levels or the value of the resulting revenue.

$$\max R = Q_B P_B + Q_L P_L$$

$$\text{such that } Q_B + Q_L = 110$$

$$Q_B = 120 - 0.18 P_B$$

$$Q_L = 165 - 0.65 P_L$$

So we get $P_B = \frac{175 - 0.65P_L}{0.18}$. Then we rewrite the objective function as

$$R = \left[120 - 0.18\frac{175 - 0.65P_L}{0.18}\right]\frac{175 - 0.65P_L}{0.18} + (165 - 0.65P_L)P_L = -2.997P_L^2 + 995.56P_L - 53.47$$

$$\frac{\partial R}{\partial P_L} = -5.994P_L + 995.56 = 0 \Rightarrow P_L = 166.09$$

Then from P_L = \$166.09 we can get P_B = \$372.44.

Question 9. A commuter is considering the benefits and costs of telecommuting, including the value of time saved, the cost of driving, and the additional cost of utilities if they work from home. The person values their time at \$12.50 per hour. The round-trip vehicle cost per commute is distributed normally with mean of \$5.50 and standard deviation of \$0.75. The number of minutes of commuting time round trip is distributed uniformly between 28 and 37 minutes. The added utility costs if the telecommute are distributed normally with mean of \$6.80 and standard deviation of \$0.40.

a. What is the expected value of the benefit/cost ratio (B/C ratio) for telecommuting for this commuter?
b. Now suppose the commuter wants to explore the possible impact of random variation on the B/C ratio. They therefore draw three random variates from the uniform (0,1) distribution, corresponding to the three variables in the B/C ratio, as follows:

Vehicle cost	0.1942
Commute time	0.7945
Utility cost	0.5504

A portion of the standard normal table is provided, giving the number of standard deviations from the mean (referred to as the "z value") corresponding to the probability value drawn from the U(0,1) distribution. What is the value of the B/C ratio based on these randomized inputs?

RAND()	Value of *z*
0.1942	−0.8626
0.5504	0.1268
0.7945	0.8221

Solution.

(a): To calculate the expected value, use the average vehicle cost of \$5.50 and utility cost \$6.80. The average length of time is the average of 28 and 37 minutes, or 32.5 minutes or 0.542 hour. The value of this time is \$6.77, and the B/C is:

$$B/C = \frac{\$5.50 + \$6.77}{\$6.80} = 1.80$$

(b): Repeating using the probabilistic data gives vehicle cost of \$4.85 and utility cost of \$6.85. The value of minutes is 28 + 0.7945 × 9 = 35.15 minutes, and the value of this time is \$7.32, and the B/C is:

$$B/C = \frac{\$4.85 + \$7.32}{\$6.85} = 1.78$$

Question 10. Vehicles are traveling on a roadway with one lane of travel in each direction. The relationship between speed and density follows a Greenshields model with $u = 60 - 0.6\,k$. Vehicles are traveling with a density of 25 veh/mi.

(a): What is the speed and flow in vehicles per hour of this traffic?

Solution.

Since you are given k = 25 veh/mi, substitute into $u(k)$ to find u and then q:

$$u(k) = u(25) = 60 - 0.6(25) = 45 mi/h$$
$$q = uk = (45)(25) = 1125 veh/h$$

A slow-moving vehicle enters the lane and travels at a constant speed of 12 mi/h. Traffic begins to back up behind this vehicle, since overtaking the slow vehicle is not possible. A shock wave forms at the end of the queue of vehicles that accumulates behind the slow-moving vehicle.

(b): What is the speed of the shock wave? Show all calculations.

Solution.

Since you are given u = 12 mi/h, rearrange the speed-density equation and substitute to find k and then q:

$$k = 100 - 1.67u = 100 - 1.67(12) = 80 veh/mi$$
$$q = uk = (12)(80) = 960 veh/h$$

$$u_w = \frac{q_2 - q_1}{k_2 - k_1}, \text{ where } k_1 = 25, k_2 = 80, q_1 = 1125, q_2 = 960$$

Therefore, $u_w = \dfrac{960 - 1125}{80 - 25} = -3.01 mi/h =\sim -3 mi/h$

Question 11. You are to create a speed-density ("u-k") curve for a lane in a roadway based on the "two-second rule" commonly recommended for safe driving, meaning that each driver should allow 2-seconds worth of space between the front of their car and the rear of the car in front. There are 5280 ft in a mile, and 3600 seconds in an hour. Assume that each car is 16-ft long.

a. Suppose the maximum density for the roadway based on this rule is calculated on the basis of vehicles maintaining 20 ft between vehicles. What is the value of the density k in veh/mi and the speed at that density in mi/h? Hint: Note that because this is not a Greenshields model, the curve does not achieve zero speed, i.e., u = 0.
b. Suppose the *u-k* curve is made up of two connected curves, one curve at low density where speed is constant at the posted speed limit (65 mi/h), and another curve where speed declines linearly from 65 mi/h with increasing density according to the 2-seconds rule. What is the value of k where the two curves intersect? Give your answer to three significant digits.
c. Draw a sketch of the resulting *u-k* curve.
d. Find the density value that maximizes flow, and the flow value in veh/h at the density value.

Solution.
(a): First calculate density and speed at maximum density value.

a. Length: 16 + 20 = 32 ft for vehicle plus space. The vehicle must travel over the 20-ft gap in 2 seconds. Therefore:

$$k = \frac{5280}{32} = 147 veh/mi$$

$$u = \frac{20\,ft}{2s} = 10\,ft/s$$

$$(10)\left(\frac{3600s/h}{5280\,ft/mi}\right) = 6.82 mi/h$$

b. Convert 65 mi/h to ft/s and solve for distance covered in 2 seconds at that speed:

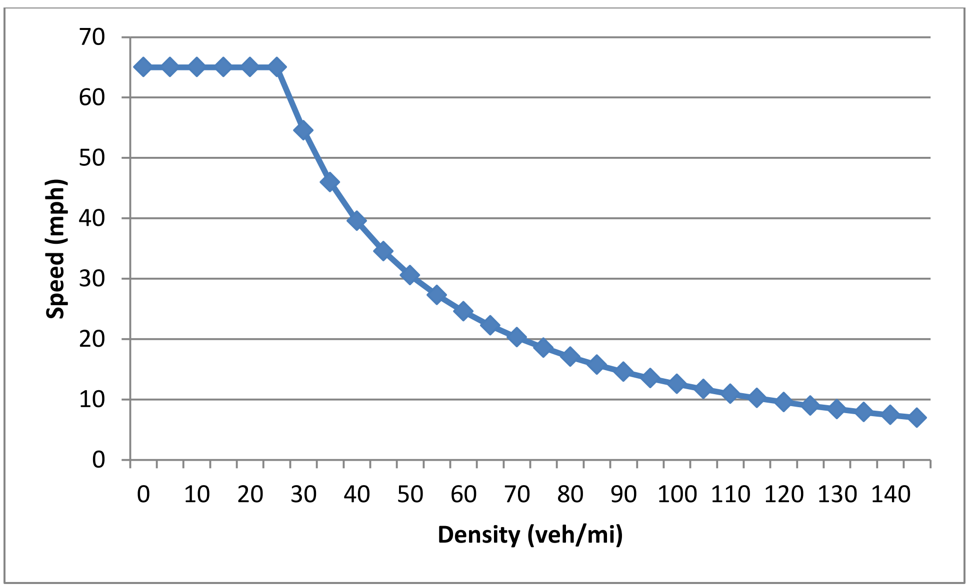
$$65 mi/h\left(\frac{5280\,ft/mi}{3600s/h}\right) = 95.3\,ft/s$$

$$95.3\,ft/s(2s) = 190.6\,ft$$

$$Lgth = Veh + Space = 16 + 190.6 = 206.6$$

$$k = \frac{5280}{206.6} = 25.5 \text{ veh/mi}$$

c. Graph of speed as a function of density has the following appearance:



d. Maximum flow occurs along a speed-density curve at the intersection between the two curves (i.e., 25.5 *v*/m, 65 mi/h). This can be observed because the maximum flow on the u = 65 mi/h portion of the curve occurs at k = 25.5 veh/mi: q = (65)(25.5) = 1658 veh/h. Also, on the other curve, q increases from that point as k decreases. For instance, at k = 20 veh/mi, if that speed were allowed on the 2-seconds rule curve, total length per vehicle including shadow is 5280/20 = 264 ft, shadow length is 264 – 16 = 248 ft, speed is 248/2 = 124 ft/s = 84.5 mi/h, and then

$$q = uk = (84.5)(20) = 1690 veh/h$$

Question 12. Two lanes approaching a construction bottleneck on a highway must narrow down to one lane. The relationship between speed and density per lane for flow both approaching and within the bottleneck can be modeled using a Greenshields model, with free-flow speed of 120 km/h and jam density of 70 veh/lane/km. Upstream from the queue that forms to enter the bottleneck, traffic approaches with a flow rate of 2700 veh/h. A shock wave forms at the back of the queue as drivers slow down from the upstream speed. What is the speed of the shock wave?

Solution:

Calculations for one lane in bottleneck:

$$q_{\max} = \frac{k_j u_f}{4} = \frac{(120)(70)}{4} = 2100 v/h$$

Calculations for two-lane queue waiting for bottleneck

$$q_2 = \frac{2100}{2} = 1050 veh/lane/h$$
$$k = \sim 60 veh/lane/km$$

Calculations upstream from queue:

$$q_1 = \frac{2700}{2} = 1350 veh/lane/h$$
$$k = \sim 14 veh/lane/km$$

Speed of shock wave:

$$u_w = \frac{\Delta q}{\Delta k} = \frac{(1050 - 1350)}{60 - 14} = \frac{-300}{46} = -6.5 km/h$$

Question 13:

Note to instructor: This problem may require additional background on the gravity model

In two-zone gravity model, the zones 1 and 2 in the model produce 800 and 1300 trips, respectively. Also, their relative attractiveness values are 2 and 3, respectively. Interzonal impedance values for the model are in the following table:

From	To 1:	To 2:
1	5	10
2	10	5

The calibration parameter for the city to which the model is applied is c = 2, and the socioeconomic adjustment factor is k_{ij} = 1 throughout the model. What are the values of the four flows Q_{IJ} predicted by the gravity model?

Solution:

Note: $F_{ij} = 1/(W_{ij}^{c})$ where the table given above displays the W_{ij}^{c} values.

Zone 1: 800 units of flow

	Aj	F_{iJ}	K	AFK	P_{ij}	Q_{ij}
1	2	0.04000	1	0.08	0.73	582
2	3	0.01000	1	0.03	0.27	218
				0.11		800

Zone 2: 1300 units of flow

	A_j	F_{iJ}	K	AFK	P_{ij}	Q_{ij}
1	2	0.01000	1	0.02	0.14	186
2	3	0.04000	1	0.12	0.86	1,114
				0.14		1,300

Table w/ all flows:

From	To 1:	To 2:
1	582	218
2	186	1,114

Question 14. For a particular roadway, the relationship between density on the roadway k measured in veh/mi and velocity v measured in mi/h is of the form:

$$k = a - b \times v$$

where a and b are random variables that are sampled from normal distributions as part of a Monte Carlo simulation. For a particular iterate of the simulation, the values of a and b are calculated by generating two values of a random variable z from the standard normal table that determine the number of standard deviations σ from the mean μ for each variable.

The following table contains the information needed to compute the values for the iterate:

Random variable	μ	σ	z
a	224	8	−2.36
b	4.05	0.38	1.07

(a): For the resulting iterate, calculate (a) v_{max} in mi/h, (b) k_{max} in veh/mi, and (c) q_{max} in veh/h.

Solution:

Use the format $a = \mu + z \times \sigma$, and similar format to calculate b, to calculate inputs into the equation for density. Then solve for q and differentiate to find v_{max}, i.e., the value that maximizes flow:

$$a = 224 - 2.36 \times 8 = 205.1$$
$$b = 4.05 + 1.07 \times 0.38 = 4.457$$
$$k = 205.1 + 4.457v$$
$$q = kv = 205.1v + 4.457v^2$$
$$\partial q / \partial v = 205.1 + 8.913v$$
$$205.1 + 8.913v_{max} = 0$$
$$v_{max} = 205.1 / 8.913 = 23.01 mi/h$$

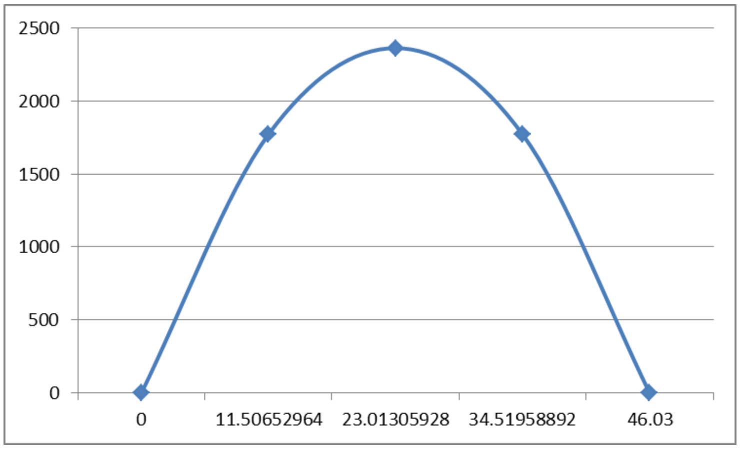
Thus the v_{max} = 23.01 mi/h is the speed that maximizes q. Substituting back into the given equations gives:

$$k_{max} = 205.1 + 4.457 \times 23.01 = 102.6 veh/mi$$
$$q_{max} = 102.6 \times 23.01 = 2360 veh/h$$

(b). [5 pts] Hand sketch a graph of q as a function of v, labeling x- and y-coordinates of appropriate points that characterize the resulting curve. Note: You are required to use your “engineering judgment” to determine which are the “appropriate” points.

Solution.

The key points that should be labeled in format (*v*,*q*) are: (0,0);(23.01, 2360);(46.03,0). The figure is shown below without the three labels:



Question 15. A city installs a bus rapid transit (BRT) system where each bus costs $600,000 and is discounted on the basis of 8% interest over 14 years. All remaining annual operating cost including wages, fuel, maintenance, and overhead amount to $85,000 per year. The bus is expected to drive 49,600 km per year.

a. What is the cost per km, including annual operating cost and annual cost of capital? Hint: use annual, not monthly discounting.

Solution: *use discounting formula:*

$$(A/P,8\%,14) = \frac{i(1+i)^N}{(1+i)^N - 1} = 0.1213$$
$$600K(0.1213) = \$72{,}778$$

Calculate cost per km by dividing all costs by total km per year:

$$\frac{\$72779 + \$85000}{49600} = \$3.18/km$$

Now suppose you wish to calculate from the government's perspective the cost per metric ton of CO_2 reduced from car drivers switching from driving their own cars (single occupant vehicles, or SOVs) to riding the BRT. Assume that ticket revenues cover the annual operating cost ($85K in part (a) of this problem), so use *only* the capital cost in calculating cost per ton. The cars average 10 km/L gasoline, and the BRT averages 1.2 km/L diesel. On average, 50 drivers switch from cars to BRT. You can ignore well-to-tank (WTT) efficiency for this problem, use tailpipe CO_2 emissions only.

b. What is the cost per metric ton of CO_2 reduced by investing in BRT?

Solution:

Use 2.4 kg CO_2 per liter for gasoline and 2.6 kg CO_2 per liter for diesel. Emissions for each car and for BRT are respectively:

$$\frac{1L}{10km}\left(\frac{2.4kg}{1L}\right) = 0.24kgCO_2/km$$
$$\frac{1L}{1.2km}\left(\frac{2.6kg}{1L}\right) = 2.17kgCO_2/km$$

Since each car emits 0.24 $kgCO_2$ per km, it follows that 50 drivers will emit 12 kg. Therefore, the savings will be 12 – 2.17 = 9.83 $kgCO_2$ per km.

Cost per km and the resulting cost per ton are the following:

$$\frac{72778}{49600} = \$1.47/km$$
$$\frac{\$1.47/km}{9.83kgCO_2/km} = \$0.149/kg = \$149/tonCO_2$$

Question 16. A new light rail (LRT) system is introduced which uses electricity from the following mix [note: 1 (metric) tonne = 1000 kg]:

Source	Output	CO_2
	[Billion kWh]	[Million tonnes]
Nuclear	30.00	0.000
Hydro	20.00	0.000
Gas	45.00	15.075
Coal	50.00	42.500
Biofuel	10.00	0.000
Wind	12.00	0.000

The system provides four round trips per hour for 16 hours per day, with an average occupancy of 65.1%. The line runs for 20 mi in each direction, and an LRT vehicle has a capacity of 360 riders and consumes 2 kWh of electricity per mile.

Among the riders of the new service, 70% are previous car commuters who traveled in single occupant vehicles (SOVs). Average commuting fuel economy is 6.76 km/L gasoline, and gasoline emits 2.35 $kgCO_2$ per liter consumed. The rest used previously existing public transportation, which can be assumed to have the same CO_2 emissions rate as the LRT system. For simplicity in an exam setting, you can assume that both car and LRT commuters travel 20 mi in each direction, thus the travelers on the LRT ride the entire length of the run, rather than boarding part way as would happen in a real-world system.

Make your calculations on a "round-trip" basis, thus a passenger who rides in one direction outbound and returns in the other direction is considered one trip, not two.

For the purposes of calculating effective CO_2 emissions, you can ignore transmission and distribution losses for the LRT, and well-to-tank (WTT) losses for private cars (i.e., consider tailpipe CO_2 emissions only).

a. What are the average CO_2 emissions per passenger round trip in the LRT, in $kgCO_2$?

b) How many tonnes of CO_2 emissions are saved per day by the new LRT system compared to the situation before it was introduced?

<u>Solution.</u>

(a): Average CO_2 emissions:

$$CO_2 / kWh = \frac{\sum TotalCO2_i}{\sum TotalkWh_i} = \frac{57.6 Bil.kg}{167 Bil.kWh} = 0.345 kgCO_2 / kWh$$

$$\text{Total electricity use per day :}$$

$$4\frac{trips}{h} \cdot 16\frac{h}{day} \cdot 40\frac{mi}{roundtrip}\left(2\frac{kWh}{mi}\right) = 5120 kWh/day$$

$$\text{Emissions per round trip :}$$

$$(4\text{trips/h})(16 h/day)(360 seats/trip) = 23{,}040 seats/day$$

$$(23{,}040 seats/day)(65.1\% occupancy) = 14{,}999 riders/day = \sim 15{,}000 riders/day$$

$$5120\frac{kWh}{day} \times 0.345\frac{kgCO_2}{kWh} = 1766 kgCO_2$$

$$1766 kgCO_2 \times \frac{1}{15000 riders/day} = 0.118 kgCO_2 / rider$$

(b): Emissions reduction: Calculate emissions for car commuters prior to LRT, then subtract new emissions to find improvement.

$$\text{Car}: 0.7 \cdot 15000 \frac{pers}{day} \cdot 40 \frac{mi}{day} \cdot \frac{1}{6.76 km/L} \cdot 2.35 \frac{kgCO_2}{L}$$

$$= 10{,}500 \frac{pers}{day} \cdot 40 \frac{mi}{day} \cdot \frac{1}{6.76 km/L} \cdot 2.35 \frac{kgCO_2}{L} =\sim 146{,}000 kgCO_2$$

Emissions when traveling by LRT :

$$\left(10{,}500 \frac{pers}{day}\right)(0.118) = 1239 kgCO_2$$

Net savings :

$$146{,}000 - 1239 = 144{,}761 kgCO_2$$

Part 3 Freight Transportation Problems

Question 1. A manufacturing company ships cartons of running shoes, each worth $600, from their production facility in East Asia to the North American market, via ocean-going container ship. In this example the company is responsible only for holding inventory cost at the origin and not at the destination. Demand in the destination market is 884 cartons per year. The cost of holding inventory is 2.2% per week, and the shipment of a container takes 5 weeks including all transit time in ports, costs $4800, and each container has a maximum capacity of 150 cartons.

a) What is the minimum total logistics cost, in your choice of per carton, per week, or per year? Solve on the basis of an integer number of cartons per shipment

<u>Solution:</u>

Since holding inventory cost is incurred only at the origin, the appropriate formula should be chosen for V*:

$$V^* = \sqrt{\frac{2FQ}{rP}} = \sqrt{\frac{2(4800)(17\,cart/week)}{(0.022)600}} = 111.1 =\sim 111$$

$V^* = 111 < 150 = V_{max}$ so we are free to choose the optimal shipment size of 111 cartons.

$$C = \frac{rPV}{2} + \frac{FQ}{V} + rPTQ$$
$$= \frac{(0.022)(600)(111)}{2} + \frac{(4800)17}{111} + (0.022)(600)(5)(17)$$
$$= \$2590$$

Therefore, the total logistics cost is $2590 per week. This translates to $134,666 per year, and since there are 884 cartons per year, the cost is $152 per carton. (Note: −1 point if students did not specifically confirm that the optimal shipment size is within the capacity of the container.)

Now suppose a new option emerges to use air freight instead of container ship. The capacity of each air freight container is 18 cartons, but the shipment size is specified at 17 cartons per shipment, shipped weekly. Each air freight container costs $1800 to ship and takes 8 days including origin and destination transit time.

b) Use the calculation of the total logistics cost via air in the same units you chose in part (a) to determine whether the company should switch to air freight or not.

$$V = 17\,cartons$$

$$C = \frac{rPV}{2} + \frac{FQ}{V} + rPTQ$$
$$= \frac{(0.022)(600)(17)}{2} + \frac{(1800)17}{17} + (0.022)(600)(1.14)(17)$$
$$= \$2169$$

Therefore, the total logistics cost via airfreight is \$2169 per week, which translates to ~\$128 per carton or \$112,800 per year. Thus since the TLC cost is lower for the same amount of shipment, the company should switch.

c) What is the most the company could pay per air freight container before the total logistics cost by air exceeds that of shipping by ocean? Show your work.

Solution.

Suppose total logistics cost by air is \$152.34 per carton, which to the nearest \$0.01 is the exact cost per carton for sea. The inventory cost by air (=\$6.60 + \$15.09 = \$21.69) does not change, so what is left is \$130.65 for this component. Solving for F:

$$\$130.65 = \frac{F}{Q} \rightarrow F = 130.65Q = 130.65(17) =\sim \$2221$$

Rounding gives *F* = \$2,221. Alternatively, you could also use the difference in weekly logistics cost of \$2590 − \$2169 = \$421. If I add this cost to the cost per airfreight container, air and sea TLC per week will be equal. Thus, the new cost is \$1800 + \$421 = \$2221.

Question 2. A tractor-trailer truck weighs 34,500 kg, has a cross-sectional area of 8.8 m^2, a drag coefficient of 0.4, and a coefficient of rolling resistance of 0.0056. Assume an air density of 1.1 kg/m^3. The truck travels a distance of 400 km at a constant speed of 112 km/h without stopping, and at that speed, the tank-to-wheel (TTW) efficiency is 26%.

a. How many liters of diesel fuel does the truck consume?
b. Suppose that the truck now travels the same distance at a reduced speed of 104 km/h to reduce CO_2 emissions. The TTW efficiency is again 26%. How many kg of CO_2 emissions are saved over the same 400-km distance compared to driving at 112 km/h?

Solution.

(a): For reference, the two speeds in the problem convert to 31.1 m/s and 28.9 m/s, respectively, which are used throughout. Substituting the known information into the tractive power equation:

$$P_{TR} = 0.5\rho A_F C_D V^3 + mgV\,C_o = 117{,}200W = 117.2kW$$

$$T = \frac{400km}{112kmh} = 3.57h$$

$$E = 418.58kWh = 1506.89MJ$$

$$E_{in} = \frac{1506.89MJ}{0.26} = 5795.72MJ$$

$$Fuel = \frac{5795.72MJ}{35.9MJ/L} =\sim 161Liter$$

(b): Repeat the calculation using the new values:

$$P_{TR} = 0.5\rho A_F C_D V^3 + mgV\,C_o = 101{,}370W = 101.37kW$$

$$T = \frac{400km}{104kmh} = 3.85h$$

$$E = (101.37)(3.85) = 389.9kWh = 1403.63MJ$$

$$E_{in} = \frac{1403.63MJ}{0.26} = 5398.59MJ$$

$$Fuel = \frac{5398.59MJ}{35.9MJ/L} =\sim 150Liter$$

Thus, the savings are 161 – 150 = 11 L, and since each liter generates approximately 2.6 kg CO_2 the savings amount to 28.9 kgCO_2.

Question 3. A container ship company operates a service on a route that is 22,300-nmi long, with a schedule whereby their ship makes a round trip in 7 weeks. For simplicity, you can ignore time spent in port, the Panama Canal, etc., and just assume that the ship cruises continuously. Non-fuel ship operating cost is $145,000 per day.

The fuel consumption in metric tons of fuel per day as a function of speed *V* in knots has the following form:

$$B = 0.519V^2$$

Fuel costs $710 per metric ton.

In addition to fuel and non-fuel cost, the container ship operator must pay a carbon tax of $50 per metric ton of CO_2. Ship fuel is approximately 85% carbon by weight. Ignore all other costs besides the three given costs.

The ship carries 25,000 TEUs and for the purposes of the problem you can assume it operates at full capacity. Also assume that the shipping line divides the total cost evenly among all the containers, and for simplicity exactly covers this cost by charges earned from the containers (i.e., no profit margin). The price charged to the shipper of the container is based on the total cost per day multiplied by the entire 7-week duration of the voyage.

A shipper of products moves a product worth $80,000 per 40-ft container (= 2 TEUs), and ships one container per week. Cost of inventory is 0.33% per week. The container takes 31 days to travel from origin to destination including time to prepare for shipping, time in motion during the line haul, and time at the destination end.

Part (a): What is the total logistics cost per container?

Solution.

First calculate the speed, fuel consumption, and fuel cost:

$$V = \frac{D}{Time} = \frac{22{,}300}{7 \times 168} = 18.96kn$$
$$U = 0.519V^2 = 0.519(18.96)^2 = 186.6 tons/day$$
$$B = C_B U = (710)(186.6) = \$132{,}465$$

The carbon tax cost is based on the fuel consumption. 85% of 186.6 tons is 158.9 tons per day. Using the ratio of 44:12 CO_2 to carbon and charging $50 per ton gives:

$$158.9\left(\frac{44}{12}\right)(\$50) = \$29{,}074$$

Since the additional cost is $145,000 per day, the combination of the three components gives a total cost of $306,539 per day.

The round trip takes 49 days, so the total cost to be covered by a full load of containers and cost per TEU is:

$$\$306{,}539 \times 49 = \$15{,}020{,}428$$
$$CostPerTEU = \frac{\$15{,}020{,}428}{25000} =\sim \$601$$

Lastly, we can plug the necessary values into the equation for total logistics cost on a weekly basis, with inventory cost of 0.33% per week, price of $80,000, flow value of one container per week, shipment size of one container, shipping time of 31 days or 4.43 weeks, and freight cost for one 40-in container equal to two TEUs = $1202:

$$TLC = rPV + rPTQ + FQ/V$$
$$= (0.0033)(80000)(1) + (0.0033)(80000)(4.43)(1) + (1202)(1)/1 = \$2635$$

(b): Short answer: Suppose the carbon tax is doubled to $100 per metric ton. All other things are equal, what is the likely impact on the operations of the container shipping line? Explain in one to two sentences. This is an open-ended question, there is more than one correct response.

Solution.

At least two responses are possible: on the one hand, the shipping company might try to slow their ships down, so that total operating cost would remain the same. However, if they felt they would lose business, they might keep operations the same and absorb the higher operating cost, if they felt that shippers did not want to see their total operating cost increase.

Question 4. A railroad company incurs a fixed cost of $200 to move boxcars over a given origin-destination pair and then a variable cost *VC* as a function of number of boxcars *X* of the following form:

$$VC(X) = 0.002X^3 - 0.34X^2 + 26X$$

Derive the equation for the marginal cost *MC* and for the average cost *AC*. Then calculate the number of boxcars *X** that optimizes the total cost incurred, either exactly or approximately, by any appropriate calculation. Hint: If you are not able to calculate an exact solution in closed form, you can still get full credit for this problem by deriving an approximate value of *X** through iteration.

Solution.

Approach is to solve for *TC, MC, AC,* and then find *X** by setting *MC = AC*:

$$TC = FC + VC = 0.002X^3 - 0.34X^2 + 26X + 200$$

$$MC = \frac{dTC}{dX} = 0.006X^2 - 0.68X + 26$$

$$AC = \frac{TC}{X} = 0.002X^2 - 0.34X + 26 + \frac{200}{X}$$

$$MC = AC$$

$$0.006X^2 - 0.68X + 26 = 0.002X^2 - 0.34X + 26 + \frac{200}{X}$$

$$0.004X^2 - 0.34X - \frac{200}{X} = 0$$

$$X^* \cong 91$$

$$0.004(X^*)^2 - 0.34X^* - \frac{200}{X*} = 0.004(91)^2 - 0.34(91) - \frac{200}{91} = -0.0138 \cong 0$$

Note that the above problem was solved with an electronic solver (in this case “GoalSeek” from Excel). In the exam, you could either have your calculator solve the problem, or program the equation into your calculator and perform trial and error until finding the closest answer.Alternatively, full credit given for trying values of 80, 90, and 100, and finding that 90 is closest to *MC – AC* = 0, thus *X** ~=90.

Question 5. An intermodal rail service provides container shipment service with a maximum capacity of 25,000 kg per container for $400 fixed cost plus $0.60 per km. A product is to be shipped 2000 km, requiring 2 days shipping time. The product is worth $8.50 per kg and demand is 350 metric tonnes per year. The cost of holding inventory is 18% per year. Each shipment of one container consumes 25 GJ from origin to destination (OD), regardless of how much product is in the container.

a. [10 points] How full should the containers be filled to minimize total logistics cost (TLC), and what is the total cost of the product per kg at the destination including product value and TLC? Use the TLC model in which holding cost is incurred at the origin, but not at the destination.

b. [5 points] Suppose the shipper of the product decides to increase the shipment time to 4 days to save energy and be greener. The new OD energy consumption is 15 GJ per container, again regardless of how full the container is. How much energy is saved per kg of product, and what is the increase in TLC per kg? Give value of increase in TLC to three significant digits.

Solution**.**

Use the formula for economic order quantity to calculate *V**. Converting demand gives 350,000 kg/year. The total cost per shipment is *F* = 400 + 2000(0.60) = $1600. The product PR = $8.50(0.18) = $1.53.

$$V^* = \sqrt{\frac{2FQ}{PR}} = \sqrt{\frac{2(1600)(350000)}{1.53}} = 27{,}056 kg / shipment$$

Since *V** is greater than the maximum capacity V_{max} = 25,000 kg, we use V_{max} as the amount to fill the container and calculate TLC on this basis. Note that the transit time is calculated in years, or 2 days/365 = 0.00548 years.

$$TLC(V) = \frac{PRV}{2FQ} + PRT + \frac{F}{V} = \frac{1.53(25000)}{2(1600)(350000)} + 1.53(0.00548) + \frac{1600}{25000} = \$0.127 / kg$$

The total cost of sale including logistics cost is then $8.50 + $0.127 = $8.627/kg.

(b): Note that for this part of the problem you need to know that the change in shipment time will not affect the chosen value of *V*. Savings are 10 GJ, therefore the amount of energy saved per kg of product is:

$$\frac{10GJ}{25000kg} = 0.0004\frac{GJ}{kg} = 400\frac{kJ}{kg}$$

The increase in logistics cost per kg equals the cost associated with 2 days or 0.00548 years in transit:

$$C = PRT = 1.53(0.00548) = \$0.00838 / kg$$

Question 6. A food processor manufactures a product at two plants (1 and 2) for three markets (A, B, C). The distances from the sources to the markets in kilometers are the following:

	A	B	C
1	1400	5300	5600
2	3800	1700	2300

Suppose 1 and 2 have a capacity of 2600 and 2400 tonnes (i.e., metric tons) per year, respectively, and the demand at 1 to 3 is 2300, 1200, and 1300 tonnes, respectively. Shipments are made 35% by rail and 65% by truck, with an energy intensity of 650 and 2250 kJ/tonne-km, respectively.

a. What is the average energy consumption in Btu per tonne-km if the 35/65 modal split holds for all possible shipments in the network?

Solution.

The energy intensity is a weighted average of the two modes:

$$\mu_{tkm} = (0.35)(650) + (0.65)(2250) = 1{,}690 kJ/tkm$$

b. Using the transportation problem in optimization as a starting point, write out the objective function and five constraint equations (not including the non-negativity constraint) specific to the given data that are necessary to form the transportation problem for finding the minimum tonne-km/minimum energy consumption origin-destination (OD) flow pattern. Hint: There is more than one way to express the five constraint equations, but your answer should clearly define the bounds on the solution space for the problem.

Solution.

Different answers are possible, but the following is representative:

$$MinZ = \sum_x 1400x_{1A} + 5300x_{1B} + 5600x_{1C} + 3800x_{2A} + 1700x_{2B} + 2300x_{2C}$$

$$SubjectTo$$

$$\sum x_{1A} + x_{1B} + x_{1C} \leq 2600$$

$$\sum x_{2A} + x_{2B} + x_{2C} \leq 2400$$

$$\sum x_{1A} + x_{2A} \geq 2300$$

$$\sum x_{1B} + x_{2B} \geq 1200$$

$$\sum x_{1C} + x_{2C} \geq 1300$$

c. Since you do not have access to Excel or Matlab describe in one or two sentences or two to three numbered steps a “heuristic” of your own design to find the optimal shipping pattern (different correct answers are possible).

Solution.

Different heuristics are possible, but here is an example:

a. Work through the destinations in order *a*, *b*, *c*, and if demand can be met from the nearest source, allocate that amount from the source to the destination.
b. If there is supply left at the affected destination, it is available for allocation to another destination. If the supply cannot be met, move on to the next destination and meet the unmet supply in the second pass.
c. Once the first pass is complete, allocate any remaining supply to meet any remaining demand in the way that minimizes total distance.

d. Find the allocation of product from sources to markets (optimal OD pattern) that minimizes total energy consumption, and show that the constraints are satisfied. Then calculate the value of total energy consumption in gigajoules [GJ] for that pattern, assuming a 35/65 rail/truck split for all shipments.

Solution**.**

Optimal pattern is shown in the table below.

Note to instructor: Since different heuristics may lead to slightly higher amounts of tonne-km, you may wish to award full credit to slightly different solutions as long as the heuristic was explained clearly, or else give most of the points for a "good" answer, but all points for the "optimal" answer.

	a	b	c	Output	Check:
1	2300	0	100	2400	2600
2	0	1200	1200	2400	2400
Shipped	2300	1200	1300		
Check	2300	1200	1300		

Resulting table of tonne-km (multiplying tonnes by distance):

In units of 1000 tonne-km:

	A	B	C
1	3220	0	560
2	0	2040	2760

Summing all volume shown in the table give a total tkm of 8.58 million, and total energy is (8.58M tonne-km)(1690kJ/tkm) = 14.5 billion kJ = 14.5 TJ.

Constraints:

Demand met at A, B, C as shown:

Plant 1: 2400 out of 2600 tonnes allocated
Plant 2: 2400 out of 2400 tonnes allocated

Question 7. A truck provides truckload service with maximum capacity of 21,000 kg for $150 fixed cost plus $0.95 per km. A product is to be shipped 1500 km; the shipping time is 3 days.

The product is worth $5.50 per kg and demand at the destination is 145 metric tonnes per year. The cost of holding inventory is 21% per year. Assume the total logistics cost (TLC) model in this case where holding inventory cost is incurred at the origin but not at the destination.

The truck consumes 23.5 MJ of energy per kilometer driven, regardless of how full it is loaded (for simplicity, we are ignoring the extra energy that a heavier truck would consume in a more accurate analysis).

A) What is the total logistics cost for the EOQ-optimal shipment size and for the full-truck shipment size (i.e., filling the truck to capacity each time) in $/kg, measured to three significant digits? (Note: two answers required.)
B) What is the energy intensity measured in kJ/tonne-km for the EOQ-optimal and full-truck options? (Note: two answers required.)

Solution.

(a): Approach—calculate V^*, then compare total logistics cost for V^* and V_{max}.

$$\text{V* case:}$$

$$F = FC + Dist \cdot VC = 150 + 0.95 * 1500 = \$1575 / shipment$$

$$PR = 1.115$$

$$Q = 145{,}000 kg / year$$

$$V^* = \sqrt{\frac{2FQ}{PR}} = \sqrt{\frac{2(1575)145000}{(5.50)0.21}} = 19886 kg / shipment$$

$$\text{Inventory cost in transit:}$$

$$PRT = (1.115)\left(\frac{3}{365}\right) = \$0.00916 / kg$$

$$C = \frac{PRV}{2Q} + PRT + \frac{F}{V}$$

$$= \frac{1.115 \cdot 19886}{2(145000)} + 0.00916 + \frac{1575}{19886}$$

$$= \$0.0765 + \$0.00916 + \$0.0792$$

$$= \$0.1648 / kg$$

Now in case each truck is loaded to the maximum 21000 kg/trip:

V_{max} case:

$$V = V_{max}$$

$$C = \frac{PRV}{2Q} + PRT + \frac{F}{V}$$

$$= \frac{1.16 \cdot 21000}{2(145000)} + 0.00916 + \frac{1575}{21000}$$

$$= 0.08074 + 0.00916 + 0.075$$

$$= \$0.1649 / kg$$

<u>Answers:</u> \$0.1648 and \$0.1649 per kg, respectively.

Note to instructor: It is advisable to allow some variation in answers since, depending on how many digits students carry in their calculations, answers may differ slightly.

(b): Energy intensity—calculate tonne-km generated, then divide energy by tonne-km to get kJ/tkm.

Total energy consumed: 23.5 MJ × 1500 km = 35,250 MJ

For V* case:

$$19886kg \frac{1tonne}{1000kg} \cdot 1500km = 29{,}829tkm$$

$$\text{Intensity:} \frac{35250MJ}{29829tkm} = 1.182 \frac{MJ}{tkm} = 1182 \frac{kJ}{tkm}$$

For V_{max} case:

$$21000kg \frac{1tonne}{1000kg} \cdot 1500km = 31{,}500tkm$$

$$\text{Intensity:} \frac{35250MJ}{31500tkm} = 1.119 \frac{MJ}{tkm} = 1119 \frac{kJ}{tkm}$$

Part 4 Alternative Transportation Energy Problems

Question 1. The operators of the Eurostar train from London to Paris wish to show that their service has lower CO_2 per passenger than a comparable short-haul jet between the two cities. The electricity mix that the Eurostar uses is from a utility grid with the following mix:

Source	Output	CO_2
	[Billion kWh]	[Million tonnes]
Nuclear	30.00	0.000
Hydro	20.00	0.000
Gas	25.00	9.075
Wind	12.00	0.000

The rate of electricity purchase requirement for the train *P* in kW as a function of speed *V* in km/h is approximated by the following function:

$$P = (1.57)V + (0.0160)V^2 + (2.07x10^{-4})V^3$$

Hint: since the equation for *P* gives the amount of electricity purchased, you can ignore all losses between the grid and the train, and between the electrical intake of the train and power delivered to the wheels.

The travel distance for both the train and the airplane is 343 km. You can assume for simplicity that the train travels the entire distance at a cruising speed of *V* = 280 km/h, and that it has room for 460 passengers.

The comparable jet between the two cities is a Boeing 737 with room for 149 passengers. It consumes 540 L of jet fuel in the first 40 km to climb to a cruising altitude of 35,000 ft, consumes 442 L to cruise at this altitude for 200 km, and uses negligible fuel to make the remaining descent into the arrival airport. Each liter of jet fuel releases 2.58 $kgCO_2$ when combusted (ignore well-to-tank CO_2 emissions).

Question: if the occupancy of the train is 65% and of the plane is 75%, what is the percent reduction in CO_2 per passenger per trip for the Eurostar compared to the jet?

Solution:

First, we determine the total power needed per hour for the train

$$P = (1.57) * 280km/h + (0.0160)(280km/h)^2 + (2.07x10^{-4})(280km/h)^3 = \frac{6,238kW}{hour}$$

Next, we determine the total energy input required for the train to travel 343km:

$$=\frac{343km}{280km/h}=1.23h*\frac{6238kw}{h}=7642kw$$

Next, we determine the total emissions for the train, using the information in Table 1. First, summing the output power provided in the table we get the following:

$$30billionkWh+20billionkWh+25billionkWh+12billionkWh=87billionkWh$$

Next, we determine the percent energy used and multiply by the total emissions provided in Table 1 and convert from tonnes to $kgCO_2$:

$$=\frac{7,642kW}{87x10^{9}kWh}=8.78x10^{-8}*9.08x10^{6}tonnesCO_2*\frac{1000kg}{tonne}=797kgCO_2$$

Next, we convert these emissions to $kgCO_2$ per passenger for the train:

$$=\frac{797kgCO_2}{\left(\frac{65}{100}\right)\times 460passengers}=\frac{2.67kgCO_2}{passenger}$$

Plane:

For the plane, we first determine the total fuel consumption and multiply by the emissions per liter of jet fuel, as given above:

$$=(540L+442L)\times\frac{2.58kgCO_2}{L}=2534kgCO_2$$

Next, we divide by expected passenger occupancy, as given above to determine total emissions per passenger:

$$=\frac{2534kgCO_2}{\left(\frac{75}{100}\right)*149passengers}=\frac{22.67kgCO_2}{passenger}$$

Finally, we calculate the reduction in emissions provided by the plane per passenger:

$$=1-\left(\frac{2.67kgCO_2}{passenger}\Big/\frac{22.67kgCO_2}{passenger}\right)\times 100\%=88.24\%$$

Summary: Based on the given power equation, power requirement for the train is 6238 kW. Electricity emissions are 0.1043 $kgCO_2$/kWh. The train travels for 1.225 hours to cover a distance of 343km, requiring 7642 kWh. Total emissions are 797 $kgCO_2$, or 2.67 $kgCO_2$/passenger for the train. The jet consumes 982 L of fuel total, leading to 2534 kg of CO_2 emissions, or 22.67 $kgCO_2$/passenger. Thus the reduction in CO_2 emissions provided by the train is 88%.

Question 2. In the U.S. in a recent year, suppose 13.2 million acres were planted in corn for use in ethanol production, resulting in 1.9 billion bushels of corn. This amount of corn required 89 trillion Btu to harvest, including all aspects: fertilizer, fuel inputs, embodied energy in equipment, etc. Each bushel results in 41 lb of corn kernels for use in ethanol production.

Processing the volume of corn thus produced into ethanol consumed 164 trillion Btu, including all aspects: embodied energy, transportation, distillation, and all other energy uses. Each gallon of ethanol requires 22.4 lb of corn kernels, and ethanol has an energy content of 75,700 Btu/gal.

a. Calculate the net energy benefit (a.k.a. NEB) achieved by this process.

Solution.

For agricultural purposes, 89 tril.Btu/1.9 bil.bushel = 47,800 Btu required per bushel of corn. Energy requirement per gallon:

$$\left(\frac{47{,}800\,Btu}{bushel}\right)\left(\frac{1\,bushel}{41\,lb.corn}\right)\left(\frac{22.4\,lbcorn}{1\,gal}\right) = 26{,}100\,Btu/gal$$

This translates into 26,100 Btu per gallon.

Production of 1 gal of ethanol is calculated based on total production and energy consumption, taking into account that 1 bushel of corn produces 1.83 gal of ethanol:

$$E_{Pdtion} = \frac{TotalEnergy}{TotalOutput} = \frac{164\,Tril.Btu}{1.9\,bil.bushel} = 86{,}300\,Btu/bushel$$

$$E_{Gallon} = \frac{86{,}300\,Btu/bushel}{1.83\,Gal/bushel} = 47{,}200\,Btu/gal$$

Therefore, the total energy per gallon including both agricultural and processing inputs is 26,100 + 47,200 = 73,300 Btu/gal. Thus the NEB ratio is

$$\frac{75{,}700}{73{,}300} = 1.03$$

Therefore NEB = NEBRatio – 1 = 0.03.

b. Suppose the acreage of corn planted for conversion into ethanol represents 15% of the total acreage planted in corn. If instead 100% of the corn acreage were converted to ethanol, the amount of gasoline consumed in that year in the U.S. is 126 billion gallons, and the energy content of gasoline is 115,400 Btu/gal, what percent of the total energy consumed in gasoline could be displaced by corn ethanol?

Solution.

Ethanol production is calculated in proportion to the ratio of existing to future production of corn for ethanol conversion:

$$(1.9 bil.bushel)(1.83 gal/bushel)\left(\frac{100\%}{15\%}\right) = 23.2 bil.gal$$

The percentage displaced is then calculated based on energy content:

$$(23.2 bil.gal)(75{,}700 Btu/gal) = 1.76 Quad.Btu$$
$$(126 bil.gal)(115{,}400 Btu/gal) = 14.54 Quad.Btu$$
$$Pct = \frac{1.76}{14.54} = 12.1\%$$

Therefore, ethanol is 1.76 Quads, gasoline is 14.54 Quads, so percentage is 12.1%.

Question 3.

A new light rail (LRT) system is introduced which uses electricity from the following mix (note: 1 (metric) tonne = 1000 kg):

Source:	Output	CO_2
	[Billion kWh]	[Million tonnes]
Nuclear	30.00	0.000
Hydro	20.00	0.000
Gas	45.00	15.075
Coal	50.00	42.500
Biofuel	10.00	0.000
Wind	12.00	0.000

The system runs once every 15 minutes from 6 AM to 11 PM, thus four times per hour for 17 hours. As a simplification, you can assume vehicles run continuously during scheduled hours and ignore access from/to the LRT storage depot and other scheduling details. The line carries a total of 15,000 riders in to the city per day (the same 15,000 riders then return via LRT to their point of origin). The line runs for 20 mi in each direction, and an LRT vehicle consumes 2 kWh of electricity per mile.

Among the riders of the new service, 70% are previous car commuters who traveled in single occupant vehicles (SOVs) an average of 28 mi round trip to work. Average commuting fuel economy is 16 mpg gasoline, and gasoline contains 8.8 $kgCO_2$. Ten percent are commuters who previously bicycled to work, with negligible CO_2 emissions. The rest used previously existing public transportation, which can be assumed to have the same CO_2 emissions rate as the LRT system.

For the purposes of calculating effective CO_2 emissions, you can ignore transmission and distribution losses for the LRT, and well-to-tank (WTT) losses for private cars (i.e., consider tailpipe CO_2 emissions only).

Answer both parts:

a. What are the average CO_2 emissions per passenger round trip in the LRT, in $kgCO_2$?

b. How many tonnes of CO_2 emissions are saved per day by the new LRT system compared to the situation before it was introduced?

<u>Solution.</u>

Part (a): Average CO_2 emissions

$$CO_2 / kWh = \frac{\sum TotalCO2_i}{\sum TotalkWh_i} = \frac{57.6 Bil.kg}{167 Bil.kWh} = 0.345 kgCO_2 / kWh$$

Total electricity use per day:

$$4\frac{trips}{h}\cdot 17\frac{h}{day}\cdot 40\frac{mi}{roundtrip}\left(2\frac{kWh}{mi}\right) = 5440 kWh/day$$

Emissions per round trip:

$$5440\frac{kWh}{day}\cdot 0.345\frac{kgCO_2}{kWh} = 1875 kgCO_2$$

$$1875 kgCO_2 \cdot \frac{1}{15000 roundtrips/day} = 0.125 kgCO_2 / RT$$

Part (b): Emissions reduction:

Calculate emissions in previous case, then subtract new emissions to find improvement

$$\text{Car}: 0.7\cdot 15000\frac{pers}{day}\cdot 28\frac{mi}{day}\cdot\frac{1}{16 mpg}\cdot 8.8\frac{kgCO_2}{gal} = 162{,}000 kgCO_2$$

$$\text{Transit}: 0.2\cdot 15000\frac{pers}{day}\cdot 0.125\frac{kgCO_2}{roundtrip} = 375\frac{kgCO_2}{day}$$

Total emissions before LRT:

$$162{,}000 + 375 = 162{,}375 kgCO_2$$

Net savings:

$$162{,}375 - 1875 = 160{,}500 kgCO_2$$

Question 4. Consider a two-car family with an old SUV and a new Hybrid which are both driven the same number of miles each week.

SUV mpg: 9

Hybrid mpg: 45

The family wants to improve fuel economy, so it can either (a) convert the Hybrid to a PHEV, or (b) take the SUV in for a tune up. By converting the Hybrid to PHEV it doubles its fuel efficiency to 90 mpg (for the purposes of this problem, ignore the resulting increase in home electric bill), or by checking the tire pressure, changing the air filter in the engine, and cleaning the fuel injectors the family could improve the SUV fuel mileage by 2 mpg to 11 mpg.

a. [15 points] If the goal is to maximize overall mpg, which should the family choose, (a) or (b)? Show calculations that support your choice.

b. [5 points] Short answer: would a non-technical layperson expect the result from part (a), or would they be surprised by it? Also, how would you explain to them the outcome in simple terms? One or two paragraphs maximum.

Solution.

Part (a): Approach—assume a specific number of miles per week and calculate effect of changes (a) or(b). Assume: 100 mi per week per vehicle.

Base case:

$$(100mi)\left(\frac{1}{9mpg}\right)+(100mi)\left(\frac{1}{45mpg}\right)=11.1+2.2=13.3gal/week$$

Case (a):

$$(100mi)\left(\frac{1}{9mpg}\right)+(100mi)\left(\frac{1}{90mpg}\right)=11.1+1.1=12.2gal/week$$

Average efficiency:

$$200mi\left(\frac{1}{12.2gal}\right)\frac{1}{2cars}=\frac{16.4}{2cars}=8.2mpgPerCar$$

Case (b):

$$(100mi)\left(\frac{1}{11mpg}\right)+(100mi)\left(\frac{1}{45mpg}\right)=9.1+2.2=11.3gal/week$$

Average efficiency:

$$200 miles \left(\frac{1}{11.3 gal} \right) \frac{1}{2 cars} = \frac{17.7}{2 cars} = 8.8 mpgPerCar$$

Answer: Case (b) is more effective.

Part (b):Answer should contain the following elements: (a) the answer is likely to be surprising to the layperson, who assumes that a PHEV is much more effective than something mundane like maintaining an SUV, (b) the answer can be explained because the Hybrid is already so efficient, it is difficult to wring any more fuel reduction out of it.

Question 5. Researcher Andreas Schaefer and his colleagues from MIT argue in a paper published in the *American Scientist* that a reasonable target for energy efficiency of passenger transportation is 1.5 MJ/pkm. A popular U.S. gasoline-powered mid-size Sedan has a curb weight of 1616 kg, cross-sectional area of 2.7 m^2, drag coefficient of 0.33, and coefficient of rolling resistance of 0.01. Suppose the vehicle is cruising on a long-distance trip on level ground for 3 hours at an average speed of 104 km/h (equivalent to the typical U.S. speed limit of 65 mi/h) through air with a density of 1.15 kg/m^3. There are two passengers in the car.

Part (a): What must the energy efficiency of the drivetrain be (i.e., percent of power available in the gasoline converted to tractive power) for the passengers in this trip to achieve the target of 1.5 MJ/pkm?

Solution:

First convert the speed to meters per second, i.e.,

$$104 km/h \left(\frac{1 kmh}{3.6 m/s} \right) = 28.9 m/s$$

Then solve for tractive power using given information:

$$\begin{aligned} P_{TR} &= 0.5 \rho A C_D V^3 + mgVC_0 \\ &= 0.5(1.15)(2.7)(0.33)(28.9)^3 + (1616)(9.8)(28.9)(0.01) \\ &= 16{,}927W = 16.93kW \end{aligned}$$

The efficiency is solved by comparing the input 1.5 MJ/pkm to output energy based on tractive effort. The problem can be solved on the basis of instantaneous power, or energy in 1 hour, or as in this case energy in 3 hours. The energy required in 3 hours of travel is:

$$E_{out} = Pt = (16.93kW)(3h) \left(\frac{3.6MJ}{kWh} \right) = 182.8MJ$$

The total energy available in 3 hours takes into account the energy per pkm, the number of passengers, and the distance traveled (104 km × 3 hours = 312 km):

$$1.5MJ / pkm(2 pers)(312km) = 936MJ$$

The efficiency is then

$$\eta_{drivetrain} = \frac{E_{out}}{E_{in}} = \frac{182.8MJ}{936MJ} = 19.5\%$$

Part (b): California and U.S. government agencies are phasing in an emissions limit of 250 gCO_2 per mile, equivalent to 156 g CO_2 per kilometer. Suppose the drivetrain efficiency is 21% (and not the answer you calculated in part (a):. Is the vehicle within this limit during this trip? Show all calculations.

Solution.

In this instance, the solution is obtained on a 1-hour basis. The energy output in part (a) in 3 hours is 182.8 MJ, so in 1 hour it is 60.9 MJ. From the list of constants and equations we get that 1 L of fuel contains 32.17MJ and emits 2.35 kgCO_2 when combusted. Calculating gives:

$$60.9MJ\left(\frac{1}{0.21}\right)\left(\frac{1L}{32.17MJ}\right)\left(\frac{2.35kgCO_2}{1L}\right)\left(\frac{1}{104km}\right) = 204.1gCO_2 / km$$

Thus the vehicle does not comply with the new standard in this situation.

Part (c). In a sentence, is the calculation in part (b) a legitimate way to evaluate whether the vehicle meets the CO_2 standard in general? Why or why not?

Solution.

No, it is not legitimate, because you must look at a complete range of possible driving speeds to evaluate the vehicle, and not just highway travel at the speed limit.

Question 6. *Note: This question could also be used within part 3 on freight transportation.*

An electric freight railroad is to be constructed that costs $2.2 billion for all capital equipment, including fixed infrastructure, locomotives, and cars. This cost is to be discounted at 8% for 20 years, with no salvage value at the end of the project lifetime. The system also consumes 550 million kWh of electricity per year, at a cost of $0.08 per kWh. Lastly, the system incurs $110 million per year in non-energy operating costs. Because of its ecological benefits, it also receives $50 million from the government per year in operating support.

Part (a): If the system charges $300 per shipment, how many shipments must it transport per year in order to exactly cover net expenditures after receiving the subsidy? You can round your answer to three significant digits.

Solution:

Calculate discounting factor and annualized capital cost:

$$A/P = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{.08(1+.08)^{20}}{(1+.08)^{20} - 1} = 0.1019$$

$$ACC = (0.1019)\$2.2B = \$224.1M$$

Next calculate energy cost, total cost per year, and net cost after deducting subsidy:

$$C_{en} = 550M \times \$0.08 = \$44M$$
$$C_{tot} = ACC + C_{en} + C_{op} = \$224.1M + \$44M + \$110M = \$378.1M$$
$$C_{net} = C_{tot} - Subsidy = \$378.1M - \$50M = \$328.1M$$

The number of shipments needed @ $300/shipment is:

$$N = \frac{\$328.1M}{\$300} =\sim 1.09M$$

Now suppose the electricity used to transport the freight is generated from the following mix of sources, with total annual output in million MWh and CO_2 emissions in million tonnes shown:

Source	MWh	CO_2 Emits
	Million	Million tonnes
Coal	37	36.63
Gas	22	7.524
Nuclear	15	0
Renew	12	0

The shipments moved by the system would otherwise be moved by truck with an emissions rate of 625 kg CO_2 per shipment. The government would like to know the cost-effectiveness of their $50 million per year investment in the system, in terms of reducing CO_2 emissions. Ignore transmission and distribution losses from the electricity sources to the railroad system.

Part (b): [10 points] What is the cost per tonne of CO_2 reduced, i.e., in $/tonne CO_2?

<u>Solution:</u>

Calculate average emissions rate by dividing total emissions by total output:

$$\mu_{CO2} = \frac{36.63 + 7.524}{37 + 22 + 15 + 12} = \frac{44.154}{86} = 513.4 kgCO_2 / MWh$$

The total emissions due to the system and emissions per shipment are therefore:

$$Emit_{tot} = 550{,}000 MWh \times 513.4 kg / MWh = 2.824E + 08 kg / year$$
$$Emit_{shipment} = 625 kg \times 1.094M = 6.83E + 08 kg / year$$

Because a shipment by truck emits 625 kg, cost per tonne reduced is then:

$$\frac{\$50M}{(683-282)\times 10\text{^}3 tonnes} = \$124.7 / tonneCO_2$$

Question 7. Cellulosic ethanol is produced by first growing a cellulosic crop and then processing it into ethanol, which has approximately 75,700 Btu of energy content per gallon.

Each acre of cellulosic crop for making the ethanol produces approximately 8 tons of harvest. Producing 1 gal of ethanol requires 10.7 lb of the crop. The energy inputs per acre in million Btu are the following:

Input	Energy
	[million Btu]
Energy input	5.72
Other input	6

On the production side, the inputs include energy consumed in the processing plant, and transportation energy to move the crop to the plant and the finished ethanol to market. The inputs per 1000 gal in million Btu are the following:

Input	Energy
	[million Btu]
Transport input	4.84
Energy input	6.45
Other input	2.12

Calculate the net energy balance ratio, also known as the NEB ratio, as well as the NEB. Give your answer to three significant digits.

<u>Solution:</u>

Yield: 8 tons = 16,000 lb per acre

Input: 10.9 lb/gal ethanol

Because the total energy per acre is 11.72 MMBtu, energy per unit of ethanol is:

$$11.72\frac{MMBtu}{acre}\left(\frac{1acre}{16{,}000lb}\right)\left(10.7\frac{lb}{gal}\right)=7837\frac{Btu}{gal}$$

The processing total is 13.41 MMBtu for 1000 gal, or 13,410 Btu/gal

Energy content of 1 gal ethanol: 75,700 Btu. Calculate NEB ratio:

$$NEB\_Ratio=\frac{EnergyOut}{EnergyIn}=\frac{75{,}700}{(7837+13410)}=\frac{75{,}700}{21247}=3.56$$

Calculate NEB (*slightly different from NEB ratio*):

$$NEB=\frac{EnergyOut-EnergyIn}{EnergyIn}=\frac{75700-21247}{21247}=3.57-1=NEB.Ratio-1=2.56$$

Question 8. A plug-in hybrid electric vehicle (PHEV) runs using a mixture of electricity and gasoline. The electricity is provided from the local grid at the following mix and rate of CO_2 emissions. Non-fossil production constitutes a mixture of renewables and nuclear which do not have any CO_2 emissions from the power plant.

Source	Production	Emissions
	bil.kWh	$lbCO_2/kWh$
Coal	95	1.8
Gas	112	0.71
Non-fossil	80	0

The PHEV can run for 30 mi on a charge, during which time it runs exclusively on electricity, thereafter it runs on gasoline. Fuel economy is 3.4 mi/kWh and 24 mi/gal.

For electricity transmitted to the vehicle, the grid transmission and distribution is on average 90% efficient. The well-to-tank efficiency of the petroleum production system from crude oil to gasoline in the tank is 88%.

The PHEV drives in the following pattern over a 5-day period:

Day	Miles
1	21
2	15
3	38
4	252
5	27

What are the total pounds of CO_2 emissions for the 5 days, including both tailpipe and upstream emissions?

Solution:

The approach here is to first divide the distance driven into EV and ICEV miles, and then calculate emissions from each.

Total miles driven on days 3 and 4 beyond the 30-mi EV range are 230 mi. The total distance driven over 5 days is 353 mi, so the EV distance is the remainder, or 123 mi.

Total emissions on the ICEV side are:

$$230mi\left(\frac{1}{24mpg}\right)\left(20\frac{lbCO_2}{gal}\right)\left(\frac{1}{0.88WTTFactor}\right) = 217.8lbCO_2$$

For the EV side, we first need to calculate average emissions per kWh. Total electricity produced is 95 + 112 + 80 = 287 bil.kWh. Calculating emissions from gas and coal and dividing by total electricity production gives:

$$112(0.71) = 79.52bil.lbCO_2 \text{ from gas}$$
$$95(1.8) = 171bil.lbCO_2 \text{ from coal}$$
$$\frac{79.52B + 171B}{287BkWh} = 0.873lbCO_2 / kWh$$

On this basis life cycle emissions on the EV side are:

$$123mi\left(\frac{1}{3.4mi / kWh}\right)\left(0.873\frac{lbCO_2}{kWh}\right)\left(\frac{1}{0.9WTTFactor}\right) = 35.1lbCO_2$$

Total emissions are the sum or 217.8 + 35.1 = 252.9 $lbCO_2$.

Part 5 Essay Questions

Question 1. Short essay, to be written in exam booklet, not in this exam question book. At a number of times during the course, we have discussed the importance of the relationship between transportation systems and economics. General examples include the following: (1) The cost and quality of transportation (sometimes affected by lack of investment) affect travel choices. (2) As a countermeasure to #1, sometimes economic investments in infrastructure, vehicle technology, etc., are used to achieve a different set of choices or different outcome. (3) In the transportation market place, both government and private companies try to adjust prices to achieve a desirable outcome (e.g., increased revenue for private companies, socially optimal outcome for government such as reduced congestion or increased use of alternative fuels, and so on). In an essay, identify and discuss three examples of this relationship between transportation and economics. The three examples do not need to be one example from each of my general examples 1 to 3 above; for instance, you could have one of each, or all three examples of #1, or any other combination you choose. Hints: (1) Specific examples of companies, cities, countries, or programs will help your essay. (2) Since the exam is closed book, you may or may not be able to recall specific quantitative values, but to the extent that you can do this correctly from memory or from your cheat sheet, it will generally help your essay. (3) Lastly, manage your time carefully: since the essay is 20% of the overall score on the exam, a good essay might take 25 to 35 minutes to brainstorm, outline, and then write out. In terms of length, the essay usually needs to be three to four sides single-spaced in the exam book, or longer, to have enough content to earn a good score. (4) I recommend jotting down some ideas early in the time, finishing the rest of the exam, then coming back at the end and writing it with the time you have remaining.

Question 2. Short essay, to be written in exam booklet, not in this exam question book. In the very first lecture of the semester, it was stated that "Transportation solutions fall into one of two areas: (1) Improving the dominant mode, or (2) Shifting demand away from the dominant mode." Taking either passenger or freight transportation as an example (but not both!), identify the dominant mode, identify problems caused by the dominant mode, and one or more alternatives for shifting demand away from the dominant mode. Then discuss possible steps and examples for improving the dominant mode or shifting demand. Hints: I am not expecting you to carry out any calculations for this essay. Also, since the exam is closed book, I am also not expecting you to necessarily be able to produce quantitative data for your essay, but if you can write qualitatively about specific examples in the real world, it will improve your score. A strong essay will discuss both solutions 1 and 2 in some detail, rather than focusing exclusively on one or the other.

Question 3. During the course of the semester, we have discussed a number of alternative transportation technologies and systems that might be applied to the problems we face with transportation today. Some examples include alternative fuels for vehicles, improving the efficiency of light-duty vehicles like cars and light trucks, developing high-speed rail, or upgrading our existing infrastructure to increase capacity.

Write an essay answer in the exam booklet in which you (1) define a specific problem that you are trying to solve and (2) evaluate the advantages and disadvantages of *two* specific examples of technologies or systems that could be used to solve this problem. Your essay should include either specific information about the technologies/systems in general or actual projects (complete, under construction, or proposed), that illustrate your perspective. For example, if providing transportation to the top of Mount Everest were your chosen problem, your chosen solutions could be (1) aerial cable cars and (2) all-terrain vehicles, and you might compare their advantages and disadvantages.

There is no right answer: you can favor either one of your chosen alternatives over the other, or you can treat them as being of equal value. Regardless, your grade will be based on the strength of your argument. You do not need to provide any equations or calculations, but specific details will strengthen your essay. As a guideline, a successful essay will likely require four pages single spaced in the exam booklet, or more, and it might take 30 to 40 minutes to compose your ideas and then convert them into the finished essay.

Question 4. Essay question: in the quest for sustainable future transportation technologies and systems, should the government play the leading role, should the private sector play the leading role, or should there be a mixture of both? Provide an essay answer in the exam booklet stating which of these three positions you take and then justifying your position. Your essay should include at least *two* specific examples of either areas of technologies and systems, or geographically situated applications, that support your position. For example, if aerial cable cars were your chosen technology or system, you could discuss either how the government or private sector could support cable cars in general, or you could discuss a specific location, such as a cable car on Mount Everest, to prove your point. There is no right answer: you can choose any one of the three positions, and your grade will be based on the strength of your argument. You do not need to provide any equations or calculations, but specific details will strengthen your essay. As a guideline, a strong essay will likely require four pages single spaced in the exam booklet, or more.

Part 6 Multiple Choice and True/False Questions

Note to instructor: Please use these multiple-choice and true/false questions with caution. These questions are provided on an "as-is" basis, and many rely on context from specific editions of transportation courses without which they cannot be posed on an exam. However, these questions might be used as a starting point for adaptation into questions that could be posed by an instructor for their specific circumstances.

Multiple Choice Questions

1. Which are the largest movers of freight in terms of ton-miles (or ton-kilometers in metric units) in the U.S. and in Europe?

a. Rail in the U.S. and trucks in Europe
b. Trucks in the U.S. and rail in Europe
c. Rail in both the U.S. and Europe
d. Trucks in both the U.S. and Europe

2. Suppose in a small European country in the year 1990 the transportation system moves 30 billion passenger-km with an energy consumption of 54 billion MJ (1 billion MJ = 1 PJ). Suppose also in the year 2000 the system moves 40 billion passenger-km, and the intensity in that year is 1.6 MJ/passenger-km. According to the Divisia analysis methodology, what is the "trended" energy consumption in the year 2000?

a. 64 PJ
b. 48 PJ
c. 72 PJ
d. 60 PJ

3. Which of the following is the primary difference between carsharing and bikesharing systems?

a. Bikesharing depends on clients' access to internet and smart phones, and carsharing doesn't.
b. In bikesharing it is easier for users to make one-way trips because it is easier for staff to reposition bicycles between stations.
c. Bikesharing works in conjunction with other modes and carsharing does not.
d. None of the above are differences between bikesharing and carsharing.

4. A light rail vehicle costs $4.6 million, which is discounted at a rate of 7.5% over 12 years. It incurs an energy cost of $127,000 per year. What is its annual capital cost to the nearest whole dollar?

a. $383,333
b. $594,678
c. $721,678
d. $791,697

5. Which of the following is identified by the report “Paying our Way” as one of the fundamental problems facing the U.S.’s deteriorating infrastructure?

a. Public transit systems in urban areas are beyond their physical capacity.
b. The number of highway lane miles grew by only 4.4% from 1980 to 2006.
c. Access to the system for users is underpriced.
d. None of the above.

6. According to the chapter on the topic of “Transportation Energy Technologies” coauthored by myself along with Lou Albright and Lars Angenent, there are five alternative end-point technologies that might eventually power the transportation system. Of the five, which three do we argue are practical for widespread use in the short- to medium-term?

a. Electricity, hydrogen, and sustainable hydrocarbons
b. Hydrogen, compressed air, and sustainable hydrocarbons
c. Compressed air, electricity, and sustainable hydrocarbons
d. Hydrogen, compressed air, and electricity

7. Which of the following is an example of a successful alternative fuel in widespread use according to the chapter by Dan Sperling and Deb Gordon?

a. Methanol from coal in California
b. Ethanol from corn in the U.S. Midwestern states
c. Use of natural gas for fueling cars in Italy
d. Ethanol from sugarcane in Brazil

8. Which of the following is a valid argument for the development of hydrogen as an alternative fuel?

a. Hydrogen vehicles can potentially achieve driving range between refueling stops that is similar to that of gasoline-powered vehicles.
b. Hydrogen fuel storage systems on board the vehicle are simple to build, unlike electric vehicles, where battery systems are complex and expensive.
c. Hydrogen produced from natural gas for use as a transportation fuel does not lead to a net increase in atmospheric CO_2.
d. None of the above arguments are valid.

9. In general, calculation of the economic order quantity or optimal shipment size from a total logistics cost perspective involves trading off which of the following?

a. Plant/warehouse inventory cost versus in-transit inventory cost
b. In-transit inventory cost versus shipping cost
c. Shipping cost versus plant/warehouse inventory cost
d. A combination of plant/warehouse and in-transit inventory cost on the one hand and shipping cost on the other hand

10. In the flow-density relationship, total flow per hour can be calculated based on density of vehicles on the roadway and which of the following?

a. Time-mean speed
b. **Space-mean speed**
c. Either time-mean or space-mean speed
d. Neither time-mean nor space-mean speed

11. Based on the America 2050 Report, which of the following is *not* an example of one of the 10 anticipated "megaregions" that will take shape in the U.S. during the 21st century?

a. Great Lakes
b. Piedmont Atlantic
c. Arizona Sun Corridor
d. Upper Great Plains—Missouri Basin

12. According to Schafer et al. in the article about the "Other Climate Threat," it is observed about daily travel patterns that:

a. As people grow wealthier, they can afford to spend more time everyday traveling.
b. Poor people are under more financial pressure so they spend more time everyday traveling than wealthy people.
c. Both low-income and high-income people spend roughly the same amount of time everyday traveling.
d. None of the above were observed in the article.

13. Considering specifically the case of the U.S., Schafer et al. note that:

a. Total transportation energy has been increased both by per capita passenger-km per year and by reduced occupancy rates in cars and airplanes as travelers have prioritized comfort and convenience.
b. Primarily, rising per capita passenger-km per year has driven up total transportation energy consumption, somewhat mitigated by increasing average occupancy in cars and aircraft.
c. The primary driver of growing transportation energy consumption has been shifting of passengers from driving cars to traveling in aircraft.
d. The primary driver of growing transportation energy consumption has been U.S. population growth, since passenger-km/person and vehicle occupancy have remained constant.

14. According to the America 2050 report, the most important reason that high-speed rail (HSR) is anticipated to be the most significant transportation mode within emerging U.S. megaregions is that:

a. HSR can reduce energy consumption and CO_2 emissions compared to highway or air modes.
b. HSR is ideally suited to providing economically competitive service in the range of 200 to 500 mi of travel.
c. Airports within megaregions will become so congested that there will be no other choice but to shift to HSR.
d. The construction of HSR systems will be the economic engine that will drive the formation of megaregions.

True-False Questions

1. According to Jose Febrillet from the Port Authority during his guest lecture, the Bayonne Bridge is being renovated and not demolished because of its national historic engineering significance as a famous steel arch bridge.

   True _XXX False ____

2. According to Dan Sperling and Deb Gordon in the book *Two Billion Cars*, Brazil provides a suitable alternative fuels model for the U.S. because the U.S. could use corn as an energy crop for ethanol in the way that Brazil uses sugarcane.

   True ____ False _ XXX

3. The National Surface Transportation Infrastructure Financing Commission (NSTIF) states that although the current fuel tax paid into the Highway Trust Fund is financially inadequate, it at least has the benefit that it is indexed to inflation and therefore increases by a small amount each year.

   True ____ False _ XXX

4. The NSTIF advocates a transition from a fuel purchase–based to mileage-based highway payment system in the future so that prices paid by transportation system users more accurately reflect their impact.

   True _ XXX False ____

5. According to Yossi Sheffi in his book on urban transport networks, user equilibrium in a network is defined as the point at which no traveler in the network can improve her/his travel time by unilaterally changing routes in the network.

   True _ XXX False ____

6. According to the discussion of alternative fuels in the textbook, use of alternative transportation modes during World War II by the U.S. traveling public represents sacrifice of convenience and comfort for another objective, in that case namely supporting the war effort.

   True _ XXX False ____

7. The film *Revenge of the Electric Car* describes Tesla Motors executive Elon Musk as following the maxim from Sung Su's *Art of War* that you should "go where your enemy isn't."

   True ____ False XXX

8. When comparing well-to-tank (WTT) and tank-to-wheel (TTW) efficiency in a well-to-wheel analysis, it is noted that the petroleum-based internal combustion engine life cycle has low WTT and high TTW efficiency, but the electric vehicle life cycle has high WTT and low TTW efficiency.

   True ____ False _ XXX

9. When studying traffic flow, "space-mean speed" is defined as the arithmetic average of the speeds of all vehicles traveling in a length of roadway that is being studied.

   True ____ False _XXX

10. According to the "America 2050 Prospectus," the desired size of a Megaregion of 200 to 500 mi of length lends itself to high-speed rail (HSR) as being the most suitable new mode of transportation to develop for travel within megaregions.

    True_XXX False ____

11. According to Schafer et al. in the article about the "Other Climate Threat," it is observed when comparing the U.S. and Tanzania that citizens spend about the same fraction of their total income on daily travel, but U.S. citizens spend more time traveling per day because they are wealthier.

    True ____ False_ XXX

12. It can be proven mathematically that the speed of the shock wave at the end of a platoon following a slow-moving vehicle is never faster than the speed of the slow-moving vehicle itself.

True_ XXX False _______

13. According to the U.S. Energy Information Administration, the country of Denmark in the year 2011 has a population of 5.53 million, a total energy consumption of 8.261×10^8 GJ, a total CO_2 emissions rate of 46.7 million metric tonnes per year, and a total GDP of $261 billion U.S. dollars. The U.S. value for CO_2 per unit of energy is 53.4 kgCO_2 per GJ, as you calculated in HW1. Based on the numbers given, the value for CO_2 per GJ of the U.S. is better for the environment than that of Denmark. Conversion factors: 1000 kg = 1 metric tonne, 1 billion GJ = 1 EJ.

True_ XXX False _______

14. According to the theory of congestion pricing, the value of the charge is the difference between the average cost equilibrium flow value (call it Q_1), which is the point where average cost AC equals inverse demand; and the marginal cost equilibrium flow value (call it Q_2), which is the point where the cost of adding one more user based on marginal cost MC equals inverse demand.

True ____ False _XXX

15. In public transportation systems, light rail transit is an example of a mode that travels primarily or exclusively on right-of-way (ROW) category A.

True ____ False _ XXX

16. The purpose of the measure of "productive capacity" in public transportation (typically measured in units of space-km/h^2 or space-mi/h^2) is to provide a metric that measures in one number both capacity, which is of interest to the operator, and travel speed, which is of interest to the passenger.

True _ XXX False ____

17. In transportation economics, consumer surplus is a measure of how much consumers save by being able to purchase transportation services at a price lower than what they would have been willing to spend to obtain those services.

True _ XXX False ____

18. In transportation economics, own price elasticity values can sometimes be positive, but cross-elasticity values must always be negative.

True ____ False _XXX